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AN ALGORITHM FOR THE INVERSION OF PARTITIONED MATRICES

HANA KAMASOVÁ

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1. INTRODUCTION

In paper [1] the following results were proved:

Let A be a square matrix of order $n \geq r$ over a field of characteristic zero, partitioned into blocks $\alpha_{i,k}$ of the type $(n_i \times n_k)$,

$$\sum_{i=1}^r n_i = \sum_{k=1}^r n_k = n.$$

Let further

$$(1,1) \quad \begin{aligned} A_1 &= \alpha_{1,1} \\ A_2 &= \begin{bmatrix} \alpha_{1,1} & \alpha_{1,2} \\ \alpha_{2,1} & \alpha_{2,2} \end{bmatrix} \\ &\vdots \\ &\vdots \\ A_{r-1} &= \begin{bmatrix} \alpha_{1,1} & \dots & \alpha_{1,r-1} \\ \vdots & & \vdots \\ \alpha_{r-1,1} & \dots & \alpha_{r-1,r-1} \end{bmatrix} \\ A_r &= A. \end{aligned}$$

Let us define the matrices $Z_{i,k}^{(p)}$ for $i, k, p = 1, 2, \dots, r$ in the following way:

1. $Z_{i,k}^{(1)} = \alpha_{i,k}$
2. $Z_{i,k}^{(p)} = Z_{i,k}^{(p-1)} - Z_{i,p-1}^{(p-1)} Z_{p-1}^{-1} Z_{p-1,k}^{(p-1)}$
for $p = 2, \dots, r$ where $Z_{p,p}^{(p)} = Z_p$.

For matrices Z_p we have

Theorem 1,1. *The matrices A_p are regular iff Z_p are regular for $p = 1, 2, \dots, r$.*

For the proof see [1].

For $i \leq p$, $k \leq p$ let us introduce $V_{i,k}^{(p)}$ as the set of subsequences of the sequence $(i, i + 1, \dots, p - 1, p, p - 1, \dots, k + 1, k)$ which have the following properties:

- 1° The first element is i , the last is k ,
- 2° Each two neighbouring elements of the subsequences are different.

Theorem 1,2. *Let the matrices $(1,1)$ be regular, let $\mathbf{A}^{-1} = [\beta_{i,k}]$ be a partitioned matrix conformal to \mathbf{A} . Then we have*

$$(1,2) \quad \beta_{i,k} = \sum (-1)^{1+m(j_1, \dots, j_s)} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1, j_2}^{(q_1)} \mathbf{Z}_{j_2}^{-1} \dots \mathbf{Z}_{j_{s-1}, j_s}^{(q_{s-1})} \mathbf{Z}_{j_s}^{-1},$$

$$(i = j_1, \dots, j_s = k) \in V_{i,k}^{(r)}$$

$$g_t = \min(j_t, j_{t+1}), \quad t = 1, \dots, s - 1$$

where $m(j_1, \dots, j_s)$ is the number of the elements of the sequence $(i = j_1, \dots, j_s = k)$.

For the proof see [1].

In this paper an algorithm for calculating the blocks of the inverse matrix of \mathbf{A} is presented. It is simultaneously an algorithm for the matrices $\mathbf{Z}_{i,k}^{(p)}$ for $i \geq p$; $k \geq p$; $i, k, p = 2, \dots, r$.

First, let us introduce the following sets:

For $i \leq p \leq k$, let us denote by $U_{i,k}^{(p)}$ the set of the subsequences of the sequence $(i, i + 1, \dots, p - 1, p, k)$ which have the properties 1°, 2°. Let $(U_{i,k}^{(p)})_p$ denote the set of those elements from $U_{i,k}^{(p)}$ which contain the term p .

Further, for $i \geq p \geq k$ let $W_{i,k}^{(p)}$ denote the set of the subsequences of the sequence $(i, p, p - 1, \dots, k + 1, k)$ which have the properties 1°, 2°.

Let $(W_{i,k}^{(p)})_p$ denote the set of those elements from $W_{i,k}^{(p)}$ which contain the term p .

