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MAL’CEV CONDITIONS FOR CONGRUENCE-REGULAR AND CONGRUENCE-
PERMUTABLE VARIETIES

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Notions. For any algebra \( \mathcal{A} = (A, F) \), an element \( a \in A \) and a relation \( R \) on \( A \), the subset \( \{ x \in A; (a, x) \in R \} \) is called a class of \( R \). \( \mathcal{A} \) is called congruence-regular, tolerance-regular, reflexive and compatible-regular if any two congruences, tolerances, reflexive and compatible relations on \( \mathcal{A} \), respectively, coincide whenever they have a class in common.

Remark. Recently, I. Chajda has given Mal’cev conditions for varieties of (i) congruence-regular and congruence-permutable algebras (see [1]); (ii) tolerance-regular algebras (see [2]).

We state that these two classes of varieties coincide and some other Mal’cev conditions hold.

Theorem. For any variety \( V \) the following conditions are equivalent:

1. \( V \) is congruence-regular and congruence-permutable;
2. \( V \) is tolerance-regular;
3. \( V \) is reflexive and compatible-regular;
4. There exist a \((2n+3)\)-ary polynomial \( t \) and ternary polynomials \( p_i \) \((i=1, \ldots, n)\) such that \( x=t(x, y, z, z, \ldots, z, p_1(x, y, z, \ldots, z, \ldots, p_n(x, y, z)) \ldots) \), \( y=t(x, y, z, p_1(x, y, z), \ldots, p_n(x, y, z)) \ldots) \), \( z=p_1(x, x, z)=\ldots=p_n(x, x, z) \);
5. There exist a \((n+3)\)-ary polynomial \( r \) and ternary polynomials \( p_i \) \((i=1, \ldots, n)\) such that \( x=r(x, y, z, z, \ldots, z) \), \( y=r(x, y, z, p_1(x, y, z), \ldots, p_n(x, y, z)) \ldots) \), \( z=p_1(x, x, z)=\ldots=p_n(x, x, z) \).