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The Rothe method and time periodic solutions to the Navier-Stokes equations and equations of magnetohydrodynamics

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THE ROTHE METHOD AND TIME PERIODIC SOLUTIONS TO THE NAVIER-STOKES EQUATIONS AND EQUATIONS OF MAGNETOHYDRODYNAMICS

Dana Lauerová

Summary. The existence of a periodic solution of a nonlinear operator equation $z' + A_0 z + B_0 z = F$ is proved. The theory developed may be used to prove the existence of a periodic solution of the variational formulation of the Navier-Stokes equations or the equations of magnetohydrodynamics. The proof of the main existence theorem is based on Rothe method in combination with the Galerkin method, using the Brouwer fixed point theorem.

Keywords: Navier-Stokes equations, periodic solutions, existence of generalized solutions.

AMS Classifications: 65N40, 35Q10.

1. INTRODUCTION

In this paper Rothe method is applied to prove the existence of a periodic solution of a nonlinear operator equation

$$(1) \qquad u' + A_0 u + B_0 u = F,$$

where A_0 is a linear differential operator continuous as an operator from the separable Hilbert space $L_2(0, T; V)$ into his dual $L_2(0, T; V')$, B_0 is a nonlinear differential operator of a special type operating on $L_2(0, T; V)$ and $F \in L_\infty(0, T; V')$.

Special properties of the nonlinear operator B_0 arise as a consequence of the fact that the equation (1) represents a general form of the variational formulation of the Navier-Stokes equations or the equations of magnetohydrodynamics. In particular, B_0 does not fulfil the condition of monotonicity, therefore it is not possible to use Rothe method in such a way as it was done e.g. in [1−4]. The most similar problems to that given by (1) were solved by R. Temam in [5] but without periodicity assumptions.

In solving our problem we first apply time-discretization to the equation (1) (with a partition of interval $\langle 0, T \rangle$ in N subintervals of length $h = T/N$) and then we look for a solution of the m-dimensional approximation of this semi-discretized problem

(the principle of the Galerkin method). A solution of the equation (1) is then obtained by simultaneous limiting processes $m \to \infty$ and $h = h_m \to 0$.

2. DEFINITIONS AND NOTATION

Let two real separable Hilbert spaces V, H be given, V being compactly and densely embedded in H. Identifying H with its dual H', we may write $V \subseteq H \subseteq V'$. The inner products in H and V are denoted by $(\cdot, \cdot)$ and $[\![\cdot, \cdot]\!]$, and the norms induced by these products are denoted by $|\cdot|$, and $[\![\cdot]\!]$, respectively. Further, let a real separable Hilbert space V_s be given, $s \in \mathbb{N}$ (s fixed), with an inner product $[\![\cdot, \cdot]\!]_s$ and the norm $[\![\cdot]\!]_s$, such that V_s is continuously and densely embedded in V. The dual pairing between V and V' as well as between V_s and V'_s is denoted by $\langle \cdot, \cdot \rangle$.

In the sequel, B-spaces $L_2(0, T; X)$, $L_\infty(0, T; X)$ will be used, where X is a B-space; their definitions may be found e.g. in [3].

We shall denote by $\mathscr{D}'(0, T; V'_s)$ the space of distributions on $\langle 0, T \rangle$ with values in V'_s. We shall understand under the derivative z' of a function $z \in L_2(0, T; V) \subset \subset \mathscr{D}'(0, T; V'_s)$ the derivative in the sense of distributions such that $z' \in L_2(0, T; V'_s)$.

Let the operator $A_0: V \to V'$ be defined by the relation

(2) $\qquad \langle A_0 z, \tilde{z} \rangle = [\![z, \tilde{z}]\!]$, $z, \tilde{z} \in V$.

A_0 is a linear continuous operator on V.

Let the operator $B_0: V \to V'_s$ be defined by the relation

(3) $\qquad B_0 z = \tilde{B}_0(z, z)$,

where $\tilde{B}_0: H \times V \to V'_s$ is a bilinear continuous operator such that

(4) $\qquad \langle \tilde{B}_0(z, \tilde{z}), \hat{z} \rangle = -\langle \tilde{B}_0(z, \hat{z}), \tilde{z} \rangle$, $z, \tilde{z} \in V$, $\hat{z} \in V_s$

and

(5) $\qquad \langle \tilde{B}_0(z, \tilde{z}), \hat{z} \rangle \leqq c |z| \, [\![\tilde{z}]\!] \, [\![\hat{z}]\!]_s$, $z \in H$, $\tilde{z} \in V$, $\hat{z} \in V_s$

hold (c is positive constant).

