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Commentationes Mathematicae Universitatis Carolinae, Vol. 3 (1962), No. 1, 31

Persistent URL: http://dml.cz/dmlcz/104906

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3, 1 (1962)

REMARK ON INTEGRATION IN COMPACT METRIC SPACES

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In [1] the following theorem was proved:

Let μ be a finite Borel measure on a compact metric space X. Then there exist x_k , $k = 1,2,\ldots$, such that, for any continuous function f,

$$\int_X f\,d\mu = \mu(X) \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f(x_k) .$$

P.C. Baayen from Amsterdam mentioned in a letter that in theorems like this one there often could be proved more about the x_k , namely:

Theorem. Let X' be an arbitrary dense set in X . Then the points x_k in the theorem can be chosen from X' .

Proof. The assertion is a simple consequence of the following lemma.

Lemma. Let $x_k' \in X$, $k = 1,2,\ldots$, and let $\lim_{k\to\infty} d(x_k;x_k')=0$, where x_k are as in [1] and d is the metric in X . Then also

$$\int_X f\,d\mu = \mu(X) \lim_{n\to\infty} \frac{1}{n} \sum_{k=1}^{n} f(x_k'),$$

for any continuous function f .

Proof. Since every continuous function on a compact space is uniformly continuous, we have $\lim_{k\to\infty} (f(x_k) - f(x_k'))= 0$, for every continous f on X . The proof follows from the well known theorems about limits.

R e f e r e n c e:

[1] Z.HEDRLÍN, On integration in compact metric spaces, CMUC, 2,4 (1961).