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EVERY GROUP IS A MAXIMAL SUBGROUP OF THE SEMIGROUP OF
RELATIONS

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The aim of this note is to extend a result of [2], namely to prove the following theorem:

Theorem: The class of maximal subgroups of semigroups of binary relations includes all groups.

This generalizes [2], Theorem 4.7 to infinite groups.^{x)}
We preserve the notation of [2] and refer to the results proved there, too.

Concerning graphs we use the notation of [1].

Proof of the theorem: Let G be an infinite group (the proof for finite case would be similar; since the finite case is solved in [2], we make this assumption for sake of brevity). By [1], there is a graph (X, R) such that $C(X, R) \simeq G$, where $C(X, R)$ is the monoid of all compatible mappings (i.e. homomorphisms) into itself. By constructions given in [1], we can assume the following about the graph (X, R) :

x) Using a different method this generalization was obtained independently by A.H. Clifford, R.J. Plemmons and B.M. Schein.

a) $|X| = |R|$ (this follows from the fact that (X, R) can be chosen without isolated points).

b) Let $V(x) = \{y \mid (x, y) \in R\}$, then $x \neq y$ implies $V(x) \not\subseteq V(y)$ and $V(y) \not\subseteq V(x)$. Similarly for $\bar{V}(x) = \{y \mid (y, x) \in R\}$.

c) $V(x) \neq \emptyset$, $V(x) \neq X$ for every $x \in X$. Similarly for $\bar{V}(x)$.

Let $\varphi: X \rightarrow R$ be a bijection. Define the relation α on $X_{01} = X \times \{0, 1\}$ ($0, 1 \notin X$) by:

$$\begin{aligned} ((x, 0), (y, 0)) \in \alpha &\iff ((x, 1), (y, 1)) \in \alpha \iff x = y, \\ ((x, 0), (y, 1)) \in \alpha &\iff x \text{ is incident with } \varphi(y), \\ ((x, 1), (y, 0)) &\notin \alpha. \end{aligned}$$

By b), c), α is reduced. Further, α is idempotent as can be easily seen. Thus by Lemma 3.4 [2] (and by its remark), the maximal subgroup H_α of \mathcal{B}_X containing α is given by $H_\alpha \simeq G_\alpha = \{\varphi \in S_{X_{01}} \mid \exists \sigma \in S_{X_{01}} \alpha \varphi = \sigma \alpha\}$.

But in this special case we have $G_\alpha = \{\varphi \mid \alpha \varphi = \varphi \alpha\}$.

Similarly as in the proof of [2], Lemma 4.2,

$$G_\alpha \simeq \{\varphi \in S_X \mid \exists \sigma \in S_X, R\varphi = \sigma R\} = G_R.$$

But obviously $G_R = A(X, R) = C(X, R) \simeq G$, by the assumption ($A(X, R)$ is the group of all automorphisms of the graph (X, R)).

I thank to Z. Hedrlín, who turned my attention to the paper [2].

References

- [1] Z. HEDRLÍN and A. PULTR: Symmetric relations (undirected)

ted graphs) with given semigroups, Monatshefte für Math. 69(1965), 318-322.

- [2] J.S. MONTAGUE and R.J. PLEMMONS: Maximal subgroups of the semigroups of relations. J. of Algebra 13(1969), 575-587.

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