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ON WEIGHTED ENTROPY OF TYPE (α, β) AND ITS GENERALIZATIONS

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1. INTRODUCTION

Consider the following model for a finite random experiment S,

(1.1)
$$S = \begin{pmatrix} a_1, a_2, \ldots, a_n \\ p_1, p_2, \ldots, p_n \\ w_1, w_2, \ldots, w_n \end{pmatrix} = \begin{pmatrix} A \\ P \\ W \end{pmatrix},$$

where $A = (a_1, a_2, \ldots, a_n)$ is the alphabet, $P = (p_1, p_2, \ldots, p_n)$ the complete probability distribution and $W = (w_1, w_2, \ldots, w_n)$ is the utility distribution, where w_i's are non-negative real numbers.

Belis and Guiaşu [1] introduced the function

(1.2)
$$H(P; W) = -\sum_{k=1}^{n} p_k w_k \log p_k\,, \quad w_k \geqq 0\,, \quad \sum_{k=1}^{n} p_k = 1\,,$$

which they considered a satisfactory measure for the average number quantity of valuable or useful information provided by a source letter.

The measure $H(P; W)$ is additive in the following sense:

(1.3)
$$H(P^*Q; W^*V) = \bar{V} H(P; W) + \bar{W} H(Q, V)\,,$$

where $Q = (q_1, \ldots, q_m)$, $0 < q_j \leqq 1$, $\sum_{j=1}^{m} q_j = 1$, and $V = (v_1, \ldots, v_m)$ is the utility distribution associated with Q, $\bar{W} = \sum_{k=1}^{n} p_k w_k$, $\bar{V} = \sum_{j=1}^{m} v_j q_j$,

$$W^*V = \{w_i v_j / w_i \in W,\ v_j \in V\}\,,$$
$$P^*Q = \{p_i q_j / p_i \in P,\ q_j \in Q\}\,.$$

*) Supported by CNPq

In this paper, we introduce and characterize a weighted entropy of type (α, β) given by

$$(1.4) \qquad H_n(P; W; \alpha, \beta) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^{n} w_k(p_k^\alpha - p_k^\beta), \quad \alpha \neq \beta, \quad \alpha, \beta > 0.$$

Some interesting properties of this measure and its further generalizations are also presented.

2. CHARACTERIZATION OF WEIGHTED ENTROPY OF TYPE (α, β)

Let a function $H_n(P; W; \alpha, \beta)$ for all $P = (p_1, \ldots, p_n)$, $p_k \geqq 0$, $\sum_{k=1}^{n} p_k = 1$, $\alpha \neq \beta$, $\alpha, \beta > 0$, satisfy the following axioms:

(I) *Continuity*: $H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta)$ is a continuous function of its arguments.

(II) *Symmetry*: $H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta)$ is symmetric with respect to the couples (p_k, w_k), $k = 1, 2, \ldots, n$.

(III) *Generalized Branching*:
If

$$p_n = p' + p'', \quad w_n = \frac{p'w' + p''w''}{p' + p''},$$

then

$$H_{n+1}(p_1, \ldots, p_{n-1}, p', p''; w_1, \ldots, w_{n-1}, w', w''; \alpha, \beta) =$$

$$= H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta) +$$

$$+ \frac{A_\alpha}{A_\alpha - A_\beta} p_n^\alpha H_2(p'/p_n, p''/p_n; w', w''; \alpha, 1) +$$

$$+ \frac{A_\beta}{A_\beta - A_\alpha} p_n^\beta H_2(p'/p_n, p''/p_n; w', w''; 1, \beta),$$

where $A_\alpha = (2^{1-\alpha} - 1)$, $A_\beta = (2^{1-\beta} - 1)$, $\alpha \neq \beta$, $\alpha, \beta > 0$.

(IV) *Uniformity*:

$$H_n(1/n, \ldots, 1/n; w_1, \ldots, w_n; \alpha, \beta) = \sum_{k=1}^{n} (w_k/n)\, a(n; \alpha, \beta).$$

(V) *Normality and Decisivity*:

$$a(2; \alpha, \beta) = 1; \; H_2(1, 0; w_1, w_2; \alpha, \beta) = 0.$$

Theorem. *The function satisfying the above axioms* (I)–(V) *is given by*

$$(2.1) \qquad H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{k=1}^{n} w_k(p_k^\alpha - p_k^\beta),$$

where $A_\alpha = (2^{1-\alpha} - 1)$, $A_\beta = (2^{1-\beta} - 1)$, $\alpha \neq \beta$, $\alpha, \beta > 0$.

