Ronald Beattie; Heinz-Peter Butzmann; Horst Herrlich

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FILTER CONVERGENCE VIA SEQUENTIAL CONVERGENCE

R. BEATTIE, H.-P. BUTZMANN, H. HERRLICH

Abstract: We examine the forgetful functor between the appropriately defined categories of filter and sequential convergence spaces. We show that it has a left adjoint, that its restriction to the category of all first countable filter convergence spaces has both a left adjoint and a right adjoint and that a suitable domain-codomain restriction is even topological. We also study some subcategories which arise in a natural way.

Key words: Filter convergence space, sequential convergence space, adjoint functor, topological functor.

Classification: 18A40, 18B99, 54A20, 54B30, 54D55

During the last 25 years sequential convergence structures have been investigated in detail (cf. e.g. [5] and [6]) and (filter) convergence structures even more thoroughly (cf. e.g. [1] and [7]). But little is known about the relations between the two. In this paper we study the forgetful functor L from the category of filter convergence spaces into that of sequential convergence spaces. We show that it has a left adjoint (which was basically defined by Frič in [3]), that its restriction to the category of all first countable spaces has not only a left adjoint but also a right adjoint, which was defined by Beattie and Butzmann in [2] and that L is even topological if we also restrict its codomain in a natural way, which was also considered by Frič and Kent in [4].

For a set X we denote by F(X) and S(X) the families of all filters and sequences on X, respectively; furthermore, P(X) denotes its power set.

Definition 1. Let X be a set. A mapping $\lambda : X \longrightarrow P(F(X))$ is called a filter convergence structure and (X,λ) a filter convergence space if the following conditions are satisfied for all $x \in X$:

(F1) $\dot{x} = \{A \subset X : x \in A\} \in \lambda(x)$

(F2) If $\mathcal{F} \in \lambda(x)$ then $\mathcal{G} \in \lambda(x)$ for all $\mathcal{G} \in F(X)$ with $\mathcal{G} \supset \mathcal{F}$.

A mapping $f:(X,\lambda) \longrightarrow (Y,\mu)$ between filter convergence spaces is called continuous if $\mathcal{F} \in \lambda(x)$ implies $f(\mathcal{F}) \in \mu(f(x))$ for all $x \in X$. We denote by F-Conv the concrete category of all filter convergence spaces and continuous mappings.

Definition 2. Let X be a set. A mapping $\mathcal{L}: X \longrightarrow P(S(X))$ is called a sequential convergence structure and $(X,\mathcal{L})$ a sequential convergence space if the following conditions are satisfied for all $x \in X$:

(S1) If $\xi \in S(X)$ and $\xi(n) = x$ for all $n \in \mathbb{N}$, then $\xi \in \mathcal{L}(x)$.

(S2) If $\xi \in \mathcal{L}(X)$, then $\eta \in \mathcal{L}(x)$ for all subsequences η of ξ.

(S3) If $\xi \in S(X)$ and $(\xi(n+1))_{n\in\mathbb{N}} \in \mathcal{L}(x)$, then $\xi \in \mathcal{L}(x)$.

A mapping $f:(X,\mathcal{L}) \longrightarrow (Y,\mathcal{M})$ between sequential convergence spaces is called continuous if $\xi \in \mathcal{L}(x)$ implies $f \circ \xi \in \mathcal{M}(f(x))$ for all $x \in X$.

We denote by S-Conv the concrete category of all sequential convergence spaces and continuous mappings.

Definition 1 gives a generalization of a "convergence space" in the sense of Fischer and Binz (see e.g. [1]) and of a "pseudotopologischer Raum" in the sense of Gähler (see e.g. [7]) and was called an L-space by Poppe ([12]). It seems to be necessary in order to treat sequential convergence spaces in the framework of filter convergence. On the other hand, our definition of a sequential convergence space is a slight restriction of the same term in the sense of Novák (see e.g. [4]), but the additional condition (S3), which means that the convergence of a sequence does not depend upon its behaviour at a finite number of elements, does not seem to be strong and avoids some unwanted pathologies. In particular it is always fulfilled if the convergence is maximal in the sense of Novák (cf. e.g. [4]). We also have to point out that our definition of a continuous mapping between sequential convergence spaces differs from that of Novák.