For $i \leq p$; $k \leq p$ let $(V_{i,k}^{(p)})_p$ denote the set of those elements from $V_{i,k}^{(p)}$ which contain the term p .

It is obvious that:

$$(1,3) \quad \text{For } i \leq k, \quad V_{i,k}^{(k)} = U_{i,k}^{(k)}.$$

$$(1,4) \quad \text{For } i \geq k, \quad V_{i,k}^{(i)} = W_{i,k}^{(i)}.$$

$$(1,5) \quad \text{For } i \leq p, \quad U_{i,p+1}^{(p)} = V_{i,p+1}^{(p+1)}.$$

$$(1,6) \quad \text{For } k \leq p, \quad W_{p+1,k}^{(p)} = V_{p+1,k}^{(p+1)}.$$

$$(1,7) \quad \text{For } i \leq p; k \leq p, \quad V_{i,k}^{(p)} \cup (V_{i,k}^{(p+1)})_{p+1} = V_{i,k}^{(p+1)}, \quad V_{i,k}^{(p)} \cap (V_{i,k}^{(p+1)})_{p+1} = \Phi.$$

$$(1,8) \quad \text{For } i \leq p < k, \quad U_{i,k}^{(p)} \cup (U_{i,k}^{(p+1)})_{p+1} = U_{i,k}^{(p+1)}, \quad U_{i,k}^{(p)} \cap (U_{i,k}^{(p+1)})_{p+1} = \Phi.$$

$$(1,9) \quad \text{For } i > p \geq k, \quad W_{i,k}^{(p)} \cup (W_{i,k}^{(p+1)})_{p+1} = W_{i,k}^{(p+1)}, \quad W_{i,k}^{(p)} \cap (W_{i,k}^{(p+1)})_{p+1} = \Phi.$$

2. RESULTS

Theorem 2.1. *Let the matrices (1,1) be regular. For $i, k, p = 1, 2, \dots, r$ let us define the following transformation:*

$$(2,1) \quad \beta_{i,k}^{(0)} = \alpha_{i,k}$$

$$(2,2) \quad \beta_{p,p}^{(p)} = (\beta_{p,p}^{(p-1)})^{-1}$$

$$(2,3) \quad \beta_{p,k}^{(p)} = -(\beta_{p,p}^{(p-1)})^{-1} \beta_{p,k}^{(p-1)} \quad \text{for } k \neq p.$$

$$(2,4) \quad \beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \quad \text{for } i \neq p.$$

$$(2,5) \quad \beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \beta_{p,k}^{(p-1)} \quad \text{for } i \neq p, k \neq p.$$

Then for $i \leq p; k \leq p;$

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1, \dots, j_s=k) \in V^{(p)}_{i,k}} (-1)^{m+1} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1, j_2}^{(q_1)} \mathbf{Z}_{j_2}^{-1} \dots \mathbf{Z}_{j_{s-1}, j_s}^{(q_{s-1})} \mathbf{Z}_{j_s}^{-1}$$

(2,6) where $q_t = \min(j_t, j_{t+1})$ for $t = 1, \dots, s-1,$

m is the number of terms of the sequence $(i = j_1, \dots, j_s = k) \in V^{(p)}_{i,k},$

(2,7) for $i > p; k > p,$

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)},$$

(2,8) for $i \leq p; k > p,$

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1, \dots, j_s=k) \in U^{(p)}_{i,k}} (-1)^{u+1} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1, j_2}^{(j_1)} \mathbf{Z}_{j_2}^{-1} \mathbf{Z}_{j_2, j_3}^{(j_2)} \dots \mathbf{Z}_{j_{s-1}, j_s}^{-1} \mathbf{Z}_{j_s}^{(j_{s-1})}$$

where u is the number of terms of the sequence

$$(i = j_1, \dots, j_s = k) \in U^{(p)}_{i,k}.$$

(2,9) For $i > p; k \leq p,$

$$\beta_{i,k}^{(p)} = \sum_{(i=j_1, \dots, j_s=k) \in W^{(p)}_{i,k}} (-1)^w \mathbf{Z}_{j_1, j_2}^{(j_2)} \mathbf{Z}_{j_2}^{-1} \mathbf{Z}_{j_2, j_3}^{(j_3)} \mathbf{Z}_{j_3}^{-1} \dots \mathbf{Z}_{j_{s-1}, j_s}^{(j_s)} \mathbf{Z}_{j_s}^{-1}$$

where w is the number of terms of the sequence

$$(i = j_1, \dots, j_s = k) \in W^{(p)}_{i,k}.$$

Particularly, for $p = r$ the partitioned matrix $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}.$

Proof by induction.

