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Aplikace matematiky, Vol. 25 (1980), No. 3, 234--240

Persistent URL: <http://dml.cz/dmlcz/103855>

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ALGORITMUS

44. MIMUN

OPTIMIZATION OF ONE-DIMENSIONAL MULTIMODAL FUNCTIONS
IN THE PRESENCE OF NOISE

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The problems of minimization in the presence of noise occur in various fields of science and engineering. But, as far as it is known to the author, among the currently available issues there is not a single publication of a computer program for solving such problems. A rather efficient and quick-operating algorithm for one-dimensional multimodal minimization in the presence of noise is proposed in [1]. This algorithm is based on the usage of a Wiener process for a statistical model of an objective function [2]. The results of investigation of a former version of this algorithm are given in [3], [4]. The algorithm for one-dimensional multimodal minimization without noise based on similar assumptions [5] is more efficient than other algorithms of analogous destination as shown in [6]. Only a brief description of this algorithm is given here to explain the meaning of formal parameters while its full description is given in [1].

Let a function $f(x)$, $a \leq x \leq b$, be minimized where only the values $z(x_i) = f(x_i) + \xi_i$ may be observed where ξ_i are independent Gaussian random numbers (noise), whose mean is equal to zero and the dispersion is σ_i^2 , i being the number of observation. Before the minimization the variance analysis of the results z_{ij} is carried out, where $z_{ij} = z(x(i))$, $x(i) = a + (b - a)(i - 1)/(m_2 - 1)$, $i = 1, \dots, m_2$, $j = 1, \dots, m_3$, i.e. at each point $x(i)$ the objective function $z(\cdot)$ is observed m_3 times. If the hypothesis of equality of $f(x(i))$ is accepted (the significance level being equal to 0.05) then the algorithm terminates indicating that the noise level is too high. If the variance of $f(\cdot)$ is significant then the dispersion of noise and the parameter of a Wiener process, chosen as a statistical model of an objective function, are estimated [1]. After that the minimization begins. To simplify the algorithm the lattice $x^i = a + (b - a)(i - 1)/(m_4 - 1)$, $i = 1, \dots, m_4$, is substituted for the interval of

minimization $[a, b]$. The additional error caused by discretization of $[a, b]$ may obviously be reduced to a desirable value choosing sufficiently large m_4 ; the value $m_4 = 101$ is large enough for many practical problems. The coordinate of the current observation is defined by the condition of maximum of the expected improvement [1, 3]. The algorithm terminates if the number of objective function evaluations reaches the maximally allowable amount or if the P-probability of evaluating the global minimum with given accuracy ε_1 exceeds 0.9; this probability is calculated according to the chosen statistical model of objective function [1]. Note that in the case when $\sigma_i/\varepsilon_1 \geq 10$ and σ_i is of the same order as variance of $f(\cdot)$, more than a thousand observations of an objective function are necessary for P to reach 0.9. On the other hand, if the noise level is not so high, a practically acceptable solution is usually obtained after 200–500 observations [1].

Using this algorithm the following remarks must be taken into account:

1. The variable *kmax* is the machine dependent constant which is initialised as 19. If the maximal real number of the user's computer is 10^k where $k < 19$ then the value of the variable *kmax* must be set equal to k .

2. The formal parameter *ifault* is the failure indicator. The normal termination of the algorithm is indicated by *ifault* = 0. If *ifault* = 1 then the cause of termination of the algorithm is a too high level of noise. *ifault* = 2 means that the number of observations reaches the maximally allowable value. The scale of values of an objective function must be chosen so that $|f(x)|$ does not exceed 10^K where $K = kmax/2$; the violation of this condition is indicated as *ifault* = 3. The termination with *ifault* = 4 means that the variance of the objective function is insignificant as shown by the results before the minimization investigation; the scale of the values of an objective function must be changed or the different number of points m_2 must be taken (for example, $2m_2 + 1$).