Before proving the theorem, we shall prove some intermediate results.

Result 1. If $q_j \geqq 0$, $j = 1, 2, \ldots, m$, $\sum_{j=1}^{m} q_j = p_k > 0$ and $w_k = \sum_{j=1}^{m} q_j u_j / \sum_{j=1}^{m} q_j$, then

$$(2.2) \qquad H_{n+m-1}(p_1, \ldots, p_{k-1}, q_1, \ldots, q_m, p_{k+1}, \ldots, p_n; w_1, \ldots, w_{k-1}, u_1, \ldots, u_m, w_{k+1}, \ldots, w_n; \alpha, \beta) = H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} p_k^\alpha H_m(q_1/p_k, \ldots, q_m/p_k; u_1, \ldots, u_m; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} p_k^\beta H_m(q_1/p_k, \ldots, q_m/p_k; u_1, \ldots, u_m; 1, \beta).$$

This result can be easily proved by induction (cf. [4]).

Result 2. If

$$q_{kj} \geqq 0, \quad j = 1, 2, \ldots, m_k, \quad \sum_{j=1}^{m_k} q_{kj} = p_k > 0, \quad k = 1, 2, \ldots, n,$$

$$\sum_{k=1}^{n} p_k = 1, \quad w_k = \frac{q_{k1} u_{k1} + \ldots + q_{km_k} u_{km_k}}{p_k},$$

then

$$(2.3) \qquad H_{m_1 + \ldots + m_n}(q_{11}, q_{12}, \ldots, q_{1m_1}; \ldots; q_{n1}, q_{n2}, \ldots, q_{nm_n}; u_{11}, u_{12}, \ldots, u_{1m_1}; \ldots; u_{n1}, u_{n2}, \ldots, u_{nm_n}; \alpha, \beta) = H_n(p_1, \ldots, p_n; w_1, \ldots, w_n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^{n} p_k^\alpha H_{m_k}(q_{k1}/p_k, \ldots, q_{km_k}/p_k; u_{k1}, \ldots, u_{km_k}; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^{n} p_k^\beta H_{m_k}(q_{k1}/p_k, \ldots, q_{km_k}/p_k; u_{k1}, \ldots, u_{km_k}; 1, \beta).$$

Proof follows from Axiom (III).

Result 3.

$$(2.4) \qquad a(n; \alpha, \beta) = \frac{A_\alpha}{A_\alpha - A_\beta} a(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} a(n; 1, \beta),$$

where

$$(2.5) \qquad a(n; \alpha, 1) = A_\alpha^{-1}(n^{1-\alpha} - 1), \quad \alpha \neq 1,$$

and

$$(2.6) \qquad a(n; 1, \beta) = A_\beta^{-1}(n^{1-\beta} - 1), \quad \beta \neq 1.$$

Proof. In (2.3), put $m_1 = m_2 = \dots = m_n = m$ and $q_{kj} = 1/nm$, $k = 1, 2, \dots, n$; $j = 1, 2, \dots, m$, then we get $p_k = 1/n$ and $w_k = \sum_{j=1}^{m} u_{kj}/m$, $k = 1, 2, \dots, n$, hence

$$(2.7) \qquad H_{nm}(1/nm, \dots, 1/nm; u_{11}, u_{12}, \dots, u_{nm}; \alpha, \beta) =$$
$$= H_n(1/n, \dots, 1/n; w_1, \dots, w_n; \alpha, \beta) +$$
$$+ \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^{n} (1/n)^\alpha H_m(1/m, \dots, 1/m; u_{k1}, \dots, u_{km}; \alpha, 1) +$$
$$+ \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^{n} (1/n)^\beta H_m(1/m, \dots, 1/m; u_{k1}, \dots, u_{km}; 1, \beta) .$$

By Axiom (IV), (2.7) reduces to

$$(2.8) \qquad a(nm; \alpha, \beta) \sum_{k=1}^{n} \sum_{j=1}^{m} \frac{u_{kj}}{nm} = a(n; \alpha, \beta) \sum_{k=1}^{n} \frac{w_k}{n} +$$
$$+ \frac{A_\alpha}{A_\alpha - A_\beta} \sum_{k=1}^{n} (1/n)^\alpha a(m; \alpha, 1) \sum_{j=1}^{m} u_{kj}/m +$$
$$+ \frac{A_\beta}{A_\beta - A_\alpha} \sum_{k=1}^{n} (1/n)^\beta a(m; 1, \beta) \sum_{j=1}^{m} u_{kj}/m$$

or

$$a(nm; \alpha, \beta) = a(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/n)^{\alpha-1} a(m; \alpha, 1) +$$
$$+ \frac{A_\beta}{A_\beta - A_\alpha} (1/n)^{\beta-1} a(m; 1, \beta) .$$

By symmetry,

$$(2.9) \qquad a(nm; \alpha, \beta) = a(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/m)^{\alpha-1} a(n; \alpha, 1) +$$
$$+ \frac{A_\beta}{A_\beta - A_\alpha} (1/m)^{\beta-1} a(n; 1, \beta) .$$

Putting $m = 1$ in (2.9) and using Axioms (V) and (III), we get (2.4).