We can now introduce the functor whose study is of main interest to us as well as its left adjoint.

Definition 3. Let (X, λ) be a filter convergence space. Define a sequential convergence structure

$\mathcal{L}(\lambda): X \longrightarrow P(S(X))$ by

$$\xi \in \mathcal{L}(\lambda)(x) \Longleftrightarrow \mathcal{F}(\xi) \in \lambda(x),$$

where $\mathcal{F}(\xi)$ denotes the Fréchet filter generated by ξ. Evidently $\mathcal{L}$ preserves continuous maps and the concrete functor L: F-Conv $\longrightarrow$ S-Conv associates with every filter convergence space (X, λ) the sequential convergence space $(X, \mathcal{L}(\lambda))$.

Definition 4. Let $(X, \mathcal{L})$ be a sequential convergence space. Define a filter convergence structure

$\varphi(\mathcal{L}): X \longrightarrow P(F(X))$ by

$$\mathcal{F} \in \varphi(\mathcal{L})(x) \text{ if and only if there is some } \xi \in \mathcal{L}(x) \text{ with } \mathcal{F}(\xi) \subset \mathcal{F}.$$

Clearly φ also preserves continuous mappings and the concrete functor F: S-Conv $\longrightarrow$ F-Conv associates with every sequential convergence space $(X, \mathcal{L})$ the filter convergence space $(X, \varphi(\mathcal{L}))$.

One can now easily establish the following:

Lemma 1. Let X be a set, λ a filter convergence structure and $\mathcal{L}$ a sequential convergence structure on X. Then the identities

$\mathrm{id}:(X, \varphi(\mathcal{L}(\lambda))) \longrightarrow (X, \lambda)$ and

$\mathrm{id}:(X, \mathcal{L}) \longrightarrow (X, \mathcal{L}(\varphi(\mathcal{L})))$

are continuous.

Theorem 1. The functor L: F-Conv $\longrightarrow$ S-Conv has F as a left adjoint.

Proof. We show that for all sequential convergence spaces $(X, \mathcal{L})$ and all filter convergence spaces (Y, μ) the following are equivalent:

(i) $f:(X, \varphi(\mathcal{L})) \longrightarrow (Y, \mu)$ is continuous

(ii) $f:(X, \mathcal{L}) \longrightarrow (Y, \mathcal{L}(\mu))$ is continuous.

This gives a natural isomorphism between the functors $\hom(F\cdot, \cdot)$ and $\hom(\cdot, L\cdot)$ and thereby proves the theorem.

(i) $\Rightarrow$ (ii): Consider the diagram

$$\begin{array}{ccc} (X, \mathcal{L}(\varphi(\mathcal{L}))) & \xrightarrow{\quad f \quad} & (Y, \mathcal{L}(\mu)) \\ & {\scriptstyle \mathrm{id}} \nwarrow \quad \nearrow {\scriptstyle f} & \\ & (X, \mathcal{L}) & \end{array}$$

then the implication follows from Lemma 1 and the fact that L is a functor.

(ii) $\Rightarrow$ (i): This time we consider the diagram

$$\begin{array}{ccc} (X,\varphi(\mathcal{L})) & \xrightarrow{\quad f \quad} & (Y,\varphi(\mathcal{L}(\mu))) \\ & {}_{f}\searrow \qquad \swarrow_{id} & \\ & (Y,\mu) & \end{array}$$

then the fact that F is a functor and again Lemma 1 give the desired result.

Since L has a left adjoint it preserves limits; it also preserves coproducts, but not quotients:

Example. Denote by ω and Ω the first infinite and the first uncountable ordinal, respectively, and endow $[0,\omega]$ and $[0,\Omega]$ with the interval topologies. Consider the set

$$T = ([0,\Omega) \times [0,\omega)) \cup \{(\Omega,\omega)\} \subset [0,\Omega] \times [0,\omega]$$

and endow it with the subspace topology τ. Then the projection

$$\pi : (T,\tau) \longrightarrow [0,\omega]$$

is a quotient map in both the category of topological spaces and th.. of filter convergence spaces. But a sequence $\xi \in S(T)$ belongs to $\mathcal{L}(\tau)((\Omega,\omega))$ if and only if it is eventually the constant (Ω,ω). On the other hand, the sequence (n) converges to ω in $[0,\omega]$.