It follows from (4) that

(6) $\qquad \langle B_0 z, z \rangle = 0$, $z \in V_s$.

We shall assume the operators A_0, B_0 to be defined on $L_2(0, T; V)$ by the relations

$\qquad (A_0 z)(t) = A_0 z(t)$

$\qquad (B_0 z)(t) = B_0 z(t)$

for a.e. $t \in \langle 0, T \rangle$.

Analogously, the operator $\tilde{B}_0$ is supposed to be defined on $L_2(0, T; H) \times \times L_2(0, T; V)$ by the relation

$$\left(\tilde{B}_0(z, \hat{z})\right)(t) = \tilde{B}_0(z(t), \hat{z}(t))$$

for a.e. $t \in \langle 0, T \rangle$.

3. EXISTENCE THEOREM

Theorem 1. *Let a function $F \in L_\infty(0, T; V')$ be given. Then there exists at least one function z satisfying the relations*

(7) $\qquad z \in L_2(0, T; V) \cap L_\alpha(0, T; H)$,

$\qquad\qquad z' \in L_2(0, T; V'_s)$,

(8) $\qquad z' + A_0 z + B_0 z = F$,

(9) $\qquad z(0) = z(T)$.

Proof. We shall apply Rothe method in the following way: Let $\{t_p\}_{p=1}^N$ be a uniform partition of $\langle 0, T \rangle$, $h = T/N$, $N \in \mathbb{N}$, $t_p = ph$. Semidiscretizing (8) we get a system of N equations

(10) $\qquad \dfrac{z^p - z^{p-1}}{h} + A_0 z^p + B_0 z^p = F^p$, $\quad p = 1, \dots, N$,

where $z^p = z(t_p)$,

$$F^p = \frac{1}{h} \int_{I_p} F(t) \, \mathrm{d}t \, , \quad I_p = (t_{p-1}, t_p\rangle, \quad p = 1, \dots, N \, .$$

It is evident that

(11) $\qquad \|F^p\|_{V'} \leqq \|F\|_{L_\infty(0,T;V')}$.

We shall use the Galerkin method to solve (10).

The space V_s being compactly embedded in H, there exists an orthonormal base $\{\omega_i\}_{i=1}^\infty$ in V_s and the set of numbers $\gamma_i > 0$, $i = 1, 2, \dots$ such that

(12) $\qquad [\![\omega_i, v]\!]_s = \gamma_i(\omega_i, v)$

for every $v \in V_s$.

For every $m \in \mathbb{N}$ we look for a sequence $\{z_m^p\}$, $p = 0, 1, \dots, N$ of the form

(13) $\qquad z_m^p = \sum_{i=1}^m \xi_i^p \omega_i$

so that the following equations are satisfied:

(14) $$\left(\frac{z_m^p - z_m^{p-1}}{h}, \omega_j\right) + [\![z_m^p, \omega_j]\!] + \langle B_0 z_m^p, \omega_j \rangle =$$
$$= \langle F^p, \omega_j \rangle, \quad j = 1, \dots, m,$$

(15) $$z_m^0 = z_m^N.$$

To prove the existence of a solution of (14) and (15) we formulate the next two lemmas:

Lemma 1. *There exists a number $R > 0$ independent of m, h, p and such that the following assertion holds: If $z_m^{p-1} \in B_m(0, R)$ for some $p \in \{1, \dots, N\}$ $(B_m(0, R)$ denotes the closed ball with center 0 and radius R in the space* lin $\{\omega_i\}_{i=1}^m$ *with the norm $|\cdot|$), and if z_m^{p-1}, z_m^p satisfy the equation* (14), *then $z_m^p \in B_m(0, R)$ as well.*

Proof of Lemma 1. Using (6) we obtain from (14) $(\varepsilon > 0)$

(16) $$|z_m^p|^2 + h[\![z_m^p]\!]^2 \leqq \frac{|z_m^{p-1}|^2}{2} + \frac{|z_m^p|^2}{2} + \frac{h\|F^p\|_{V'}^2}{2\varepsilon} + \frac{\varepsilon h [\![z_m^p]\!]^2}{2}.$$

For $0 < \varepsilon < 2$ and R such that

(17) $$R \geqq c_1 \|F\|_{L_\infty(0,T;V')} \sqrt{\left(\frac{1}{\varepsilon(2-\varepsilon)}\right)}$$

we get easily from (16) that the desired implication

(18) $$|z_m^{p-1}| \leqq R \Rightarrow |z_m^p| \leqq R$$

holds $(c_1$ denotes the constant of the embedding of V in H).