Also from (2.8) and (2.9), we have

$$(2.10) \quad a(m; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/m)^{\alpha-1} a(n; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} (1/m)^{\beta-1} a(n; 1, \beta) =$$
$$= a(n; \alpha, \beta) + \frac{A_\alpha}{A_\alpha - A_\beta} (1/n)^{\alpha-1} a(m; \alpha, 1) + \frac{A_\beta}{A_\beta - A_\alpha} (1/n)^{\beta-1} a(m; 1, \beta) .$$

The expression (2.10) together with (2.4) gives

$$(2.11)\qquad A_\alpha\{a(m;\alpha,1)[1-(1/n)^{\alpha-1}]+[(1/m)^{\alpha-1}-1]\,a(n;\alpha,1)\}=$$
$$=A_\beta\{[1-(1/n)^{\beta-1}]\,a(m;1,\beta)+[(1/m)^{\beta-1}-1]\,a(n;1,\beta)\}\,.$$

Put $n=2$ in (2.11), then

$$(2.12)\qquad A_\alpha\{(1-2^{1-\alpha})\,a(m;\alpha,1)-(1-m^{1-\alpha})=$$
$$=A_\beta\{(1-2^{1-\beta})\,a(m;1,\beta)-(1-m^{1-\beta})\}=C\ (\text{say})\,.$$

For $m=1$, (2.12) gives $C=0$, which immediately gives (2.5) and (2.6). (2.4) together with (2.5) and (2.6) gives

$$(2.13)\qquad a(n;\alpha,\beta)=\frac{A_\alpha}{A_\alpha-A_\beta}\,\frac{n^{1-\alpha}-1}{A_\alpha}+\frac{A_\beta}{A_\beta-A_\alpha}\,\frac{n^{1-\beta}-1}{A_\beta}=$$
$$=\frac{n^{1-\alpha}-n^{1-\beta}}{A_\alpha-A_\beta}\,,\quad \alpha\neq\beta\,.$$

Result 4.

$$(2.14)\qquad H_2(p_1,p_2;w_1,w_2;\alpha,\beta)=(2^{1-\alpha}-2^{1-\beta})^{-1}\sum_{k=1}^{2}w_k(p_k^\alpha-p_k^\beta)\,,\quad \alpha\neq\beta\,,$$

for all $p_1, p_2\in[0,1]$ with $p_1+p_2=1$.

Proof. In (2.3) set $n=2$, $m_1=r$, $m_2=s-r$, $q_{kj}=1/s$, $k=1,2$; $j=1,2,\ldots,m$,

$$p_1=\sum_{j=1}^{m_1}p_{kj}=\sum_{j=1}^{r}p_{kj}=\frac{r}{s}\,,\quad p_2=\frac{s-r}{s}\,,$$

$$w_1=\frac{1}{p_1}\sum_{j=1}^{m_1}q_{1j}u_{1j}=\frac{1}{r}\sum_{j=1}^{r}u_{1j}\,,\quad w_2=\frac{1}{s-r}\sum_{j=1}^{s-r}u_{2j}\,;$$

then

$$(2.15)\qquad H_s(1/s,\ldots,1/s;u_{11},\ldots,u_{1r},u_{21},\ldots,u_{2,s-r};\alpha,\beta)=$$
$$=H_2(p_1,p_2;w_1,w_2;\alpha,\beta)+$$
$$+\frac{A_\alpha}{A_\alpha-A_\beta}\sum_{k=1}^{2}p_k^\alpha H_{m_k}(q_{k1}/p_k,\ldots,q_{km_k}/p_k;u_{k1},\ldots,u_{km_k};\alpha,1)+$$
$$+\frac{A_\beta}{A_\beta-A_\alpha}\sum_{k=1}^{2}p_k^\beta H_{m_k}(q_{k1}/p_k,\ldots,q_{km_k}/p_k;u_{k1},\ldots,u_{km_k};1,\beta)=$$
$$=H_2(p_1,p_2;w_1,w_2;\alpha,\beta)+$$
$$+\frac{A_\alpha}{A_\alpha-A_\beta}[p_1^\alpha H_r(1/r,\ldots,1/r;u_{11},\ldots,u_{1r};\alpha,1)+$$

