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PŘEDBĚŽNÁ SDĚLENÍ

TWO-STRIP RHEOLOGICAL MODEL FOR SIMPLE ORTHOTROPIC VISCOELASTIC BODIES

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The complexity of the two-dimensional rheological behavior of anisotropic viscoelastic bodies needs, in some cases, a special representation. Therefore, the author has introduced the two-dimensional rheological models, of which the two-strip model on Fig. 1 for orthotropic viscoelastic bodies is the most simple. It consists of the Hookean elastic zone H and the Newtonian viscuous zone N. In the direction of the x-axis, it behaves like a Kelvin solid and in the direction of the y-axis like a Maxwell liquid.

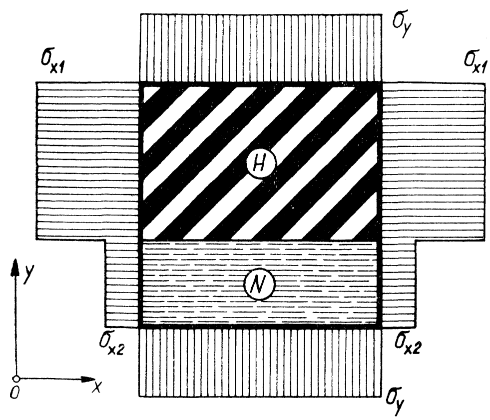

Fig. 1. Two-Strip Rheological Model for a Viscoelastic Body.

The stress σ_x is decomposed into the component σ_{x1} acting in the elastic zone and into σ_{x2} acting in the viscous zone.

The normal strain rates at the plane-stress state are given by

$$(1)\quad \frac{\mathrm{d}\varepsilon_x}{\mathrm{d}t} = \frac{1}{E_x}\left(\frac{\mathrm{d}\sigma_{x1}}{\mathrm{d}t} - \mu_x \frac{\mathrm{d}\sigma_y}{\mathrm{d}t}\right) = \frac{1}{\lambda_x}(\sigma_{x2} - \nu_x\sigma_y),$$

$$(2)\quad \frac{\mathrm{d}\varepsilon_y}{\mathrm{d}t} = \frac{1}{E_y}\left(\frac{\mathrm{d}\sigma_y}{\mathrm{d}t} - \mu_y \frac{\mathrm{d}\sigma_{x1}}{\mathrm{d}t}\right) + \frac{1}{\lambda_y}(\sigma_y - \nu_y\sigma_{x2}),$$

where λ_x, λ_y are the coefficients of normal viscosity,

μ_x, μ_y the elastic Poisson ratios,

ν_x, ν_y the viscuous Poisson ratios.

Introducing $\sigma_{x2} = \sigma_x - \sigma_{x1}$ into (1),

$$\frac{d\sigma_{x1}}{dt} + \frac{E_x}{\lambda_x}\sigma_{x1} = \frac{E_x}{\lambda_x}(\sigma_x - \nu_x\sigma_y) + \mu_x\frac{d\sigma_y}{dt}, \tag{3}$$

and hence

$$\sigma_{x1} = e^{-(E_x t/\lambda_x)}\left\{\int_0^t\left[\frac{E_x}{\lambda_x}(\sigma_x - \nu_x\sigma_y) + \mu_x\frac{d\sigma_y}{d\tau}\right]e^{E_x\tau/\lambda_x}\,d\tau + \sigma_{x10}\right\}. \tag{4}$$

Substituting (4) into (1) and performing the integration, one obtains

$$\varepsilon_x = e^{-(E_x t/\lambda_x)}\left[\int_0^t\left(\frac{\sigma_x - \nu_x\sigma_y}{\lambda_x} + \frac{\mu_x}{E_x}\frac{d\sigma_y}{d\tau}\right)e^{E_x\tau/\lambda_x}\,d\tau + \frac{\sigma_{x10}}{E_x}\right] - \frac{\mu_x}{E_x}\sigma_y, \tag{5}$$

i.e.

$$\varepsilon_x = e^{-(E_x t/\lambda_x)}\left\{\int_0^t\frac{1}{\lambda_x}\left[\sigma_x - (\mu_x + \nu_x)\sigma_y\right]e^{E_x\tau/\lambda_x}\,d\tau + \frac{\sigma_{x10}}{E_x}\right\}, \tag{6}$$

which also represents the solution of the following first-order linear differential equation

$$\lambda_x\frac{d\varepsilon_x}{dt} + E_x\varepsilon_x = \sigma_x - (\mu_x + \nu_x)\sigma_y, \tag{7}$$

corresponding to the rheological equation of a Kelvin solid.

