Pavel Pudlák

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(Preliminary communication)

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THE OBSERVATIONAL PREDICATE CALCULUS AND COMPLEXITY OF COMPUTATIONS
(Preliminary communication)
Pavel PUDLÁK, Praha

Abstract: A close connection between the languages nondeterministically recognizable in polynomial time and projectively definable classes of finite structures is shown. A hierarchy of projective classes of structures is introduced and studied.

Key words: Computational complexity, recognizability in polynomial time, projective (pseudoelementary) classes of finite structures.

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Introduction. By the classical observational predicate calculus we mean the predicate calculus with the usual syntax (predicates, function symbols, connectives, classical quantifiers) but with the semantics modified by allowing only finite models. A \textit{variety} of the type $t$ is a class $K$ of (finite) models closed under isomorphisms. $K$ is \textit{projective} if there is a sentence $\varphi$ of a richer type such that a model $M$ of the type $t$ is in $K$ iff it can be expanded to a model of $\varphi$. We say that $\varphi$ projectively defines $K$ (cf.[2],[3],[4]).

x) Sometimes, "pseudoelementary" is used instead of "projective".
In this paper, a close connection between the languages recognizable by the nondeterministic Turing automata working in a polynomial time and the projective varieties of finite models is shown. For this purpose, a hierarchy of projective varieties is introduced. Further, a result of S.A. Cook [1] about the mentioned languages is used to prove that the hierarchy of projective varieties is strictly increasing.

**Notation.** 1. The complexity of a sentence $\varphi$ is the number of the quantifiers contained in $\varphi$.

2. When we speak about recognizability of a variety $K$ of oriented graphs, we mean the recognizability of the codes of elements of $K$. The code of an oriented graph $<M,R>$ is a word of the length $|M|^2$ in an alphabet $A$, $|A| = 4$, in which the cardinality of $M$ and the incidence matrix is marked.

3. The code of a word $\alpha \in \{0,1\}^+$ in the variety of all graphs is every graph which is the union of a strict linear ordering on some set $M$, $|M| = |\alpha|$, and some loops which mark presence of 1 in $\alpha$. For an $L \subseteq \{0,1\}^+$ we denote by $\text{Cod}(L)$ the variety of codes of words contained in $L$.

**Definition.** By $\mathcal{L}_N(m^k)$ we denote the set of all languages in $\{0,1\}$ recognizable nondeterministically in the time $m^k$.

By $\mathcal{N}_k$ we denote the set of all varieties of graphs recognizable in time $m^k$.

By $\mathcal{G}_k$ we denote the set of all varieties of graphs
projectively definable by a sentence of complexity $k$.

Theorem 1. For every $k \geq 2$, $L \in \mathcal{F}_N(m^{2k})$ iff

$\text{Cod}(L) \in \mathcal{NP}_k$.

Theorem (S.A. Cook [1]). For every $1 \leq k < n$,

$\mathcal{F}_N(m^k) \not\subseteq \mathcal{F}_N(m^n)$.

Corollary 2. For every $2 \leq k < n$, $\mathcal{NP}_k \not\subseteq \mathcal{NP}_n$.

Theorem 3. For every $k \geq 2$, $\mathcal{NP}_k \subseteq \mathcal{NP}_{3/2,k} \subseteq \mathcal{NP}_{6k}$.

Corollary 4. A variety of graphs is projective iff it is recognizable in a polynomial time.

Corollary 5. For every $k \geq 2$, $\mathcal{NP}_k \not\subseteq \mathcal{NP}_{6k+1}$.

Lemma 6. For every $k \geq 2$, $n \geq 1$, $\mathcal{NP}_k = \mathcal{NP}_{k+1}$ implies $\mathcal{NP}_{n,k} = \mathcal{NP}_{n,(k+1)}$.

Lemma 7. The variety of complete graphs is of complexity 2 but not 1.

Corollary 8. For every $k \geq 1$, $\mathcal{NP}_k \not\subseteq \mathcal{NP}_{k+1}$.

Remark 9. The assertion of Corollary 8 holds for structures of any type $t$, whenever $t$ contains at least one predicate or function symbol of arity at least 2.

Corollary 4 is due to L. Lovász, D.S. Johnson and P. Gács [4]. The importance of this theorem is derived from the following consequence of it:

The class of all projective varieties is closed under complements iff the class of all languages recognizable nondeterministically in a polynomial time is closed under complements.
The original proof of this theorem in [4] uses methods different from our ones. Complete proofs are contained in the author's master thesis.

References


Matematicko-fyzikální fakulta
Karlová universita
Sokolovská 83, 18600 Praha 8
Československo

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