As the above example shows, L has no right adjoint. But if we suitably restrict the domain category of L, then the restricted functor has a concrete left- and a concrete right adjoint:

Definition 5. A filter convergence structure λ on a set X and the filter convergence space (X,λ) are called first countable if for every $x \in X$ and every $\mathcal{F} \in \lambda(x)$ there is a $\mathcal{G} \in \lambda(x)$ with a countable base such that $\mathcal{G} \subset \mathcal{F}$.

We denote by <u>F-Conv</u>^{0} the full subcategory of <u>F-Conv</u> whose objects are the first countable filter convergence spaces and by E the embedding E:<u>F-Conv</u>$^{0} \longrightarrow$ <u>F-Conv</u>.

<u>Proposition 1</u>. <u>F-Conv</u>^{0} is a coreflective modification of <u>F-Conv</u>, i.e. the embedding E:<u>F-Conv</u>$^{0} \longrightarrow$ <u>F-Conv</u> has a concrete right adjoint R.

<u>Proof</u>. Given a filter convergence space (X, λ) we define

$\lambda^{0}: X \longrightarrow P(F(X))$ by

$$\mathcal{F} \in \lambda^{0}(x) \Longleftrightarrow \text{there is a filter } \mathcal{G} \in \lambda(x) \text{ with a countable base such that } \mathcal{G} \subset \mathcal{F}.$$

It is easy to see that the concrete functor R:<u>F-Conv</u> $\longrightarrow$ <u>F-Conv</u>^{0} which associates with any filter convergence space (X, λ) the filter convergence space (X, λ^{0}) is the desired right adjoint of E.

It is clear that the codomain restriction F' of F is a left adjoint of $L \circ E$. We shall now construct its right adjoint.

<u>Definition 6</u>. Let $(X, \mathcal{L})$ be a sequential convergence space. Define a filter convergence structure

$$\gamma(\mathcal{L}): X \longrightarrow P(F(X))$$

by stating that a filter $\mathcal{F} \in F(X)$ belongs to $\gamma(\mathcal{L})(x)$ for some $x \in X$ if and only if there is a filter $\mathcal{G} \subset \mathcal{F}$ with a countable base satisfying the following condition:

(SC) $\xi \in S(X)$ and $\mathcal{F}(\xi) \supset \mathcal{G}$ implies $\xi \in \mathcal{L}(x)$.

As we shall show in the following Lemma, γ preserves continuous maps and the concrete functor G:<u>S-Conv</u> $\longrightarrow$ <u>F-Conv</u>^{0} associates with every sequential convergence space $(X, \mathcal{L})$ the filter convergence space $(X, \gamma(\mathcal{L}))$.

Lemma 2. Let $(X,\mathcal{L})$ and $(Y,\mathcal{M})$ be sequential convergence spaces and $f:(X,\mathcal{L}) \longrightarrow (Y,\mathcal{M})$ be a continuous mapping. Then $f:(X,\gamma(\mathcal{L})) \longrightarrow (Y,\gamma(\mathcal{M}))$ is continuous.

Proof. For every $x \in X$ and every $\mathcal{F} \in \gamma(\mathcal{L})(x)$ there is a filter $\mathcal{G} \subset \mathcal{F}$ with a monotone countable base $\{G_n : n \in \mathbb{N}\}$ satisfying the condition (SC). We claim that $f(\mathcal{G})$ also satisfies (SC):

Take a sequence $\eta \in S(Y)$ such that $\mathcal{F}(\eta) \supset f(\mathcal{G})$. Then for every $n \in \mathbb{N}$ there is a $k_n \in \mathbb{N}$ with

$$f(G_n) \supset \{\eta(k) : k \geq k_n\}.$$

So for every $k \geq k_1$ one can choose $r_k \in \mathbb{N}$ and $\xi_k \in G_{r_k}$ such that $\eta(k) = f(\xi_k)$ and that $r_k \geq n$ if $k \geq k_n$.
But then $\mathcal{F}((\xi_n)_{n \geq k_1}) \supset \mathcal{G}$ and so $(\xi_n)_{n \geq k_1} \in \mathcal{L}(x)$, implying

$$(\eta(n))_{n \geq k_1} = (f(\xi_n))_{n \geq k_1} \in \mathcal{M}(f(x)).$$

By (S3) we have then $\eta \in \mathcal{M}(f(x))$.