$$+ p_2^{\alpha} H_{s-r}(1/(s-r), \dots, 1/(s-r); u_{21}, \dots, u_{2,s-r}; \alpha, 1)] +$$

$$+ \frac{A_\beta}{A_\beta - A_\alpha} [p_1^{\beta} H_r(1/r, \dots, 1/r; u_{11}, \dots, u_{1r}; 1, \beta) +$$

$$+ p_2^{\beta} H_{s-r}(1/(s-r), \dots, 1/(s-r); u_{21}, \dots, u_{2,s-r}; 1, \beta)],$$

$$H_2(p_1, p_2; w_1, w_2; \alpha, \beta) = \frac{\sum_{j=1}^{r} u_{1j} + \sum_{j=1}^{s-r} u_{2j}}{s} a(s; \alpha, \beta) -$$

$$- \frac{A_\alpha}{A_\alpha - A_\beta} [p_1^{\alpha} \sum_{j=1}^{r} (u_{1j}/r)\, a(r; \alpha, 1) + p_2^{\alpha} \sum_{j=1}^{s-r} (u_{2j}/(s-r))\, a(s-r; \alpha, 1)] -$$

$$- \frac{A_\beta}{A_\beta - A_\alpha} [p_1^{\beta} \sum_{j=1}^{r} (u_{1j}/r)\, a(r; 1, \beta) + p_2^{\beta} \sum_{j=1}^{s-r} (u_{2j}/(s-r))\, a(s-r; 1, \beta)] =$$

$$= \frac{a(s; \alpha, \beta)}{s} [r w_1 + (s-r) w_2] -$$

$$- \frac{A_\alpha}{A_\alpha - A_\beta} [p_1^{\alpha} w_1 a(r; \alpha, 1) + p_2^{\alpha} w_2\, a(s-r; \alpha, 1)] -$$

$$- \frac{A_\beta}{A_\beta - A_\alpha} [p_1^{\beta} w_1 a(r; 1, \beta) + p_2^{\beta} w_2 a(s-r; 1, \beta)].$$

The expression (2.15) together with (2.4), (2.5) and (2.6) gives (2.14) for rationals. By Continuity Axiom (I), the above result is valid for all reals p_1 and $p_2 \in [0, 1]$. With the help of Axiom (III) it is easy to prove that

$$H_n(p_1, \dots, p_n;\ w_1, \dots, w_n; \alpha, \beta) = (A_\alpha - A_\beta)^{-1} \sum_{k=1}^{n} (p_k^{\alpha} - p_k^{\beta})\, w_k\,, \quad \alpha \neq \beta\,,$$

which completes the proof of Theorem.

3. PROPERTIES OF $H_n(P; W; \alpha, \beta)$

1. *Nonnegativity*: $H_n(P; W; \alpha, \beta) \geqq 0$ for $\alpha, \beta > 0$.
2. If $w_1 = w_2 = \dots = w_n = w$, then

$$H_n(P; W; \alpha, \beta) = w(2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^{n} (p_k^{\alpha} - p_k^{\beta}) = H_n(P; \alpha, \beta)\,,$$

which is an entropy of type (α, β) (cf. Sharma and Taneja [3], Taneja [4]).

3.

$$H_{n+1}(p_1, \dots, p_n, 0; w_1, \dots, w_n, w_{n+1}; \alpha, \beta) = H_n(p_1, \dots, p_n; w_1, \dots, w_n; \alpha, \beta)$$

whatever the weights $w_1, \dots, w_n, w_{n+1}$ and the probabilities $p_1, \dots, p_n$ are.

4. The measure $H_n(P; W; \alpha, \beta)$ is a convex $\cap$ function of the probability distribution $(p_1, \ldots, p_n)$, provided one of the parameters α and β (>0) is greater than unity and the other is less than or equal to unity, i.e., either $\alpha > 1$, $0 < \beta \leqq 1$ or $\beta > 1$, $0 < \alpha \leqq 1$ (cf. Sharma and Taneja [3], Taneja [4]).

5. *Linearity*: For $\alpha, \beta > 0$, the function $H_n(P; W; \alpha, \beta)$ is linear with respect to the utilities W, i.e., if $W' \in R^+$, $W'' \in R^+$, $\lambda_1 \in R^+$, $\lambda_2 \in R^+$, then $H_n(P; \lambda_1 W' + \lambda_2 W''; \alpha, \beta) = \lambda_1 H_n(P; W'; \alpha, \beta) + \lambda_2 H_n(P; W''; \alpha, \beta)$.