1. For $p = 1$, $\beta_{1,1}^{(0)} = \mathbf{Z}_1$ is regular and we have by (2,1)–(2,5)

$$\beta_{i,k}^{(1)} = \begin{bmatrix} \mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} \\ \mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2, & \mathbf{Z}_{23}^{(2)} & & \mathbf{Z}_{2r}^{(2)} \\ \vdots & \vdots & & & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} & \mathbf{Z}_{r2}^{(2)}, & \dots & \dots & \mathbf{Z}_{rr}^{(2)} \end{bmatrix}.$$

For $p = 2$, $\beta_{2,2}^{(1)} = \mathbf{Z}_2$ is regular by Theorem 1,1 and we obtain

$$\beta_{i,k}^{(2)} = \begin{bmatrix} \mathbf{Z}_1^{-1} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & -\mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}, \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_2^{-1}, \\ \mathbf{Z}_{31}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{32}^{(2)}\mathbf{Z}_2^{-1}, \\ \vdots & \vdots \\ \mathbf{Z}_{r1}^{(1)}\mathbf{Z}_1^{-1} - \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}\mathbf{Z}_{21}^{(1)}\mathbf{Z}_1^{-1}, & \mathbf{Z}_{r2}^{(2)}\mathbf{Z}_2^{-1}, \\ \\ -\mathbf{Z}_1^{-1}\mathbf{Z}_{13}^{(1)} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots & -\mathbf{Z}_1^{-1}\mathbf{Z}_{1r}^{(1)} + \mathbf{Z}_1^{-1}\mathbf{Z}_{12}^{(1)}\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ -\mathbf{Z}_2^{-1}\mathbf{Z}_{23}^{(2)}, & \dots & -\mathbf{Z}_2^{-1}\mathbf{Z}_{2r}^{(2)} \\ \mathbf{Z}_3, & \dots & \mathbf{Z}_{3r}^{(3)} \\ \\ \mathbf{Z}_{r3}^{(3)}, & \dots & \mathbf{Z}_{rr}^{(3)} \end{bmatrix}.$$

It is obvious that Theorem 2,1 is true for $p = 1, 2$.

2. Let us suppose that our theorem is true for p ; then $\beta_{p+1,p+1}^{(p)} = \mathbf{Z}_{p+1}$, therefore by Theorem 1,1 $\beta_{p+1,p+1}^{(p)}$ is regular.

Proof of 2,6:

a) Let $i < p + 1$; $k < p + 1$; then by (2,5)

$$\begin{aligned} \beta_{i,k}^{(p+1)} &= \beta_{i,k}^{(p)} - \beta_{i,p+1}^{(p)}(\beta_{p+1,p+1}^{(p)})^{-1}\beta_{p+1,k}^{(p)} = \sum_{V^{(p)}_{i,k}} (-1)^{m_1+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_k^{-1} - \\ &- \left(\sum_{U^{(p)}_{i,p+1}} (-1)^{u+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1,p+1}}^{(j_{s-1})} \mathbf{Z}_{p+1}^{-1} \left(\sum_{W^{(p)}_{p+1,k}} (-1)^w \mathbf{Z}_{p+1,j_2}^{(j_2)} \dots \mathbf{Z}_k^{-1} \right) \right) = I + II. \end{aligned}$$

After performing the multiplication in II we obtain obviously a sum where we summarize over all sequences of the set $(V_{i,k}^{(p+1)})_{p+1}$. The sign of every summand in II is $(-1)^{u+w}$. Since the term $p + 1$ is included both in number u and in number w , it is clear that $(-1)^{u+w} = (-1)^{v+1}$ where v is the number of terms of the sequence from $(V_{i,k}^{(p+1)})_{p+1}$.

