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ANNOUNCEMENTS OF NEW RESULTS

ON THE INTEGRATION ON σ-CLASSES

M. Navara, P. Pták (Technical University of Prague, Suchbátarova 2, 16627 Praha 6, Československo), received 9.4. 1980

We announce a partial solution to two problems posed by S. Gudder in the paper "A generalized measure and probability theory for the physical sciences", Foundations of Probability Theory, Statistical Inference and Statistical Theories of Science, Vol. III, 121-141(1976). Suppose (Ω, C) is a σ-class and f and g are finitely valued measurable functions on (Ω, C). Suppose m is a probability measure on (Ω, C).
1. If $f \leq g$ then $\int_\Omega f\, dm \leq \int_\Omega g\, dm$. The same is valid if f, g have countable number of values which converge to zero.
2. If any of f, g has at most five values and f+g is measurable, then $\int_\Omega f\, dm + \int_\Omega g\, dm = \int_\Omega (f+g)\, dm$.

SOME SEQUENTIAL PROPERTIES

(M. Hušek, Charles University, Sokolovská 83, 18600 Praha 8, Československo), received 29.4. 1980

R. Frič and V. Koutník asked in their contribution on Top. Colloq. in Budapest, August 1978, whether there are sequentially complete completely regular spaces X, Y such that $C^*(X) = C^*_s(X)$ (C^*_s stands for sequentially continuous maps), $C^*(Y) = C^*_s(Y)$, $C^*(X)$ and $C^*(Y)$ are isomorphic and X, Y are not homeomorphic. The space $X = [0,1]^{\omega_1}$ and the Σ-product $Y = \{\{x_\alpha\} \in X \mid |\{\alpha \mid x_\alpha \neq 0\}| \leq \omega\}$ solve the problem (the required properties follow from Mazur's results in Fund. Math. 39(1952), 229-238). This pair also solves their Problem 5 whether for $X \subset \beta Y$, $\beta X = \beta Y$ one has $X \subset \sigma Y$ (here σY is the sequential closure of Y in βY). Similarly one can construct disjoint sets X, Y in $[0,1]^{\omega_1}$ such that $C^*(X) =$

$= C_s^*(X)$, $C^*(Y) = C_s^*(Y)$ and $\sigma X = \sigma Y = [0,1]^{\omega_1}$: $X = \{\{x_\alpha\} \in [0,1]^{\omega_1} \mid |\{\alpha \mid x_\alpha \text{ rational}\}| \leq \omega\}$, $Y = \{\{x_\alpha\} \in [0,1]^{\omega_1} \mid |\{\alpha \mid x_\alpha \text{ irrational}\}| \leq \omega\}$.

OPEN IMAGES OF ORDERABLE SPACES

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The problem of van Wouwe was solved. Every suborderable space is an open continuous image of an orderable space. As corollaries of the procedure one gets generalizations to higher cardinals of known results for first countable spaces (linearly uniformizable space is a topological space compatible with a uniformity having a monotone base, a generalized Baire space is a box-product $(\kappa^\lambda)_\lambda$ where κ is a regular infinite cardinal with the discrete topology, a caterpillar space is a topological T_1-space having a monotone base of neighborhoods at each point):
(1) Every caterpillar space is an open continuous image of a subspace of a generalized Baire space. (2) Every completely linearly uniformizable space is an open continuous image of a generalized Baire space.