Introducing

$$\sigma_{x1} = E_x\varepsilon_x + \mu_x\sigma_y, \tag{8}$$

obtained from (1), into (2), one obtains, after integration,

$$\varepsilon_y = \frac{1 - \mu_x\mu_y}{E_y}\sigma_y - \frac{\mu_y E_x}{E_y}\varepsilon_x + \frac{1}{\lambda_x}\int_0^t\left[(1 + \mu_x\nu_y)\sigma_y - \nu_y\sigma_x + \nu_x E_x\varepsilon_x\right]d\tau. \tag{9}$$

After substituting from (6), this equation becomes

(10)

$$\varepsilon_y = \frac{1 - \mu_x\mu_y}{E_y}\sigma_x - \frac{\mu_y E_x}{E_y}e^{-(E_x t/\lambda_x)}\left\{\int_0^t\frac{1}{\lambda_x}\left[\sigma_x - (\mu_x + \nu_x)\sigma_y\right]e^{E_x\tau/\lambda_x}\,d\tau + \frac{\sigma_{x10}}{E_x}\right\} +$$
$$+ \frac{1}{\lambda_x}\int_0^t\left((1 + \mu_x\nu_y)\sigma_y - \nu_y\sigma_x + \nu_y E_x\,e^{-(E_x\tau/\lambda_x)}\left\{\int_0^\tau\frac{1}{\lambda_x}\left[\sigma_x - (\mu_x + \nu_x)\sigma_y\right].\right.\right.$$
$$\left.\left. .\,e^{E_x\tau/\lambda_x}\,d\tau + \frac{\sigma_{x10}}{E_x}\right\}\right)d\tau.$$

From (7)

$$\sigma_x = E_x \varepsilon_x + \lambda_x \frac{d\varepsilon_x}{dt} + (\mu_x + \nu_x)\sigma_y ; \tag{11}$$

introducing this into (9) yields after differentiation,

$$\frac{d\sigma_y}{dt} + \frac{(1 - \nu_x \nu_y)E_y}{(1 - \mu_x \mu_y)\lambda_y}\sigma_y = \frac{E_y}{1 - \mu_x \mu_y}\left[\frac{d\varepsilon_y}{dt} + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\frac{d\varepsilon_x}{dt}\right], \tag{12}$$

corresponding to the rheological equation of a Maxwell liquid; hence

$$\sigma_y = e^{-C_y t}\left\{\frac{E_y}{1 - \mu_x \mu_y}\int_0^t \left[\frac{d\varepsilon_y}{d\tau} + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\frac{d\varepsilon_x}{d\tau}\right] e^{C_y \tau}\, d\tau + \sigma_{y0}\right\}, \tag{13}$$

i. e.

$$\sigma_y = \frac{E_y}{1 - \mu_x \mu_y}\left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\varepsilon_x\right] - \tag{14}$$

$$- \left(\frac{E_y}{1 - \mu_x \mu_y}\right)^2 \frac{1 - \nu_x \nu_y}{\lambda_y} e^{-C_y t}\int_0^t \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\varepsilon_y\right] e^{C_y \tau}\, d\tau + \sigma_{y0} e^{-C_y t},$$

where $C_y = (1 - \nu_x \nu_y) E_y / (1 - \mu_x \mu_y) \lambda_y$ is the reciprocal relaxation time.

Substituting (14) into (11) yields

$$\sigma_x = E_x \varepsilon_x + \frac{(\mu_x + \nu_x) E_y}{1 - \mu_x \mu_y}\left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\varepsilon_x\right] + \lambda_x \frac{d\varepsilon_x}{dt} - (\mu_x + \nu_x) \,. \tag{15}$$

$$\cdot \left(\frac{E_y}{1 - \mu_x \mu_y}\right)^2 \frac{1 - \nu_x \nu_y}{\lambda_y} e^{-C_y t}\int_0^t \left[\varepsilon_y + \left(\frac{\mu_y E_x}{E_y} + \frac{\nu_y \lambda_x}{\lambda_y}\right)\varepsilon_x\right] e^{C_y \tau}\, d\tau + \sigma_{y0}\, e^{-C_y t}.$$

The assumption of full compactness of the two-strip model yields, for the symmetrical shear stress and strain, the following expression

$$\sigma_{xy} = \sigma_{yx} = G\varepsilon_{xy} + \eta \frac{d\varepsilon_{xy}}{dt} . \tag{16}$$

If however, the vertical boundaries of the model on Fig. 1 are free to deflect, the body behaves at the vertical shear like a Kelvin solid and the shear stress σ_{xy} is given by (16); for the horizontal shear, the equation of the Maxwell liquid is valid

$$\frac{d\sigma_{yx}}{dt} + \frac{G}{\eta}\sigma_{yx} = G\frac{d\varepsilon_{yx}}{dt} \tag{17}$$

from which

$$\sigma_{yx} = e^{-(Gt/\eta)} \left(\int_0^t G \frac{d\varepsilon_{yx}}{d\tau} e^{G\tau/\eta} \, d\tau + \sigma_{yx0} \right). \tag{18}$$

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