Lemma 2. *Let R satisfy* (17). *Then for every $m \in \mathbb{N}$ there exists $h = h_m$ such that the following assertion holds: If $z_m^{p-1} \in B_m(0, R)$ then there exists a unique function $z_m^p \in B_m(0, R)$ such that z_m^{p-1}, z_m^p satisfy the equation* (14) *(with $h = h_m$). Moreover, z_m^p depends continuously on z_m^{p-1}.*

To prove Lemma 2 we shall use the theorem on local existence of a solution of an equation depending on a parameter (Theorem 3.4.1, Chap. I, [6]) which we present here as Lemma 3:

Lemma 3. *Let $(X_1, d_1), (X_2, d_2)$ be two complete metric spaces, let $x_0 \in X_1$, $p_0 \in X_2, \alpha, \beta > 0, 0 \leqq \lambda < 1$ and suppose that*

  (i) $G: B(x_0, \alpha; X_1) \times B(p_0, \beta; X_2) \to X_1$ *is continuous;*

  (ii) $d_1(G(x_1, p), G(x_2, p)) \leqq \lambda\, d_1(x_1, x_2)$ *for $x_1, x_2 \in B(x_0, \alpha; X_1)$ and $p \in B(p_0, \beta: X_2)$;*

  (iii) $d_1(G(x_0, p), x_0) < \alpha(1 - \lambda)$ *for $p \in B(p_0, \beta; X_2)$.*

Then the equation $x = G(x, p)$ *has a unique solution* $x = x^*(p) \in B(x_0, \alpha; X_1)$ *for any* $p \in B(p_0, \beta; X_2)$. *Moreover,* $x^*: B(p_0, \beta; X_2) \to B(x_0, \alpha; X_1)$ *is continuous.*

Proof of Lemma 2. Using (13) we can rewrite the equation (14) in the form

$$(19) \qquad \xi^p = \xi^{p-1} + hA^{-1}\left[\varphi^p - \hat{A}\xi^p - B(\xi^p)\,\xi^p\right]$$

where ξ^p and φ^p are vectors, the coordinates of which are respectively ξ_i^p and (F^p, ω_i), $i = 1, \ldots, m$, while $\hat{A}$, A and $B(\xi^p)$ are matrices, the elements of which are respectively $(\hat{A})_{ij} = [\![\omega_j, \omega_i]\!]$, $(A)_{ij} = (\omega_j, \omega_i)$ and $B(\xi^p)_{ij} = \langle \tilde{B}_0(z_m^p, \omega_j), \omega_i \rangle$, $i = 1, \ldots, m$, $j = 1, \ldots, m$. To prove the existence of a solution of the equation (19) we use Lemma 3:

Let the mapping $G: \mathbb{R}^m \times \mathbb{R}^m \to \mathbb{R}^m$ be defined by the relation

$$(20) \qquad G(\xi^p, \xi^{p-1}) = \xi^{p-1} + hA^{-1}\left[\varphi^p - \hat{A}\xi^p - B(\xi^p)\,\xi^p\right].$$

Without going in details, we may ensure that the conditions (i)−(iii) of Lemma 3 be fulfilled by making h dependent on m. In particular, it can be easily shown that for every $m \in \mathbb{N}$ there exists $h = h_m$ sufficiently small $(h_m \to 0,\ m \to \infty)$ such that G defined by (20) satisfies the conditions (i)−(iii) of Lemma 3 (with $\alpha > \beta > 0$). Moreover, we choose (for every m) $h = h_m$ so that $h_m = T/N_m$, where $N_m \in \mathbb{N}$. Thus, the partition of the interval $\langle 0, T\rangle$ is made dependent on m.

