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ALGORITMY

36. SNEDECOR

AN ALGORITHM FOR FISHER - SNEDECOR'S F-TEST WITHOUT APPLICATION OF CRITICAL VALUES

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The algorithm suggested in this paper computes the probability that Fisher-Snedecor's test statistic will exceed the value F actually observed, i.e.

(1) $$\alpha_{m,n}(F) = \frac{\left(\frac{m}{n}\right)^{m/2}}{B\left(\frac{m}{2}, \frac{n}{2}\right)} \int_F^\infty y^{m/2-1}\left(1 + \frac{m}{n} y\right)^{-(m+n)/2} \mathrm{d}y ,$$

where m, n is the pair of numbers of degrees of freedom. This probability may be hence called the significance degree, similarly as in the case of Student's t-statistic treated in our previous paper [1]. Since the latter represents a special case of the present problem (with $m = 1, F = t^2$), the features of the algorithm and further remarks made in [1] apply here, too (except the distinction between one-sided and two-sided tests) and will not be repeated.

Analogously as in [1], the relation

(2) $$\alpha_{m,n}(F) = A_{m,n}(x)$$

with

(3) $$x = \left(1 + \frac{m}{n} F\right)^{-1},$$

(4) $$A_{m,n}(x) = \frac{1}{B\left(\frac{m}{2}, \frac{n}{2}\right)} \int_0^x y^{n/2-1}(1 - y)^{m/2-1} \mathrm{d}y$$

holds and the algorithm is based on the following recurrence relations and initial conditions

$$(5)\qquad A_{m,n}(x) = A_{m,n-2}(x) - \frac{\Gamma\left(\frac{m+n}{2} - 1\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} x^{n/2-1}(1-x)^{m/2}$$

$(m > 0,\ n > 2)$,

$$(6)\qquad A_{m,n}(x) = A_{m-2,n}(x) + \frac{\Gamma\left(\frac{m+n}{2} - 1\right)}{\Gamma\left(\frac{m}{2}\right)\Gamma\left(\frac{n}{2}\right)} x^{n/2}(1-x)^{m/2-1}$$

$(m > 2,\ n > 0)$,

$$(7)\qquad A_{1,1}(x) = \frac{2}{\pi}\arcsin\sqrt{x}\,,\quad A_{2,1}(x) = \sqrt{x}\,,\quad A_{2,2}(x) = x\,,$$

$$(8)\qquad A_{m,n}(x) = 1 - A_{n,m}(1-x)\,.$$

The last relation follows from the definition, the remainder can be proved by differentiation (the relations (5) and (6) being equivalent due to (8)).

If the statistic F has its usual form

$$(9)\qquad F = \left(\frac{y}{m}\right)\Big/\left(\frac{z}{n}\right),$$

where y, z are certain sampling characteristics, then the transform (3), which is used instead of the statistic F, may be evaluated directly from y, z in the form

$$(10)\qquad x = z/(y+z)\,.$$

```
real procedure SNEDECOR(x, m, n); value x; real x; integer m, n;
begin
real a, b, c, d, e, f; integer i;
procedure G;
        begin c := c × x;
              for f := e step 2 until i do
                  begin a := a + b; d := b × c; b := d/f; c := c + 2 × x end f
        end G;
procedure H;
        begin x := 1 − x; G; b := −d; c := i + 1; e := 3; i := n;
              x := 1 − x; G
        end H;
procedure P; begin b := sqrt(x); c := 1; H end;
```

```
procedure Q; begin b := 1; c := n; G; a := a × (1 − x) ↑ (n ÷ 2) end;
if n > (n ÷ 2) × 2
  then
  begin i := m;
        if m > (m ÷ 2) × 2
        then
        begin a := 0·63661977 × arcsin(sqrt(x));
              b := 0·63661977 × sqrt((1 − x) × x); c := 2;
              d := b; e := 3; H
        end
        else begin a := 0; e := 2; P end
  end
  else
  begin a := 0; e := 2;
        if m > (m ÷ 2) × 2
        then
        begin i := n; n := m; m := i; x := 1 − x; P; x := 1 − x;
              n := m; m := i; a := 1 − a
        end
        else
        if m > n
        then
        begin i := n; n := m; Q; n := i; a := 1 − a end
        else
        begin i := m; x := 1 − x; Q; x := 1 − x end
  end;
SNEDECOR := a
end SNEDECOR
```

The result is obtained with the accuracy of at least about 5 decimal places. We give some check values:

$$
\begin{aligned}
SNEDECOR\ (0{\cdot}3, 1, 1) &= 0{\cdot}36901\\
SNEDECOR\ (0{\cdot}25, 1, 10) &= 0{\cdot}00027\\
SNEDECOR\ (0{\cdot}75, 1, 19) &= 0{\cdot}02099\\
SNEDECOR\ (0{\cdot}5, 4, 10) &= 0{\cdot}10937\\
SNEDECOR\ (0{\cdot}4, 10, 6) &= 0{\cdot}58010\\
SNEDECOR\ (0{\cdot}7, 3, 8) &= 0{\cdot}38890\\
SNEDECOR\ (0{\cdot}6, 4, 9) &= 0{\cdot}28109\\
SNEDECOR\ (0{\cdot}1, 3, 1) &= 0{\cdot}39582\\
SNEDECOR\ (0{\cdot}2, 5, 11) &= 0{\cdot}00143\\
SNEDECOR\ (0{\cdot}3, 7, 3) &= 0{\cdot}55292\\
SNEDECOR\ (0{\cdot}75, 10, 1) &= 0{\cdot}99973
\end{aligned}
$$

The program has been tested in the symbolic language MOST [3] and implemented in the Biophysical Institute, Faculty of General Medicine, Charles University Prague for the computer ODRA 1013 [4].

[1] *Režný, Z., Jirkovský, J.:* STUDENT. An algorithm for Student's *t*-test without application of critical values, Aplikace matematiky 19 (1974), 133—135.

[2] *Janko, J.:* Statistical Tables (in Czech), Publ. House of the Czechoslovak Acad. of Sci., Prague 1958.

[3] *Szczepkowicz, J.:* Programming in the autocode MOST 1 (in Polish), Elwro Publication 03-VI-1, Wroclaw.

[4] *Černý, V., Půr, J.:* Programmer's Manual on Automatic Computer ODRA 1013 (in Czech), Kanc. stroje n. p., Hradec Králové 1967.