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## MULTI-POLARIZED GRAPHS

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## 1. INTRODUCTION

Zítek in [1] introduced a widely applicable notion of polarized graphs. Zelinka [2]–[8] extended the theory and its applications. The purpose of this remark is to generalize the notion of the polarized graph and to show an example of its application.

The graph  $G = (V, H)$  with the vertex set  $V$  and the edge set  $H$  is called partially polarized if there exists a set  $W \subset V$  such that every  $w \in W$  has two poles  $w^+$  and  $w^-$  (i.e.  $w$  is a couple  $(w^+, w^-)$ ). The graph is called polarized if  $W = V$ .

If  $w \in W$  and  $h = (w, v)$  or  $(v, w)$ , then one of the poles  $w^+$  or  $w^-$  with which  $h$  is incident is indicated. Therefore we write  $h = (w^+, v)$ , etc.

On a partially polarized or polarized graph one can define an admissible path  $v_0, h_1, v_1, \dots, v_{n-1}, h_n, v_n$  as a path with the property  $v_i \in W \Rightarrow h_{i-1}$  is incident with  $v_i^+$  and  $h_i$  with  $v_i^-$  or  $h_{i-1}$  is incident with  $v_i^-$  and  $h_i$  with  $v_i^+$ .

The graph  $G = (V, H)$  is called *multi-polarized* if

1. for every vertex  $v \in V$ , a pole set  $P(v) = \{v^{(1)}, \dots, v^{(n(v))}\}$  is defined.
2. for every  $h \in H$ ,  $h = (v, w)$  a pole from  $P(v)$  as well as one from  $P(w)$  is indicated with which  $h$  is incident; thus we shall write the edge  $h$  in the form  $h = (v^{(i)}, w^{(j)})$ .
3. for every  $v \in V$ ,  $n(v) \geq 2$ , a set of ordered pairs  $R(v) \subset P^2(v)$  is given. The set  $R(v)$  is called the set of *forbidden transitions* in  $v$ .

We do not specify whether  $G$  is oriented or not (or mixed), because the following considerations are possible in all the three cases.

The path  $v_0, h_1, v_1, \dots, v_{m-1}, h_m, v_m$  on  $G$  is said to be admissible if the condition  $(v_s^{(j)}, v_s^{(k)}) \notin R(v_s)$  holds for every  $h_s = (v_{s-1}^{(i)}, v_s^{(j)})$ ,  $h_{s+1} = (v_s^{(k)}, v_{s+1}^{(k)})$ .

We see that

- for  $n(v) \equiv 1$  and  $R(v) \equiv \emptyset$  we obtain the case of ordinary graphs
- for  $n(v) \equiv 2$  and  $R(v) = \{(v^+, v^+), (v^-, v^-)\}$  the case of polarized graphs.

## 2. WEIGHTED MULTI-POLARIZED GRAPHS

A multi-polarized graph, which may be denoted by  $G = (V, H, P, R)$ , is called weighted if for every  $v \in V$ , each  $(v^{(i)}, v^{(j)}) \in P^2(v) - R(v)$  is assigned a weight  $\gamma(v^{(i)}, v^{(j)}) \in \langle 0, \infty \rangle$  and every  $h \in H$  a weight  $\gamma(h) \in \langle 0, \infty \rangle$ .

For an admissible path  $C = (v_0, h_1, \dots, h_n, v_n)$  where  $h_s = (v_{s-1}^{(i_s)}, v_s^{(j_s)})$  we define a weight  $\gamma(C)$  as follows:

$$\gamma(C) = \sum_{s=1}^n [\gamma(v_{s-1}^{(i_s)}, v_s^{(j_s)}) + \gamma(v_s^{(i_{s+1})}, v_s^{(j_s)})].$$

An admissible path  $C = (v = v_0, h_1, v_1, \dots, h_n, v_n = w)$  is said to be minimal if  $\gamma(C)$  is minimal among all possible weights of admissible paths from  $v$  to  $w$ .

The problem of finding the minimal admissible path from  $v$  to  $w$  in a weighted polarized graph is a generalization of that for polarized graph which can be obtained putting  $R(v) = \{(v^+, v^+), (v^-, v^-)\}$ ,  $\gamma \equiv 0$  on  $P^2(v) - R(v)$  and  $\gamma \equiv 1$  on  $H$ .

## 3. APPLICATION

Let us consider a urban street network with only the right-of-way signs on all crossings (i.e., we suppose that no other means are used for the control of crossings — no traffic lights, near-hand side rule, or roundabouts).

Let us denote by  $V$  the set of crossings,  $H$  the set of street segments,  $P(v)$  the set of entries to a crossing  $v$  and  $R(v)$  the set of all forbidden passages through  $v$ . Let  $\gamma$  express the average time necessary for passing through a crossing or through a segment of a street.

Solving a traffic assignment problem for this net one can suppose that a driver chooses the minimal admissible path and thus it is necessary to solve the problem mentioned in 2.

## 4. SOLUTION

The solution of the minimal admissible path problem is simple. We adjoin to  $G = (V, H, P, R, \gamma)$  a new graph  $\bar{G} = (\bar{V}, \bar{H})$  as follows:

1.  $\bar{V} = \bigcup_{v \in V} P(v)$ ,
2.  $\bar{H} = H \cup \bigcup_{v \in V} [P^2(v) - R(v)]$ .

On  $G$  we can let the definition of  $\gamma$  without any change and call it the length of edges.

To find the minimal admissible path in  $G$  from  $v$  to  $w$  is the same as to find the path of minimal length in  $\bar{G}$  from the set  $P(v)$  to the set  $P(w)$ , i.e., the minimal path from the shortest paths from  $v^{(i)}$  to  $w^{(j)}$ . The algorithm for this problem can be found in almost every monograph on the graph theory.

#### References

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#### Súhrn

### MULTIPOLARIZOVANÉ GRAFY

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V článku sa zavádza pojem multipolarizovaného grafu, ktorý je zovšeobecnením pojmu polarizovaného grafu, zavedeného Zítikom [1] a študovaného najmä Zelinkom [2–8]. Takýto graf môže mať vrcholy aj s viac, než dvojma pólmi. Cesta na multipolarizovanom grafe môže prechádzať vrcholom len cez „dovolenú“ dvojicu pólův.

Ďalej sa študuje problém minimálnej cesty a aplikácia na cestnú dopravu.

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