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ANNOUNCEMENTS OF NEW RESULTS

SOME THEOREMS ON THE LATTICE OF LOCAL INTERPRETABILITY TYPES

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In [2], J. Mycielski introduced the lattice of multidimensional local interpretability types of theories (for next shortly type) and he posed some problems (see also a joint manuscript [1] with A. Ehrenfeucht). By next three theorems we solved two of them. $|T|$ denotes type of a theory T and $\leq$ denotes ordering in the lattice.

Theorem 1: A type is meet-irreducible iff it contains a complete theory.

Corollary of the proof: If S is a finitely axiomatizable and essentially undecidable theory and R is its recursively axiomatizable extension then type $|R|$ is not meet-irreducible.

Theorem 2: For each two types t, s such that $s \nleq t$ there exists a meet-irreducible type $t' \geq t$ such that still $s \nleq t'$.

Corollary: For each type t which is not maximal there exists a meet-irreducible type $t' \geq t$ which is still not maximal.

Theorem 3: Each type contains a theory with a finite language.

Also some results about mutual multidimensional interpretability of various theories of order are proved. Let us define six theories in the language $\{=;\prec\}$ (= is equality and in the following theories we assume implicitly the axioms of equality, $x \preceq y$ stands for $x \prec y \vee x = y$):

PO: $\forall x;\ \neg x \prec x + \forall x,y,z;(x \prec y \,\&\, y \prec z) \rightarrow x \prec z + \forall x \exists y; x \prec y$

POD: $PO + \forall x\, \forall y \succ x\, \exists t \succ x; y \succ t + \forall x\, \exists y; y \prec x$

POS: $PO + \exists! x\, \forall y; x \preceq y + \forall x\, \forall y \succ x\, \exists z \succ x\, \forall t \succ x; z \preceq y \,\&\, (t \preceq y \rightarrow z \preceq t)$
$+ \forall x\, \forall y \prec x\, \exists z \prec x\, \forall t \prec x; y \preceq z \,\&\, (y \preceq t \rightarrow t \preceq z)$

LO: $PO + \forall x,y; x \preceq y \vee y \preceq x$; LOD: POD + LO; LOS: POS + LO

In the following theorem $\leq_1$ denotes 1-dimensional interpretability and $\perp$ denotes incomparability with respect to multidimensional (!) interpretability.

Theorem 4: (i) PO < LO, POS, POD (ii) LO < LOS, LOD
(iii) POS < LOS (iv) POS $\perp$ POD, LO, LOD
(v) LOS $\perp$ POD, LOD (vi) $|LO| = |LOS| \wedge |LOD|$
(vii) for each theory T: $LO \leq T \rightarrow LO \leq_1 T$.

References: [1] A. Ehrenfeucht, J. Mycielski: Theorems and problems on the lattice of local interpretability, manuscript
[2] J. Mycielski: A lattice of interpretability types of theories, Journal of Symbolic Logic 42, 1977

A POSSIBLE MODAL REFORMULATION OF COMPREHENSION SCHEME

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In various set theories the Cantor's comprehension is reformulated (Quine's NF) or replaced by a set of axioms (ZF,GB).