For G defined by (20) the assertion of Lemma 3 can be formulated as follows:

For every $\xi^{p-1} \in B(0, \beta, \mathbb{R}^m)$ $(B(0, \beta, \mathbb{R})$ denotes the closed ball with center 0 and radius β in the space $\mathbb{R}^m)$ there exists one and only one $\xi^p \in B(0, \alpha, \mathbb{R}^m)$ $(\alpha > \beta)$ such that ξ^{p-1}, ξ^p satisfy (19) (with $h = h_m$) and, moreover, ξ^p depends continuously on ξ^{p-1}.

To interpret this result in the space of functions z_m^{p-1}, z_m^p, we choose β sufficiently large so that

$$B(0, R, \mathbb{R}_\gamma^m) \subset B(0, \beta, \mathbb{R}^m)$$

where $\mathbb{R}_\gamma^m$ denotes the space $\mathbb{R}^m$ with the norm $|\varrho|_{m,\gamma} = \sqrt{(\sum_{i=1}^{m}(\varrho_i^2/\gamma_i))}$, $(\gamma_i$ were defined in (12)).

Thus, $\xi^{p-1} \in B(0, \beta, \mathbb{R}^m)$ for $z_m^{p-1} \in B_m(0, R)$, and we may define a continuous mapping $\mathcal{T}_m^p$ $(m \in \mathbb{N},\ p \in \{1, \ldots, N_m\})$ in the following way:

$$\mathcal{T}_m^p: B_m(0, R) \to B_m(0, k\alpha)$$

$$z_m^{p-1} \mapsto z_p^m,$$

where k is the constant of the embedding of V_s in H, $\alpha > \beta > 0$, and z_m^p is the unique solution of the equation (14). From Lemma 1 we obtain that $z_m^p = \mathcal{T}_m^p(z_m^{p-1}) \in B_m(0, R)$. Thus, the mapping $\mathcal{T}_m^p$ maps the closed ball $B_m(0, R)$ continuously into itself. This completes the proof of Lemma 2.

Define a mapping

$$\mathscr{P}_m = \mathscr{T}_m^{N_m} \circ \ldots \circ \mathscr{T}_m^1$$

$$\mathscr{P}_m(z_m^0) = z_m^{N_m}$$

such that in the sequence

(21) $\qquad \{z_m^0, \ldots, z_m^{N_m}\}$

each two neighbouring elements satisfy the equation (14) (with $h = h_m$), each element is uniquely determined by its predecessor and depends on it continuously. $\mathscr{P}_m$ maps the closed ball $B_m(0, R)$ continuously into itself. From the Brouwer fixed point theorem we obtain the existence of a function z_{0m} such that $\mathscr{P}_m(z_{0m}) = z_{0m}$. We put $z_m^0 = z_{0m}$ and for this initial function there exists a unique sequence (21) with the properties mentioned above. The following estimate takes place:

(22) $\qquad |z_m^p| \leqq R, \quad p = 0, 1, \ldots, N_m, \quad m \in \mathbb{N}.$

4. APRIORI ESTIMATES AND LIMITING PROCESSES

Using the sequence (21) with $z_m^0 = z_{0m}$ we define functions

(23) $\qquad z_m(t) = z_m^p, \quad t \in I_p, \quad I_p = ((p - 1) h_m, p h_m\rangle, \quad p = 1, \ldots, N_m,$

$\qquad z_m(0) = z_m^0,$

(24) $\qquad Z_m(t) = \left(\dfrac{t - (p - 1) h_m}{h_m}\right) z_m^p + \left(\dfrac{p h_m - t}{h_m}\right) z_m^{p-1},$

$\qquad t \in I_p, \quad p = 1, \ldots, N_m,$

$\qquad Z_m(0) = z_m^0.$

Further, we define

$$F_m(t) = F^p, \quad t \in I_p, \quad p = 1, \ldots, N_m.$$

In the following lemma standard methods are used to prove some of the apriori estimates (cf. e.g. $[4]$, $[5]$).