Since $U_{i,p+1}^{(p)}$ contains only increasing sequences and $W_{p+1,k}^{(p)}$ contains only decreasing

sequences, we obtain for every $\mathbf{Z}_{j_t, j_{t+1}}^{(q_t)}$ in II clearly $q_t = \min(j_t, j_{t+1})$. By relation (1,7) we can therefore write

$$\beta_{i,k}^{(p+1)} = \sum_{(i=j_1, \dots, j_s=k) \in \mathcal{V}_{i,k}^{(p+1)}} (-1)^{m+1} \mathbf{Z}_{j_1}^{-1} \mathbf{Z}_{j_1, j_2}^{(q_1)} \dots \mathbf{Z}_{j_s}^{-1}.$$

$$q_t = \min(j_t, j_{t+1})$$

b) Let $i = p + 1$; $k = p + 1$; then

$$\beta_{p+1, p+1}^{(p+1)} = (\beta_{p+1, p+1}^{(p)})^{-1} = \mathbf{Z}_{p+1}^{-1}.$$

c) Let $i = p + 1$; $k < p + 1$; then

$$\beta_{p+1, k}^{(p+1)} = -(\beta_{p+1, p+1}^{(p)})^{-1} \beta_{p+1, k}^{(p)} = -\mathbf{Z}_{p+1}^{-1} \left(\sum_{\mathcal{W}_{p+1, k}^{(p)}} (-1)^w \mathbf{Z}_{p+1, j_1}^{(j_1)} \dots \mathbf{Z}_k^{-1} \right),$$

therefore by (1,6)

$$\beta_{p+1, k}^{(p+1)} = \sum_{\mathcal{V}_{p+1, k}^{(p+1)}} (-1)^{m+1} \mathbf{Z}_{p+1}^{-1} \dots \mathbf{Z}_k^{-1}.$$

d) Let $i < p + 1$; $k = p + 1$; then

$$\beta_{i, p+1}^{(p+1)} = \left(\sum_{U_{i, p+1}^{(p)}} (-1)^{u+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1}, p+1}^{(j_{s-1})} \right) \mathbf{Z}_{p+1}^{-1}$$

and by (1,5) we get

$$\beta_{i, p+1}^{(p+1)} = \sum_{\mathcal{V}_{i, p+1}^{(p+1)}} (-1)^{m+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{p+1}^{-1}.$$

Proof of (2,7):

Let $i > p + 1$; $k > p + 1$; then

$$\beta_{i, k}^{(p+1)} = \beta_{i, k}^{(p)} - \beta_{i, p+1}^{(p)} (\beta_{p+1, p+1}^{(p)})^{-1} \beta_{p+1, k}^{(p)} = \mathbf{Z}_{i, k}^{(p+1)} - \mathbf{Z}_{i, p+1}^{(p+1)} \mathbf{Z}_{p+1}^{-1} \mathbf{Z}_{p+1, k}^{(p+1)} = \mathbf{Z}_{i, k}^{(p+2)}.$$

Proof of (2,8):

a) Let $i < p + 1 < k$; then

$$\begin{aligned} \beta_{i, k}^{(p+1)} &= \sum_{U_{i, k}^{(p)}} (-1)^{u_1+1} \mathbf{Z}_i^{-1} \mathbf{Z}_{i, j_2}^{(i)} \dots \mathbf{Z}_{j_{s-1}}^{-1} \mathbf{Z}_{j_{s-1}, k}^{(j_{s-1})} - \\ &- \left(\sum_{U_{i, p+1}^{(p)}} (-1)^{u_2+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1}, p+1}^{(j_{s-1})} \right) \mathbf{Z}_{p+1}^{-1} \mathbf{Z}_{p+1, k}^{(p+1)} = \\ &= \sum_{U_{i, k}^{(p)}} (-1)^{u_1+1} \mathbf{Z}_i^{-1} \mathbf{Z}_{i, j_2}^{(i)} \dots \mathbf{Z}_{j_{s-1}, k}^{(j_{s-1})} + \sum_{(U_{i, k}^{(p+1)})_{p+1}} (-1)^{u_3+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{p+1}^{-1} \mathbf{Z}_{p+1, k}^{(p+1)} \end{aligned}$$

where $u_3 = u_2 + 1$ is the number of terms of the sequence $(i = j_1, \dots, j_s = k) \in (U_{i, k}^{(p+1)})_{p+1}$.