For easy reference we state the following rather obvious facts as

Lemma 3. Let X be a set, $\mathcal{L}$ a sequential convergence structure and Λ a first countable filter convergence structure on X. Then the following identities are continuous:

$$\text{id}: (X,\mathcal{L}(\gamma(\mathcal{L}))) \longrightarrow (X,\mathcal{L}) \quad \text{and}$$
$$\text{id}: (X,\Lambda) \longrightarrow (X,\gamma(\mathcal{L}(\Lambda))).$$

Theorem 2. The functor $L \circ E$: F-Conv$^{0} \longrightarrow$ S-Conv has a concrete left adjoint (the codomain restriction $F' = R \circ F$ of F) and G as a concrete right adjoint.

Proof. Let (X,Λ) be a first countable filter convergence space and $(Y,\mathcal{M})$ a sequential convergence space. This time we show for every mapping $f: X \longrightarrow Y$ the equivalence of:

(i) $f:(X.\mathcal{L}(\lambda)) \longrightarrow (Y,\mathcal{M})$ is continuous

(ii) $f:(X,\lambda) \longrightarrow (Y,\gamma(\mathcal{M}))$ is continuous.

(i) $\Longrightarrow$ (ii): Since $f:(X,\gamma(\mathcal{L}(\lambda))) \longrightarrow (Y,\gamma(\mathcal{M}))$ is continuous, the implication follows from Lemma 3.

(ii) $\Longrightarrow$ (i): Since $f:(X,\mathcal{L}(\lambda)) \longrightarrow (Y,\mathcal{L}(\gamma(\mathcal{M})))$ is continuous, we can apply Lemma 3 again.

The functor $L \circ E$ is not topological, since it is not surjective. But if we restrict not only the domain of L but also its codomain, then the resulting surjective restriction has not only concrete right- and left adjoints but is even topological. Now the objects in the range of $L \circ E$ can be characterized in different ways:

Proposition 2. For a sequential convergence space $(X,\mathcal{L})$ the following are equivalent:

(i) There is a first countable filter convergence structure λ on X with $\mathcal{L} = \mathcal{L}(\lambda)$.

(ii) There is a filter convergence structure λ on X with $\mathcal{L} = \mathcal{L}(\lambda)$.

(iii) $\mathcal{L}(\varphi(\mathcal{L})) = \mathcal{L}$.

(iv) $\mathcal{L}(\gamma(\mathcal{L})) = \mathcal{L}$.

(v) If $\xi, \eta \in S(X)$ and $\mathcal{F}(\xi) = \mathcal{F}(\eta)$, then $\xi \in \mathcal{L}(x)$ for some $x \in X$ if and only if $\eta \in \mathcal{L}(x)$.

(vi) If $\xi \in \mathcal{L}(x)$ for some $x \in X$, then $\xi \circ \alpha \in \mathcal{L}(x)$ for every finite-to-one sequence $\alpha \in S(\mathbb{N})$.

Proof. (i) $\Longrightarrow$ (ii) $\Longrightarrow$ (v) and (iv) $\Longrightarrow$ (i) are clear.

(v) $\Longrightarrow$ (vi): Choose any $x \in X$, $\xi \in \mathcal{L}(x)$ and a finite-to-one sequence $\alpha \in S(\mathbb{N})$. There is a subsequence β of $id_{\mathbb{N}}$ such that

$$\{\beta(k):k\in\mathbb{N}\} = \{\alpha(k):k\in\mathbb{N}\}.$$

An easy calculation shows then that $\mathcal{F}(\xi\circ\beta) = \mathcal{F}(\xi\circ\alpha)$, which gives the implication.