Lemma 4. *The sequences of functions $\{z_m\}_{m=1}^{\infty}$, $\{Z_m\}_{m=1}^{\infty}$ are bounded in the space $L_2(0, T; V) \cap L_\infty(0, T; H)$. Moreover,*

(25) $\qquad z_m - Z_m \to 0 \quad in \quad L_2(0, T; H), \quad m \to \infty.$

Proof. From (22) we immediately obtain

(26) $\qquad \|z_m\|_{L_\infty(0,T;H)} \leqq R$

(27) $\qquad \|Z_m\|_{L_\infty(0,T;H)} \leqq R$

The equation (14) implies

$$\sum_{p=1}^{N_m}\left(|z_m^p|^2 - |z_m^{p-1}|^2 + |z_m^p - z_m^{p-1}|^2\right) + 2h_m\sum_{p=1}^{N_m}[\![z_m^p]\!]^2 \leq$$

$$\leq \sum_{p=1}^{N_m} h_m\left(\|F^p\|_{V'}^2 + [\![z_m^p]\!]^2\right).$$

Using (11) and the periodicity condition (15), we conclude that

(28) $$\sum_{p=1}^{N_m}|z_m^p - z_m^{p-1}|^2 \leq d_1,$$

(29) $$\|z_m\|_{L_2(0,T;V)}^2 = \sum_{p=1}^{N_m} h_m[\![z_m^p]\!]^2 \leq d_1,$$

(30) $$\|Z_m\|_{L_2(0,T;V)}^2 \leq 2\sum_{p=1}^{N_m} h_m[\![z_m^p]\!]^2 \leq 2d_1$$

where $d_1 = T\|F\|_{L_\infty(0,T;V')}^2$.

The condition (25) is easily proved if we notice that

$$z_m(t) - Z_m(t) = \frac{ph_m - t}{h_m}\left(z_m^p - z_m^{p-1}\right), \quad t \in I_p,$$

and that (28) holds.

Lemma 5. *The sequence $\{dZ_m/dt\}_{m=1}^\infty$ is bounded in the space $L_2(0, T; V_s')$.*

Proof. The proof of this lemma is based on a special choice of the base $\{\omega_i\}_{i=1}^\infty$ in the space V_s (relation (12)): For the functions (23), (24) the equation (14) is of the form

(31) $$\left(\frac{dZ_m(t)}{dt}, \omega_j\right) + [\![z_m(t), \omega_j]\!] + \langle\tilde{B}_0(z_m(t), z_m(t)), \omega_j\rangle = \langle F_m(t), \omega_j\rangle,$$

for a.e. $t \in \langle 0, T\rangle$, $j = 1, \ldots, m$.

Define a projector

$$P_m: H \to \lin\{\omega_i\}_{i=1}^m,$$

(32) $$P_m(v) = \sum_{i=1}^m \gamma_i(v, \omega_i)\,\omega_i.$$

For $v \in V_s$ we have from (31)

(33) $$\left(\frac{dZ_m(t)}{dt}, v\right) = \langle F_m(t), P_m v\rangle - [\![z_m(t), P_m v]\!] - \langle\tilde{B}_0(z_m(t), z_m(t)), P_m v\rangle.$$

It follows from (32) that

(34) $[\![P_m v]\!]_s \leq [\![v]\!]_s$

and, consequently,

(35) $[\![P_m v]\!] \leq c_3 [\![v]\!]_s$.

Thus, from (33) we get

$$|(Z'_m(t), v)|^2 \leq \tilde{c}\{\|F_m(t)\|^2_{V'} + [\![z_m(t)]\!]^2 + |z_m(t)|^2 [\![z_m(t)]\!]^2\} [\![v]\!]^2_s$$

and this implies, by virtue of (11), (26) and (29), the assertion of Lemma 5.

Lemmas 4 and 5 enable us to assert that there exist subsequences $\{z_\mu\} \subset \{z_m\}$, $\{Z_\mu\} \subset \{Z_m\}$ such that

(36) $z_\mu \rightharpoonup z$ (weak convergence in $L_2(0, T; V)$ and

$*$-weak convergence in $L_\infty(0, T; H)$),

(37) $Z_\mu \rightharpoonup Z$ (weak convergence in $L_2(0, T; V)$ and

$*$-weak convergence in $L_\infty(0, T; H)$),

(38) $Z'_\mu \rightharpoonup Z'$ (weak convergence in $L_2(0, T; V'_s)$).

The theorem on compact embedding (Theorem 5.1, Chap. I, [7]) implies that

(39) $Z_\mu \to Z$ strongly in $L_2(0, T; H)$.

From (25) and (36)−(39) it follows that

(40) $z = Z$

and

(41) $Z'_\mu \rightharpoonup z'$ (weak convergence in $L_2(0, T; V'_s)$).

From the definition of A_0 we have

(42) $A_0 z_\mu \rightharpoonup A_0 z$ (weak convergence in $L_2(0, T; V')$).

