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STABLE GLOBAL ATTRACTORS in E^2 x)

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Introduction. In [1] and [2] topological results for a point-set study of dynamical systems in E^2 are obtained and utilized in stability study. Incidentally, Theorem 2 of [1] does not require property $\mathcal{S}$ for we may obtain desired sections from the following: Let (X, π) be a dynamical system on a locally compact (Hausdorff) space X and let $X^* = X \cup \{\omega\}$, $\omega \notin X$, denote the one-point compactification of X. Then there is a dynamical system (X^*, π^*) on X^* with the property that $\pi^*(x, t) = \pi(x, t)$ for every $x \in X$ and every $t \in \mathcal{R}$.

Our results are in the notation of [3] and we recall, in particular, that if M is a (positive) stable attractor (positively asymptotically stable) which is compact in a dynamical system on a Hausdorff space, then for each x in the region of attraction, $A(M)$, which is not in M we must have $\Lambda^-(x) \cap A(M) = \phi$.

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Theorem. Let (E^2, π) be a dynamical system on Euclidean 2-space and let $M \subset E^2$ be compact, invariant, (positively) stable, and a (positive) global $(A(M) = E^2)$ attractor. For each $x \in \mathrm{Bd}\, M$ one of the following holds:

(i) x is a rest point;

(ii) $\gamma(x)$ is a simple closed curve;

(iii) $\Lambda^-(x)$ is a (non-empty) continuum of rest points.

Proof: Assume $x_0 \in \mathrm{Bd}\, M$ is such that x_0 is not a rest point and $\gamma(x_0)$ is not a simple closed curve. $\mathrm{Bd}\, M$ is compact and invariant, hence, $\Lambda^-(x_0) \neq \emptyset$ and $\Lambda^-(x_0) \subset \mathrm{Bd}\, M$. Suppose $p \in \Lambda^-(x_0)$ is not a rest point. Let Σ be a transversal at p with associated $\eta > 0$, i.e., $\pi(\Sigma \times \{t\}) \cap \Sigma = \emptyset$ for every $0 < |t| \leq \eta$. Now $\gamma^-(x_0)$ has a countable infinite number of intersections with Σ; say $\{x_k\}_{k=1}^{\infty}$ in order along $\gamma^-(x_0)$ where $x_k = \pi(x_0, \theta_k)$ and $0 > \theta_1 > \theta_2 > \ldots > \theta_n > \ldots$. By assumption $\gamma^-(x_0)$ is not the complete trajectory through x_0 and, therefore, $\{x_k\}_1^{\infty}$ has a unique limit point x_∞ in Σ and it is easy to see $x_\infty = p$.

Let C_k, $k = 1, 2, 3, \ldots$ denote the simple closed curve consisting of arcs of $\gamma^-(x_0)$ and Σ between x_k and x_{k+1}. Then $p \notin C_k$ for any k. Let G_k, $k = 1, 2, 3, \ldots$ be that component of $E^2 \setminus C_k$ which contains p. For every k, G_k is negatively invariant and unbounded. To see the latter, suppose for $k_0 \geq 1$, G_{k_0} is bounded. Since $p \in G_{k_0}$ and G_{k_0} is open in E^2, there is a $y \in G_{k_0} \cap (E^2 \setminus M)$ and $\overline{\gamma^-(y)} \subset \overline{G_{k_0}}$ is compact. Hence, $\Lambda^-(y) \neq \emptyset$ but this is impossible since y lies in $E^2 \setminus M$.

Finally, $x_0 \notin G_k$ for each $k \geq 1$. Denote the bounded component of $E^2 \setminus C_k$ by D_k. Then $x_0 \in D_1$, and there is a $y \in (E^2 \setminus M) \cap D_1$ such that $\Lambda^-(y) = \emptyset$, $\gamma^-(y) \cap G_k \neq \emptyset$ for each $k = 2n+1$, $n = 0, 1, 2, \dots$, and $\gamma^-(y) \cap C_k \neq \emptyset$ for each $k = 2n+1$, $n = 0, 1, 2, \dots$. This means $\gamma^-(y)$ has an infinite number of intersections with Σ and that $p \in \Lambda^-(y)$.

Again this is impossible since $y \in E^2 \setminus M$. Hence, if $p \in \Lambda^-(x_0)$, then p is a rest point. Since $x_0 \in \mathrm{Bd}\, M$; $\Lambda^-(x_0)$ is a (non-empty) continuum and the proof is complete.

Corollary. Under the same hypothesis as the theorem, if $\mathrm{Bd}\, M$ contains no rest points, M is topologically a closed 2-cell.

Proof: For each $x \in \mathrm{Bd}\, M$, $\gamma(x)$ is a simple closed curve by the theorem. But any continuum in the plane which is the disjoint union of (more than one) simple closed curves is topologically an annulus [4]. Therefore, $\mathrm{Bd}\, M$ is a simple closed curve. Let

D, C be components of $E^2 \setminus Bd\,M$ with C unbounded. Then $\overline{D} = M$ and the proof is complete.

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