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Commentationes Mathematicae Universitatis Carolinae, Vol. 19 (1978), No. 1, 147--149

Persistent URL: http://dml.cz/dmlcz/105841

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REFLECTIVE MAC NEILLE COMPLETIONS OF FIBRE-SMALL CATEGORIES
NEED NOT BE FIBRE-SMALL
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Abstract: See Title

Key words: Initial completion, universal initial completion, Mac Neille completion, semi-topological functor, topologically-algebraic functor, fibre-smallness, strong fibre-smallness.

AMS: 18D30, 18A35 Ref. Z.: 2.726.23

Mac Neille completions have been defined in [2]. Categories having reflective Mac Neille completions, have been characterized by Wischnewsky and Tholen [8], Hoffmann [5], Adámek [1] and by Herrlich and Strecker [3] as those $(\mathbb{A},U)$, for which $U$ is semi-topological. Categories, having fibre-small Mac Neille completions, have been characterized by Adámek [1] and by Herrlich and Strecker [41 as those $(\mathbb{A},U)$, which are strongly fibre-small. The title statement provides a negative answer to a problem posed by Adámek [11, p. 22. The example is as follows.

Let $(\Omega,\preceq)$ be a large complete lattice. Let $X$ be the following category:

Objects: $X_0$, $B_\alpha$, $C_\alpha$, $D_\alpha$ for all $\alpha \in \Omega$

Morphisms:

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Composition is uniquely determined by the fact that morphism classes $\text{hom}(X,Y)$ contain at most one element.

Let $\mathcal{A}$ be the subcategory of $X$, obtained by removing $X_0$, $\text{id}_{X_0}$, all $r_\alpha$, $p_\alpha$, $q_\alpha$, $e_\alpha$, and all $h_{\alpha\beta}$ with $\beta > \alpha$, and let $U: \mathcal{A} \to X$ be the embedding functor. Then $U$ is not only semi-topological, but even topologically-algebraic in the sense of Y.H. Hong [7] and S.S. Hong [6], i.e. any $U$-source has some (generating, initial)-factorization

$$X \xrightarrow{f_i} UA_i = X \xrightarrow{g} UA \xrightarrow{U_{\text{mi}}} UA_i$$

as indicated by the following table:

<table>
<thead>
<tr>
<th>$f_i$</th>
<th>$UA_i$</th>
<th>$g$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1) $X = B_\alpha$ and ${f_i\mid i \in I} \cap {r_\alpha } \cup {h_{\alpha\beta} \mid \beta &gt; \alpha}$</td>
<td>$\phi$</td>
<td>$\text{id}<em>{B</em>\alpha}$</td>
</tr>
</tbody>
</table>
| (2) $X = B_\alpha$ and $\{f_i\mid i \in I\} \cap (\{r_\alpha \} \cup \{h_{\alpha\beta} \mid \beta > \alpha\}) \neq \phi$ | $r_\alpha$ | $

Hence, by [31], $(\mathcal{A}, U)$ has not only a reflective Mac Neille
completion but even a reflective universal initial completion. Since the $g_\beta : X_0 \rightarrow UD_\beta$ are pairwise non-equivalent semi-final $U$-morphisms, $(A, U)$ is not strongly fibre-small. Hence its Mac Neille completion is not fibre-small.

References


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(Oblatum 21.12. 1977)