Lemma 6. *There exists a subsequence of* $\tilde{B}_0(z_\mu, z_\mu)$ *that converges weakly to* $\tilde{B}_0(z, z)$ *in* $L_2(0, T; V'_s)$.

Proof. Using the properties of $\tilde{B}_0$ and the fact that $z_\mu \to z$ in $L_2(0, T; H)$, it can be easily proved that

(43) $\langle \tilde{B}_0(z_\mu, z_\mu), v \rangle \to \langle \tilde{B}_0(z, z), v \rangle$ for $v \in L_\infty(0, T; V_s)$.

Since (26) and (29) hold, the sequence $\tilde{B}_0(z_\mu, z_\mu)$ is bounded in $L_2(0, T; V'_s)$. Thus, there exists a subsequence of $\tilde{B}_0(z_\mu, z_\mu)$ that converges weakly in the space $L_2(0, T; V'_s)$. For simplicity it will be denoted by $\tilde{B}_0(z_\mu, z_\mu)$ again. By virtue of (43) the assertion of Lemma 6 is proved.

Lemma 7. *The sequence* $\{F_\mu\}$ *converges to* F *in* $L_2(0, T; V')$.

The proof of this lemma is quite analogous to that of Lemma 4.9, Chap. III, [5], therefore we do not present it here.

The system of equations (31) can be rewritten in a form suitable for the limiting process:

$$(44) \qquad \left(\frac{dZ_\mu(t)}{dt}, \omega_j\right) + \langle A_0\, z_\mu(t), \omega_j\rangle + \langle \tilde{B}_0(z_\mu(t), z_\mu(t)), \omega_j\rangle =$$

$$= \langle F_\mu(t), \omega_j\rangle, \quad j = 1, \ldots, \mu.$$

Let $\{g_i\}_{i=1}^\infty$ denote the base in $L_2(0, T)$. From (44), using Lemmas 6 and 7 and the relations (41) and (42), we obtain by the limiting process $\mu \to \infty$, for every $i, j \in \mathbb{N}$, the equation

$$(45) \qquad \langle z', g_i w_j\rangle_t + \langle A_0 z, g_i w_j\rangle_t + \langle B_0 z, g_i w_j\rangle_t = \langle F, g_i w_j\rangle_t,$$

where $\langle\cdot, \cdot\rangle_t$ denotes the dual pairing between $L_2(0, T; V_s)$ and $L_2(0, T; V_s')$. This implies that (8) is fulfilled. The periodicity condition (9) is also satisfied since it follows from (37), (40) and (41) that

$$(46) \qquad Z_\mu(t) \rightharpoonup z(t) \quad \text{(weak convergence in } V_s'\text{) for every } t \in \langle 0, T\rangle,$$

and, simultaneously,

$$Z_\mu(0) = Z_\mu(T) \quad \text{for all } \mu.$$

This completes the proof of Theorem 1.

Remark. Specifying the operators A_0, B_0 and the spaces V_s, V and H, we can use Theorem 1 to prove the existence of a periodic solution of a variational formulation of the Navier-Stokes equations and the equations of magnetohydrodynamics, for details see [8].

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Souhrn

ROTHEHO METODA A PERIODICKÉ ŘEŠENÍ NAVIER-STOKESOVÝCH ROVNIC A ROVNIC MAGNETICKÉ HYDRODYNAMIKY

DANA LAUEROVÁ

V článku je dokázána existence periodického řešení nelineární operátorové rovnice $z' + A_0 z + B_0 z = F$, která vzniká zobecněním variační formulace úlohy Navier-Stokesových rovnic nebo rovnic magnetické hydrodynamiky. Vhodným spojením Rotheho metody, Galerkinovy metody a Brouwerovy věty je dokázána hlavní existenční věta.

Резюме

МЕТОД РОТЕ И ПЕРИОДИЧЕСКИЕ РЕШЕНИЯ УРАВНЕНИЙ НАВИЕРА-СТОКСА И УРАВНЕНИЙ МАГНЕТОГИДРОДИНАМИКИ

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В статье доказывается существование периодического решения нелинейного операторного уравнения $z' + A_0 z + B_0 z = F$. Сформулированную теорему существования можно непосредственно применить к доказательству существования периодического решения вариационной формулировки уравнений Навиера-Стокса или уравнений магнетогидродинамики. Главным методом доказательства является метод Роте в комбинации с методом Галеркина при использовании теоремы Броуэра.

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