Commentationes Mathematicae Universitatis Carolinae

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Existence and multiplicity results for nonlinear noncoercive equations

Commentationes Mathematicae Universitatis Carolinae, Vol. 22 (1981), No. 3, 635--636

Persistent URL: http://dml.cz/dmlcz/106105

VARIETIES OF SUBREGULAR ALGEBRAS ARE DEFINABLE BY A MAL'CEV CONDITION

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In [1], J. Timm introduced the concept of subregular algebra as follows: An algebra $\mathfrak{A}$ is called <u>subregular</u> if any congruence Θ on $\mathfrak{A}$ is uniquely determined by its classes $[b]\Theta$, $b \in \mathfrak{B}$, for every subalgebra $\mathfrak{B}$ of $\mathfrak{A}$.

<u>Theorem</u>. For any variety V, the following conditions are equivalent:

(1) Every algebra $\mathfrak{A} \in V$ is subregular;

(2) There exist unary polynomials $u_1,\dots,u_n$, ternary polynomials $p_1,\dots,p_n$ and 4-ary polynomials $s_1,\dots,s_n$ such that

$x = s_1(x,y,z,u_1(z))$

$s_i(x,y,z,p_i(x,y,z)) = s_{i+1}(x,y,z,u_{i+1}(z))$ for $1 \le i < n$

$y = s_n(x,y,z,p_n(x,y,z))$

$u_i(z) = p_i(x,x,z)$ for $1 \le i \le n$;

(3) There exist unary polynomials $u_1,\dots,u_n$ and ternary polynomials $p_1,\dots,p_n$ such that

$(u_i(z) = p_i(x,y,z),\ 1 \le i \le n) \Longleftrightarrow x = y.$

R e f e r e n c e s

[1] J. TIMM: On regular algebras, in Contributions to universal algebra, Proceedings of the Colloquium held in Szeged, 1975. Coll. Math. Soc. J. Bolyai, Vol. 17. Norht-Holland, Amsterdam 1977, pp. 503-514.

EXISTENCE AND MULTIPLICITY RESULTS FOR NONLINEAR NONCOERCIVE EQUATIONS

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We assume that $L:D(L) \subset L^2(\Omega) \longrightarrow L^2(\Omega)$ is linear selfadjoint operator with dense domain D(L) and closed range R(L). Let 0 be an eigenvalue of L and let for the corresponding eigenspace $\dim N(L) < +\infty$; $L^2(\Omega) = N(L) \oplus R(L)$. We assume that the functions in N(L) satisfy the "unique continuation property" (i.e. the only function w N(L) which is vanishing on the set of positive measure in Ω is $w \equiv 0$). Let $K: R(L) \longrightarrow R(L)$ (the right inverse of L) be compact.

Let $G: L^2(\Omega) \longrightarrow L^2(\Omega)$ be the Nemytskii operator associated with g (i.e. $G(u)(x) = g(u(x))$, $x \in \Omega$), where $g: \mathbb{R} \longrightarrow \mathbb{R}$ is a continuous odd bounded function with continuous derivative g' on $\mathbb{R}$, $c = \|K\| \sup_{z \in \mathbb{R}} g'(z) < 1$ and $\int_0^{+\infty} |g(z)| dz < +\infty$.

<u>Theorem</u>. For $f_2 \in R(L)$ either

(i) for each $w \in N(L)$ there exists precisely one $v(w) \in R(L)$ such that $u = w + v(w)$ is solution of the equation $Lu + G(u) = f_2$ and there is no solution of $Lu + G(u) = f$ with $f = f_1 + f_2$, $f_1 \in N(L)$, $f_1 \neq 0$;

or (ii) the equation $Lu + G(u) = f_2$ has at least one solution and there is a real number $T(f_2) > 0$ such that the equation $Lu + G(u) = f_1 + f_2$ has at least two distinct solutions if $0 < \|f_1\| < T(f_2)$.

In distinction from the previous papers dealing with such a type of nonlinearity we assume nothing about the limits $\gamma(a)_{\pm} = \lim_{z \to \pm\infty} \inf z^{\alpha} \min_{s \in \langle \pm a, \pm z \rangle} g(s)$. The functions $g(s) = se^{-s^2}$ and $g(s) = \sin(s)e^{-s^2}$ can be given as an example.