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THE HEWITT REALCOMPACTIFICATION OF A PRODUCT

(Preliminary communication)

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This brief communication deals with the equality $v(P \times Q) = vP \times vQ$, where v is the Hewitt realcompactification (all spaces under consideration are uniformizable Hausdorff topological spaces). The results published below complete in a way those results due to W.W. Comfort and S. Negrepontis ([1], [2]) provided that measurable cardinals exist.

The symbol m_1 stands for the first measurable cardinal. By [3], a space P is said to be pseudo-m_1-compact if every locally finite disjoint family of open sets in P is of nonmeasurable cardinal.

Theorem 1. Let P be a discrete space. Then $v(P \times Q) = vP \times vQ$ if and only if either $\mathrm{card}\, P < m_1$ or $\mathrm{card}\, Q < m_1$.

Corollary. If P is not a pseudo-m_1-compact space and $\mathrm{card}\, Q \geq m_1$ then $v(P \times Q) \neq vP \times vQ$.

Theorem 2. Let P be a locally compact realcompact space. Then $v(P \times Q) = vP \times vQ$ if and only if either $card\ \mathcal{P} < m_1$ or $\mathcal{Q}$ is pseudo-m_1-compact.

The preceding result completes Theorem 2.2 from [2] : If $\mathcal{P}$ is a locally compact realcompact space of nonmeasurable cardinal, then $v(\mathcal{P} \times \mathcal{Q}) = v\mathcal{P} \times v\mathcal{Q}$ for each space $\mathcal{Q}$.

If we restrict ourselves to spaces of measurable cardinals, then Theorems 1 and 2 assert that (under the assumptions stated) $v(\mathcal{P} \times \mathcal{Q}) = v\mathcal{P} \times v\mathcal{Q}$ if and only if $\mathcal{P} \times \mathcal{Q}$ is pseudo-m_1-compact. In general, only one implication of this assertion is true:

<u>Theorem 3</u>. Let $card\ \mathcal{P} \geq m_1$ and $card\ \mathcal{Q} \geq m_1$. If $v(\mathcal{P} \times \mathcal{Q}) = v\mathcal{P} \times v\mathcal{Q}$ then $\mathcal{P} \times \mathcal{Q}$ is pseudo-m_1-compact.

<u>Corollary</u>. Let $\mathcal{P}$ be a locally compact realcompact space and $\mathcal{Q}$ be a pseudo-m_1-compact space. Then $\mathcal{P} \times \mathcal{Q}$ is pseudo-m_1-compact.

Unlike the Čech-Stone compactification where noncompact spaces $\mathcal{P}$ exist such that $\beta(\mathcal{P} \times \mathcal{Q}) = \beta\mathcal{P} \times \beta\mathcal{Q}$ for every compact space $\mathcal{Q}$, the situation does not hold any more (at least for nonmeasurable cardinals) if we replace compact by realcompact and β by v.

<u>Theorem 4</u>. If $\mathcal{P}$ is not realcompact and if $card\ \mathcal{P} < m_1$ then there is a realcompact space $\mathcal{Q}$ such that $v(\mathcal{P} \times \mathcal{Q}) \neq v\mathcal{P} \times \mathcal{Q}$.

The following theorem generalizes Theorem 4.5 from [2]. Its converse holds under conditions of a type, e.g., if $\mathcal{P} \times \mathcal{Q}$ is a k'-space.

Theorem 5. Let $\mathcal{P}$ be a k'-space and either $\upsilon\mathcal{Q}$ be locally compact or $\upsilon\mathcal{P} \times \upsilon\mathcal{Q}$ be a k'-space. If either $\mathcal{Q}$ is pseudo- m_1 -compact or every compact subset of $\mathcal{P}$ is of nonmeasurable cardinal and if either $\mathcal{P}$ is pseudo- m_1 -compact or every compact subset of $\upsilon\mathcal{Q}$ is of nonmeasurable cardinal, then $\upsilon(\mathcal{P} \times \mathcal{Q}) = \upsilon\mathcal{P} \times \upsilon\mathcal{Q}$.

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