Pavel Drábek
Existence and multiplicity results for nonlinear noncoercive equations

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ANNOUNCEMENTS OF NEW RESULTS

VARIETIES OF SUBREGULAR ALGEBRAS ARE DEFINABLE BY A MAL'CEV CONDITION

Jaromír Duda (616 00 Brno 16, Kroftova 21, Československo), received 28.4. 1981

In [1], J. Timm introduced the concept of subregular algebra as follows: An algebra $\mathcal{A}$ is called subregular if any congruence $\Theta$ on $\mathcal{A}$ is uniquely determined by its classes $[b]_{\Theta}$, $b \in \mathcal{A}$, for every subalgebra $\mathcal{B}$ of $\mathcal{A}$.

Theorem. For any variety $V$, the following conditions are equivalent:

1. Every algebra $\mathcal{A} \in V$ is subregular;
2. There exist unary polynomials $u_1, \ldots, u_n$, ternary polynomials $p_1, \ldots, p_n$ and 4-ary polynomials $s_1, \ldots, s_n$ such that
   
   \[ x = s_1(x,y,z,u_1(z)) \]
   
   \[ s_i(x,y,z,p_i(x,y,z)) = s_{i+1}(x,y,z,u_{i+1}(z)) \text{ for } 1 \leq i < n \]
   
   \[ y = s_n(x,y,z,p_n(x,y,z)) \]
   
   \[ u_i(z) = p_i(x,x,z) \text{ for } 1 \leq i \leq n; \]
3. There exist unary polynomials $u_1, \ldots, u_n$ and ternary polynomials $p_1, \ldots, p_n$ such that
   
   \[ (u_i(z) = p_i(x,y,z), 1 \leq i \leq n) \iff x = y. \]

References


EXISTENCE AND MULTIPLICITY RESULTS FOR NONLINEAR NONCOERCIVE EQUATIONS

Pavel Drábek (Katedra matematiky VŠSE, Nejedlého sady 14, 306 14 Plzeň), received 13.5. 1981

We assume that $L:D(L) \subset L^2(\Omega) \to L^2(\Omega)$ is linear self-adjoint operator with dense domain $D(L)$ and closed range $R(L)$. Let $0$ be an eigenvalue of $L$ and let for the corresponding eigenspace $\dim N(L) < + \infty$; $L^2(\Omega) = N(L) \oplus R(L)$. We assume that the functions in $N(L)$ satisfy the "unique continuation property" (i.e. the only function $w \in N(L)$ which is vanishing on
Let \( K : \mathbb{R}(L) \to \mathbb{R}(L) \) (the right inverse of \( L \)) be compact.

Let \( G : \mathbb{L}^2(\Omega) \to \mathbb{L}^2(\Omega) \) be the Nemytskii operator associated with \( g \) (i.e. \( G(u)(x) = g(u(x)), \ x \in \Omega \)), where \( g : \mathbb{R} \to \mathbb{R} \) is a continuous odd bounded function with continuous derivative \( g' \) on \( \mathbb{R} \), \( c = \| K \| \sup_{z \in \mathbb{R}} g'(z) < 1 \) and \( \int_0^{+\infty} |g(z)|dz < +\infty \).

**Theorem.** For \( f_2 \in \mathbb{R}(L) \) either

(i) for each \( w \in \mathbb{N}(L) \) there exists precisely one \( v(w) \in \mathbb{R}(L) \) such that \( u = w + v(w) \) is solution of the equation \( Lu + G(u) = f_2 \) and there is no solution of \( Lu + G(u) = f \) with \( f = f_1 + f_2 \), \( f_1 \in \mathbb{N}(L), f_1 \neq 0 \); or

(ii) the equation \( Lu + G(u) = f_2 \) has at least one solution and there is a real number \( T(f_2) > 0 \) such that the equation \( Lu + G(u) = f_1 + f_2 \) has at least two distinct solutions if \( 0 < \| f_1 \| < T(f_2) \).

In distinction from the previous papers dealing with such a type of nonlinearity we assume nothing about the limits

\[
\gamma(s) = \lim_{x \to +\infty} \inf_{\beta \in \{a, b\}} \lambda^\beta(s).
\]

The functions \( g(s) = ae^{s^2} \) and \( g(s) = \sin(s)e^{-s^2} \) can be given as an example.