Using relation (1,8) we obtain

$$\beta_{i, k}^{(p+1)} = \sum_{U_{i, k}^{(p+1)}} (-1)^{u+1} \mathbf{Z}_i^{-1} \dots \mathbf{Z}_{j_{s-1}, k}^{(j_{s-1})}.$$

b) Let $i = p + 1 < k$; then

$$\beta_{p+1,k}^{(p+1)} = -\mathbf{Z}_{p+1}^{-1} \mathbf{Z}_{p+1,k}^{(p+1)} = \sum_{U_{p+1,k}^{(p+1)}} (-1)^{u+1} \mathbf{Z}_{p+1}^{-1} \cdots \mathbf{Z}_{p+1,k}^{(p+1)},$$

for $U_{p+1,k}^{(p+1)}$ contains only one element – the sequence $(p + 1, k)$.

Proof of (2,9):

a) Let $i > p + 1 > k$; then

$$\begin{aligned} \beta_{i,k}^{(p+1)} &= \sum_{W_{i,k}^{(p)}} (-1)^{w_1} \mathbf{Z}_{i,j_2}^{(j_2)} \mathbf{Z}_{j_2}^{-1} \cdots \mathbf{Z}_{j_{s-1},k}^{(k)} \mathbf{Z}_k^{-1} - \\ &- \mathbf{Z}_{i,p+1}^{(p+1)} \sum_{W_{p+1,k}^{(p)}} (-1)^{w_2} \mathbf{Z}_{p+1,j_2}^{(j_2)} \cdots \mathbf{Z}_{j_{s-1},k}^{(k)} \mathbf{Z}_k^{-1} = \\ &= \sum_{W_{i,k}^{(p)}} (-1)^{w_1} \mathbf{Z}_{i,j_2}^{(j_2)} \cdots \mathbf{Z}_k^{-1} + \sum_{(W_{i,k}^{(p+1)})_{p+1}} (-1)^{w_3} \mathbf{Z}_{i,p+1}^{(p+1)} \cdots \mathbf{Z}_k^{-1} \end{aligned}$$

where $w_3 = w_2 + 1$ is the number of terms of the sequence $(i = j_1, \dots, j_s = k) \in (W_{i,k}^{(p+1)})_{p+1}$.

By (1,9) we obtain

$$\beta_{i,k}^{(p+1)} = \sum_{W_{i,k}^{(p+1)}} (-1)^w \mathbf{Z}_{i,j_2}^{(j_2)} \cdots \mathbf{Z}_k^{-1}.$$

b) Let $i > p + 1 = k$; then

$$\beta_{i,p+1}^{(p+1)} = \mathbf{Z}_{i,p+1}^{(p+1)} \mathbf{Z}_{p+1}^{-1}.$$

For $p = r$, formulas (2,6) for $\beta_{i,k}^{(r)}$ and (1,2) for $\beta_{i,k}$ are identical, thus the partitioned matrix $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}$.

3. EXAMPLE

An algorithm for the calculation of the inverse matrix of a quasi-triangular matrix.

Theorem 3,1. Let the matrices (1,1) be regular. Let $\alpha_{i,k} = 0$ for $i > k$. Let $\mathbf{A}^{-1} = [\beta_{i,k}]$ be a partitioned matrix conformal to \mathbf{A} . Then

$$(3,1) \quad \mathbf{Z}_{i,k}^{(p)} = \alpha_{i,k} \quad \text{for } i \geq p; k \geq p; p \geq 2,$$

$$(3,2) \quad \beta_{i,k} = \sum_{(i=j_1, \dots, j_s=k) \in V_{i,k}^{(k)}} (-1)^{m+1} \alpha_{j_1,j_1}^{-1} \alpha_{j_1,j_2} \alpha_{j_2,j_2}^{-1} \cdots \alpha_{j_s,j_s}^{-1} \quad \text{for } i \leq k$$

where m is the number of terms of the sequence $(i = j_1, \dots, j_s = k) \in V_{i,k}^{(k)}$,

$$(3,3) \quad \beta_{i,k} = 0 \quad \text{for } i > k.$$

For the proof see [2].