3. The algorithm calls the auxiliary **real procedure** *ndtr*, which is the ALGOL version of *SUBROUTINE NDTR* [7] and which calculates the value of the Gaussian distribution function.

Algorithm *mimun*:

procedure *mimun*(*be, en, sn1, am, m, f, e1, e, nf, xm, ym, ifault,*
 anm, b, c, y);

comment: *be* ... input ... start of interval of optimization,
 en ... input ... end of interval of optimization,
 sn1 ... input ... if(*sn1* > 0) then *sn1* is variance of noise,
 if(*sn1* ≤ 0) then variance of noise to be
 evaluated by algorithm,
 am ... input ... if *am* = 1 then minimization,
 if *am* = -1 then maximization,

m ... input ... *m*[1] ... maximal allowed number of observations of objective function *f*,
m[2] ... number of observation points for parameters estimation, it is recommended = 6,
m[3] ... number of observations at each point for parameters estimation, it is recommended = 5,
m[4] ... number of points of lattice. it is recommended = 101,
f ... input ... objective function,
e1 ... input ... if(*e1* ≥ 0) then *e1* is required accuracy of *ym*, if(*e1* < 0) then required accuracy is equal to $\sqrt{\text{variance of noise}/\text{abs}(e1)}$,
e ... output ... estimation of mean-root-square error of *ym*,
nf ... output ... number of observations of *f*,
xm ... output ... estimation of optimum point,
ym ... output ... estimation of optimum,
ifault ... output ... failure indicator
anm, *b*, *c*, *y* ... workspace, dimension of these arrays ≥ *m* [4];
value *be*, *en*, *sn1*, *am*, *e1*; **integer** *nf*, *ifault*;
real *be*, *en*, *sn1*, *am*, *e1*, *e*, *xm*, *ym*; **integer array** *m*;
array *anm*, *b*, *c*, *y*; **real procedure** *f*;
begin integer *n*, *kmax*, *k*, *k1*, *n1*, *n2*, *j*, *k3*, *k4*, *km*, *km2*;
real *dt*, *cv*, *eps2*, *an*, *pp*, *amax*, *ym1*, *p1*, *p2*, *p3*, *p4*, *p5*, *p6*, *cv2*, *ym2*, *sn2*,
aw, *av1*, *av2*, *am1*, *v1*, *v2*, *c1*, *c2*, *pr*, *pr1*, *ppab*, *va*, *d*, *d1*,
a11, *a12*, *a21*, *a22*, *eb*, *sf*, *eps1*, *eps3*;
real procedure *av*(*k*, *co*);
comment: auxiliary procedure for *mimun*: calculates conditional mean and variance of Wiener process;
integer *k*; **real** *co*;
begin integer *i*; **real** *a*, *p*;
a := *y*[*k*]; *p* := *b*[*k*]; *co* := 1.0; *i* := *k*;
