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A remark to the paper of M. Froda-Schechter: Préordres et équivalences dans l'ensemble des familles d'un ensemble

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The aim of this remark is to deepen the knowledge about the lattice of all classes $\mathcal{R}_e(\mathcal{L})$ from the preceding paper [1]. We use the notation introduced in [1]. Cardinal and ordinal operations with ordinal types are denoted as in [2].

Let $G$ be a partially ordered set. If $X \subseteq G$, $x \in X$, $x \leq y \Rightarrow y \in X$ then $X$ is an end of $G$. The set of all ends is denoted by $\mathcal{E}(G)$. $\mathcal{E}(G)$ is supposed to be ordered by inclusion, i.e. $X, Y \in \mathcal{E}(G), X \subseteq Y \Rightarrow X \subseteq Y$. Now, we shall deal with type of $\mathcal{E}(G)$. Let $f$ be an isotonic mapping of $G$ into $\{0, 1\}$, $0 < 1$. The set of all $g \in G$, for which $f(g) = 1$ is an end. On the other hand, if $X$ is an end and $h(x) = 1$ for $x \in X$, $h(x) = 0$ for $x \in X$, then $h$ is an isotonic mapping of $G$ into $\{0, 1\}$. Hence we get immediately that the ordinal type of $\mathcal{E}(G)$ is $2^\gamma$, where $\gamma$ is an ordinal type of $G$ and $2$ is an ordinal type of $\{0, 1\}$. This result can be also easily obtained from general considerations in [3] (theorem 5.4). Especially, if $\mathcal{P}(E)$ is ordered by means of inclusion, $\varepsilon$ the type of an antichain with cardinal number $\text{card} \ E$, the ordinal type of $\mathcal{E}(\mathcal{P}(E))$ is $2^{2\varepsilon}$. In following, we put $\mathcal{E} = \mathcal{E}(\mathcal{P}(E))$.

Put $\mathcal{R}_e = \{\mathcal{R}_e(\mathcal{L}) : \mathcal{L} \subseteq \mathcal{P}(E)\}$ and order $\mathcal{R}_e$ by (D 10) from § 5 in [1]. According to (3.2) in [1], $\mathcal{M}_e(\mathcal{L}) = \{M \in \mathcal{P}(E) : \exists L \subseteq M\}$ is the greatest element in $\mathcal{R}_e(\mathcal{L})$. Clearly $\mathcal{M}_e(\mathcal{L}) \in \mathcal{E}$. If $\mathcal{L} \in \mathcal{E}$, then $\mathcal{M}_e(\mathcal{L}) = \mathcal{L}$. Thus a mapping $f$ which maps $\mathcal{R}_e(\mathcal{L})$ on $\mathcal{M}_e(\mathcal{L})$ is an one-to-one mapping of $\mathcal{R}_e$ on $\mathcal{E}$.

Let $\mathcal{R}_e(\mathcal{L}_1) < \mathcal{R}_e(\mathcal{L}_2)$. Then there exist $\mathcal{L}_1 \in \mathcal{R}_e(\mathcal{L}_1)$ and $\mathcal{L}_2 \in \mathcal{R}_e(\mathcal{L}_2)$ such that $\mathcal{L}_1 \subseteq \mathcal{L}_2$. Thus $\mathcal{M}_e(\mathcal{L}_1) = \mathcal{M}_e(\mathcal{L}_1) \supseteq \mathcal{M}_e(\mathcal{L}_2) = \mathcal{M}_e(\mathcal{L}_2)$.

On the contrary if $\mathcal{L}_1, \mathcal{L}_2 \in \mathcal{E}$, $\mathcal{L}_1 \subseteq \mathcal{L}_2$ it is $\mathcal{R}_e(\mathcal{L}_2) < \mathcal{R}_e(\mathcal{L}_1)$.

This implies that $f$ is an antiisomorphism.

By [2] I, § 7 it is $\tilde{\alpha}^\varepsilon = \tilde{\alpha}^\varepsilon$, where $\tilde{\alpha}$ is an ordinal type of a set which is antiisomorphic to a set of the type $\alpha$. Thus

$$(a) \quad 2^{2\varepsilon} = 2^{\varepsilon}.$$
Hence we get

*The ordinal type of $\mathcal{R}_e$ is $2^\omega$.*

From (a) it follows that the following assertion may be added to (5.3) in [1].

*The set of all classes e-superior is a lattice which is isomorphic to $\mathcal{R}_e$."

**REFERENCES**