Lemma. Let the matrices (1,1) be regular, let $\alpha_{i,k} = 0$ for $i > k$. Then $\beta_{i,k}^{(p)} = 0$ for $i > k$; $p = 1, 2, \dots, r$.

Proof. For $\beta_{i,k}^{(p)}$ formulas (2,6)–(2,9) are true for $p = 1, 2, \dots, r$. In the case of a quasi-triangular matrix, these formulas have by (3,1)–(3,3) the following form:

(2,6)' For $i \leq k \leq p$,

$$\beta_{i,k}^{(p)} = \sum_{V_{i,k}(\alpha)} (-1)^{m+1} \alpha_{i,i}^{-1} \dots \alpha_{k,k}^{-1};$$

for $k < i \leq p$, $\beta_{i,k}^{(p)} = 0$.

(2,7)' For $i > p$; $k > p$,

$$\beta_{i,k}^{(p)} = \mathbf{Z}_{i,k}^{(p+1)} = \alpha_{i,k}.$$

(2,8)' For $i \leq p$; $k > p$,

$$\beta_{i,k}^{(p)} = \sum_{U_{i,k}(\alpha)} (-1)^{u+1} \alpha_{i,i}^{-1} \dots \alpha_{j_{s-1},k}.$$

(2,9)' For $i > p$; $k \leq p$,

$$\beta_{i,k}^{(p)} = \sum_{W_{i,k}(\alpha)} (-1)^w \alpha_{i,j_2} \alpha_{j_2,j_2}^{-1} \dots \alpha_{k,k}^{-1}.$$

Since $W_{i,k}^{(p)}$ contains only decreasing sequences, it is $j_t > j_{t+1}$ for all $t = 1, \dots, s - 1$; thus $\alpha_{j_t, j_{t+1}} = 0$. Therefore $\beta_{i,k}^{(p)} = 0$ for $i > k$.

The transformation formulas:

$$\beta_{i,k}^{(0)} = \alpha_{i,k},$$

$$\beta_{p,p}^{(p)} = (\beta_{p,p}^{(p-1)})^{-1},$$

$$\beta_{p,k}^{(p)} = -(\beta_{p,p}^{(p-1)})^{-1} \beta_{i,k}^{(p-1)} \quad \text{for } k > p,$$

$$\beta_{p,k}^{(p)} = 0 \quad \text{for } k < p,$$

$$\beta_{i,p}^{(p)} = \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \quad \text{for } i < p,$$

$$\beta_{i,p}^{(p)} = 0 \quad \text{for } i > p,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} - \beta_{i,p}^{(p-1)} (\beta_{p,p}^{(p-1)})^{-1} \beta_{p,k}^{(p-1)} \quad \text{for } i < p < k,$$

$$\beta_{i,k}^{(p)} = \beta_{i,k}^{(p-1)} \quad \text{for } i > p; k > p \quad \text{and for } i < p; k < p.$$

References

- [1] *H. Kamasová, A. Šimek*: Metoda inverze matice rozdělené na bloky, Aplikace matematiky, 2, 14 (1969), 105–114.
- [2] *H. Kamasová, A. Šimek*: Inversion of quasi-triangular matrices, Aplikace matematiky, 2, 15 (1970), 146–148.

Souhrn

ALGORITMUS PRO INVERZI BLOKOVÝCH MATIC

HANA KAMASOVÁ

V článku je uveden algoritmus pro výpočet inverzní matice k blokové matici rozdělené na $r \times r$ bloků.

Věta. *Nechť matice (1,1) jsou regulární. Pro $i, k, p = 1, 2, \dots, r$ definujme transformace (2,1)–(2,5). Potom platí (2,6)–(2,9). Speciálně pro $p = r$ bloková matice $[\beta_{i,k}^{(r)}] = \mathbf{A}^{-1}$.*

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