for *i* := *i* - 1 **while** *co* × *eps2* < *p* ∧ *i* > 0 **do**
if *anm*[*i*] > 0.0 **then begin** *a* := *a* + *p* × *y*[*i*]; *co* := *co* + *p*; *p* := *p* × *b*[*i*]
end;
p := *c*[*k*]; *i* := *k*;
for *i* := *i* + 1 **while** *co* × *eps2* < *p* ∧ *i* ≤ *n* **do**
if *anm*[*i*] > 0.0 **then begin** *a* := *a* + *p* × *y*[*i*]; *co* := *co* + *p*; *p* := *p* × *c*[*i*]
end;
av := *a*/*co*;
end *av*;
real procedure *fi*(*x*, *nr*);

```

value x, nr; integer nr; real x;
comment: auxiliary procedure for mimun;
begin integer k; real a;
a := 0.0; for k := 1 step 1 until nr do a := a + f(x);
fi := a/(nr × cv)
end fi;
procedure updata;
comment: auxiliary procedure for mimun:
updates array of parameters c, b;
begin integer k, k1, kp, kp1; real b1, bs, cs;
kp := 1; kp1 := n; bs := cs := b[1] := c[n] := 1.0;
for k := 2 step 1 until n do
  begin if anm[k] > 0.0 then
    begin b1 := dt/((dt/anm[kp] + (k - kp) × an × bs) × anm[k]);
      bs := bs × b1 + 1.0; b[k] := b1; kp := k
    end;
    k1 := n + 1 - k; if anm[k1] > 0.0 then
      begin b1 := dt/((dt/anm[kp1] + (kp1 - k1) × an × cs) × anm[k1]);
        cs := cs × b1 + 1.0; c[k1] := b1; kp1 := k1
      end
    end
  end
end updata;
real procedure ndtr(x); value x; real x;
comment: Gaussian distribution function, algol version of subroutine ndtr:
system/360 scientific subroutine package;
begin real t, d, p, ax; ax := abs(x); t := 1.0/(1.0 + 0.2316419 × ax);
d := 0.3989423 × exp(-x × x/2.0); p := 1.0 - d × t × (((1.330274 × t -
1.821256) × t + 1.781478) × t - 0.3565638) × t + 0.3193815);
if x > 0 then ndtr := p else ndtr := 1.0 - p
end Any other procedure of analogous destination may be used instead of ndtr;
kmax := 19; pp := 2.0; eps2 := 0.001; ifault := 0;
n := m[4]; n1 := m[2]; n2 := m[3]; cv := 1.0; amax := 10 ↑ (kmax ÷ 2 - 1);
eb := (en - be)/(n - 1); p2 := p5 := p6 := 0.0; ym1 := amax; an := 1/(n - 1);
for k := 1 step 1 until n do begin y[k] := 0.0; anm[k] := -1.0/amax end;
for k := 1 step 1 until n1 do
  begin p3 := 0.0; for k1 := 1 step 1 until n2 do
    begin p4 := fi(be + eb × (((n - 1) × (k - 1)) ÷ (n1 - 1)), 1);
      if abs(p4) < amax then begin p3 := p3 + p4; p2 := p2 + p4 × p4 end
      else begin ifault := 3; go to fin end
    end;
    y[k] := p3/n2; if ym1 > p3 then ym1 := p3;
    p5 := p5 + p3; p6 := p6 + p3 × p3
  end