4. GENERALIZED MEASURES OF WEIGHTED ENTROPY

The measure $H(P; W)$ satisfies the additivity in the sense

$$H(P*Q; W*V) = \bar{V} H(P; W) + \bar{W} H(Q; V), \tag{4.1}$$

where $\bar{W} = \sum_{k=1}^{n} w_k p_k$, $\bar{V} = \sum_{j=1}^{m} v_j q_j$.

In general, regard $\bar{W}$ and $\bar{V}$ as functions of probabilities and utilities and consider the generalized additivity in the sense

$$H(P*Q; W*V) = G(Q; V)\, H(P; W) + G(P; W)\, H(Q; V), \tag{4.2}$$

where P, Q, W, V, $P*Q$ and $U*V$ have their usual meaning.

Write

$$H(P; W) = \sum_{k=1}^{n} h(p_k, w_k), \quad G(P; W) = \sum_{k=1}^{n} g(p_k, w_k), \tag{4.3}$$

where h and g are continuous real functions defined on the set $[0, 1] \times [0, 1]$.

The expressions (4.2) and (4.3) together give

$$\sum_{k=1}^{n} \sum_{j=1}^{m} h(p_k q_j, w_k v_j) = \sum_{k=1}^{n} \sum_{j=1}^{m} g(p_k, w_k)\, h(q_j, v_j) + \sum_{k=1}^{n} \sum_{j=1}^{m} g(q_j, v_j)\, h(p_k, w_k). \tag{4.4}$$

Real continuous solutions of the functional equation (4.4) (cf. Sharma and Gupta [2]) are the following three sets of solutions:

(i) $h(p, w) = p^\alpha w^\beta (c_1 \log p + c_2 \log w)$, $g(p, w) = p^\alpha w^\beta$,

(ii) $h(p, w) = (1/2k)\,(p^\alpha w^\beta - p^\gamma w^\delta)$, $g(p, w) = \frac{1}{2}(p^\alpha w^\beta + p^\alpha w^\beta)$, and

(iii) $h(p, w) = (1/R)\, p^\alpha w^\beta \sin(\gamma \log p + \delta \log w)$,

$g(p, w) = p^\alpha w^\beta \cos(\gamma \log p + \delta \operatorname{iog} w)$,

where $\alpha > 0$, $\beta \geqq 0$, γ, δ, c_1, c_2, k and R are real constants.

Now take $c_2 = 0$ in (i), $\beta = \delta$ in (ii), $\delta = 0$ in (iii), only for the function h, and introduce the boundary condition $h(1, \frac{1}{2}) = 1$. Then the above three sets of solutions with regard to (4.3) reduce to

$$(4.5) \qquad H_l(P; W) = -2^{\alpha-1} \sum_{k=1}^{n} p_k^\alpha w_k^\beta \log p_k ,$$

$$(4.6) \qquad H_p(P; W) = (2^{1-\alpha} - 2^{1-\beta})^{-1} \sum_{k=1}^{n} w_k^\beta (p_k^\alpha - p_k^\beta) , \quad \alpha \neq \gamma , \quad \alpha, \gamma > 0 ,$$

$$(4.7) \qquad H_s(P; W) = -\frac{2^{\alpha-1}}{\sin \gamma} \sum_{k=1}^{n} p_k^\alpha w_k^\beta \sin (\gamma \log p_k) , \quad \gamma \neq 0 .$$

The limiting case of both the measures (4.6) and (4.7) when $\gamma \to \alpha$ in (4.6) and $\gamma \to 0$ in (4.7) is the measure (4.5). Thus, (4.5) is the weighted entropy or useful information for $\alpha = \beta = 1$.

These measures (4.5), (4.6) and (4.7) are the measures parallel to that studied earlier by Sharma and Taneja [3]. A more detailed study of these measures will appear elsewhere.

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Souhrn

ENTROPIE TYPU (α, β) S VAHOU A JEJÍ ZOBECNĚNÍ

GUR DIAL, INDER JEET TANEJA

Belis a Guiasu studovali zobecnění Shannonovy entropie, tj. entropii s vahou nebo užitečnou entropii. V tomto článku je definována a charakterizována entropie typu (α, β) s vahou a jsou studovány její vlastnosti. Je pojednáno též o dalších zobecněních, která zahrnují entropii s vahou s více parametry.

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