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SHORT BRANCHES IN RUDIN-FROLÍK ORDER

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Rudin-Frolík order of types of ultrafilters in βN has the following properties:

(1) each type of ultrafilters has at most $2^{\aleph_0}$ predecessors, [2],

(2) the cardinality of each branch is at least $2^{\aleph_0}$.

Thus, in Rudin-Frolík order the cardinality of branches can be only $2^{\aleph_0}$ or $(2^{\aleph_0})^+$. It was shown in [1] that there exists a chain order - isomorphic to $(2^{\aleph_0})^+$. Hence, the existence of a branch of cardinality $(2^{\aleph_0})^+$ is proved.

The following result solves the problem of the existence of a branch having smaller cardinality.

Theorem. In Rudin-Frolík order there exists an unbounded chain order-isomorphic to ω_1.

By the properties (1) and (2) the branch containing this chain has cardinality $2^{\aleph_0}$.

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RESULTS ON DISJOINT COVERING SYSTEMS ON THE RING OF INTEGERS

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A system of congruence classes

(1) $a_1(\text{mod } n_1),\ a_2(\text{mod } n_2),\ \dots,\ a_k(\text{mod } n_k)$

will be called a disjoint covering system (DCS) if for every integer x there is exactly one $i \in \{1, 2, \dots, k\}$ such that $x \equiv a_i(\text{mod } n_i)$. The integers $n_1, n_2, \dots, n_k$ will be called moduli of (1) and their least common multiple will be called the common modulus of (1).

If $k > 1$ then no two moduli of (1) are relatively prime. This condition can be expressed in the form

(2) $$\bigwedge_{i=1}^{k} \bigwedge_{j=1}^{k} \varphi(n_i, n_j)$$

where $\varphi(x, y)$ is the formula

$$\exists z \exists u \exists v\ (z \neq 1 \wedge z.u = x \wedge z.v = y)$$

Consider more generally the formulae of the form