```

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end;
nf :=  $n1 \times n2$ ;  $p5 := p5/nf$ ;  $p6 := p6/n2$ ;  $sf := \text{abs}(p6 - p5)/(n1 - 1)$ ;
if  $sn1 > 0.0$  then  $sn2 := sn1$  else  $sn2 := \text{abs}(p2 - p6)/(n1 \times (n2 - 1))$ ;
 $sf := \text{abs}(p6 - p5)/(n1 - 1)$ ;
if  $sf < sn2 \times 2.5$  then begin ifault := 1; go to fin end;
comment: estimation of parameters;
 $p1 := y[1]$ ;  $cv2 := 0.0$ ;
for  $k := 2$  step 1 until  $n1$  do
  begin  $p2 := y[k]$ ;  $cv2 := cv2 + (p2 - p1) \uparrow 2$ ;  $p1 := p2$ 
  end;
 $cv := \text{sqrt}(cv2)$ ; if  $cv < 1.0/amax$  then begin ifault := 4; go to fin end;
 $dt := sn2/cv2$ ;
for  $k := 1$  step 1 until  $n1$  do
  begin  $k1 := ((n - 1) \times (k - 1)) \div (n1 - 1) + 1$ ;
   $y[k1] := y[k]/cv$ ;  $anm[k1] := n2$ 
  end;
if  $e1 > 0.0$  then  $eps3 := e1/cv$  else  $eps3 := \text{sqrt}(dt/\text{abs}(e1))$ ;
 $eps1 := eps3/pp$ ;
comment: begin of optimization;
 $ym1 := ym1/(cv \times n2)$ ;  $v1 := 0.0$ ; update;
 $lopt: ppab := 1.0$ ; if  $v1 \geq eps1$  then  $ppab := 0.0$ ;  $ym2 := av2 := av(1, c2)$ ;
 $km := km2 := 1$ ;  $pr := 0.0$ ;
comment main loop, computing of point of current observations;
for  $k := 1$  step 1 until  $n$  do
  begin if  $k < n \wedge anm[k] > 0.0$  then
    begin  $c1 := c2$ ;  $av1 := av2$ ;  $k3 := k$ ;
    for  $j := k + 1$  step 1 until  $n$  do if  $anm[j] > 0.0$  then
      begin  $av2 := av(j, c2)$ ; if  $av2 < ym2$  then
        begin  $ym2 := av2$ ;  $km2 := j$ 
        end;
       $k4 := j$ ;  $a11 := 1.0/c1$ ;  $a12 := a11 \times b[k3]$ ;  $a21 := a11 \times c[k3]$ ;
       $a22 := 1.0/c2$ ; go to l1
    end
  end;
  l1:  $d := (k - k3)/(k4 - k3)$ ;  $d1 := (1 - d)$ ;  $aw := av1 \times d1 + av2 \times d$ ;
   $va := \text{sqrt}(d \times d1 \times (k4 - k3)/(n - 1) + (d1 \times (d1 \times a11 + d \times a21)/$ 
   $anm[k3] + d \times (d1 \times a12 + d \times a22))/anm[k4]) \times dt$ ;
  if  $k = km2$  then  $v2 := va$ ;
   $am1 := ym1 - aw$ ;  $p1 := -0.2 \times am1$ ;
  comment: computing of probability of finding
  global optimum with required accuracy ppab;
  if  $v1 < eps1 \wedge va \geq 1.5 \times v1 \wedge ppab \geq 0.9$  then

```

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ppab := ppab × (1 - ndtr((am1 - eps3)/va));
comment computing of mean improvement;
va := va × 7.0; if va > p1 then
  begin pr1 := am1 × 0.65 × exp(-0.443 × (0.75 - am1/va) ↑ 2) +
    va × 0.3989 × exp(-(am1 × am1)/(2.0 × va × va));
    if pr1 > pr ∧ va > eps1 then begin km := k; pr := pr1 end
  end
end main loop;
if ppab ≥ 0.9 then go to l2; d := anm[km];
j := 0.1 × d + 1.0; d1 := d + j; p4 := fi(be + eb × (km - 1), j);
if abs(p4) > amax then begin ifault := 3; go to fin end;
y[km] := (y[km] × d + am × j × p4)/d1; anm[km] := d1;
nf := nf + j; ym1 := ym2; v1 := v2; updata;
if nf < m[1] then go to lopt; ifault := 2;
l2: ym := ym2 × cv/am; xm := be + (km2 - 1) × eb; e := v2 × pp × cv;
fin:
end;

```

Example: The test function:

real procedure f(x); **value** x; **real** x;

comment: test function for mimun, integer

parameter kun must be declared in driver program

and initialised there as kun = 127;

begin real a, b; **integer** i;

comment: generation of pseudo-random number a;

kun := kun × 3125; kun := kun - entier(kun/67108864) × 67108864;

a := kun/33554432 - 1.0; b := 0.0;

for i := 1 **step** 1 **until** 5 **do**

b := b - i × sin((i + 1) × x + i);

f := a + b

end

was minimized with the input parameters: be = -10.0, en = 10.0, am = 1.0, sn1 = -1.0, e1 = -5.0, m[1] = 5000, m[2] = 6, m[3] = 5, m[4] = 101. The following results were obtained (computer BESM-6):

xm = 5.800000000, ym = -12.07391983, nf = 86, e = 0.2136057320, ifault = 0.

The FORTRAN codes of this algorithm are available from the author.

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