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REMARK ON INTEGRATION IN COMPACT METRIC SPACES

Zdeněk HEDRLÍN , Praha

In [1] the following theorem was proved:

Let μ be a finite Borel measure on a compact metric space X . Then there exist x_k , $k = 1, 2, \dots$, such that, for any continuous function f ,

$$\int_X f d\mu = \mu(X) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(x_k).$$

P.C. Baayen from Amsterdam mentioned in a letter that in theorems like this one there often could be proved more about the x_k , namely:

Theorem. Let X' be an arbitrary dense set in X . Then the points x_k in the theorem can be chosen from X' .

Proof. The assertion is a simple consequence of the following lemma.

Lemma. Let $x'_k \in X$, $k = 1, 2, \dots$, and let $\lim_{k \rightarrow \infty} d(x_k; x'_k) = 0$, where x_k are as in [1] and d is the metric in X . Then also

$$\int_X f d\mu = \mu(X) \lim_{n \rightarrow \infty} \frac{1}{n} \sum_{k=1}^n f(x'_k),$$

for any continuous function f .

Proof. Since every continuous function on a compact space is uniformly continuous, we have $\lim_{k \rightarrow \infty} (f(x_k) - f(x'_k)) = 0$, for every continuous f on X . The proof follows from the well known theorems about limits.

R e f e r e n c e :

- [1] Z.HEDRLÍN, On integration in compact metric spaces, CMUC, 2,4 (1961).