<u>(vi) $\Rightarrow$ (iii)</u>: Take any $x\in X$. By Lemma 1 we know that

$$\mathcal{L}(x) \subset \mathcal{L}(\varphi(\mathcal{L}))(x).$$

So assume that $\xi \in \mathcal{L}(\varphi(\mathcal{L}))(x)$. Then there is a sequence $\eta\in\mathcal{L}(x)$ with $\mathcal{F}(\xi)\supset\mathcal{F}(\eta)$ and so for every $k\in\mathbb{N}$ there is a natural number $r_k\in\mathbb{N}$ such that

$$\{\eta(n):n\geq k\} \supset \{\xi(n):n\geq r_k\}$$

This gives a finite-to-one sequence $\alpha\in S(\mathbb{N})$ with

$$\xi(n) = \eta(\alpha(n)) \quad \text{for all } n\geq r_1,$$

which implies $\xi\in\mathcal{L}(x)$ by (vi).

<u>(iii) $\Rightarrow$ (iv)</u>: By Lemma 3 we have

$$\mathcal{L}(\gamma(\mathcal{L}))(x) \subset \mathcal{L}(x) \quad \text{for all } x\in X.$$

So assume that $\xi\in\mathcal{L}(x)$, then for every sequence $\eta\in S(X)$ with $\mathcal{F}(\eta)\supset\mathcal{F}(\xi)$ we get $\mathcal{F}(\eta)\in\varphi(\mathcal{L})(x)$ and hence

$$\eta\in\mathcal{L}(\varphi(\mathcal{L}))(x) = \mathcal{L}(x).$$

Therefore $\mathcal{F}(\xi)$ satisfies (SC), giving the desired result.

Following Frič and Kent ([4]) we define:

<u>Definition 7</u>. A sequential convergence space $(X,\mathcal{L})$ satisfies the condition (FL) if one of the equivalent conditions of Proposition 2 is fulfilled.

We denote by <u>S-Conv</u>^{0} the full subcategory of <u>S-Conv</u> whose objects are the sequential convergence spaces satisfying the condition (FL) and by E':<u>S-Conv</u>$^{0}\longrightarrow$ <u>S-Conv</u> the corresponding embedding functor.

<u>Remark</u>. Lemma 1 and Lemma 3 show that both (F,L) and ($L\circ E$,G) are Galois connections of the third kind as they were introduced

and studied by Herrlich and Hušek in [10]. And the results of the following proposition, which summarizes the most important relations between the different modifications, are actually a part of 3.4 in [10]:

Proposition 3

(1) For every sequential convergence space $(X,\mathcal{L})$ the following identities are continuous:

$$(X,\mathcal{L}(\gamma(\mathcal{L}))) \xrightarrow{id} (X,\mathcal{L}) \xrightarrow{id} (X,\mathcal{L}(\varphi(\mathcal{L}))).$$

Either identity is a *homeomorphism if and only if* $(X,\mathcal{L})$ satisfies the condition (FL) and in this case the diagram collapses.

(2) For every first countable filter convergence space (X,λ) the identities

$$(X,\varphi(\mathcal{L}(\lambda))) \xrightarrow{id_1} (X,\lambda) \xrightarrow{id_2} (X,\gamma(\mathcal{L}(\lambda)))$$

are continuous. The mapping id_1 is a homeomorphism if and only if there is a sequential convergence structure $\mathcal{L}$ on X with $\lambda = \varphi(\mathcal{L})$ and id_2 is a homeomorphism if and only if there is a sequential convergence structure $\mathcal{L}$ on X with $\lambda = \gamma(\mathcal{L})$.

After these preparations we are ready to prove the main result of this paper.

Theorem 3. The domain-codomain restriction L^o:F-Conv$^o \longrightarrow$ S-Conv of L is topological.

Proof. Let $(X,\mathcal{L})$ be a sequential convergence space satisfying the condition (FL), (X_i, λ_i) a family of first countable filter convergence spaces and $(f_i:(X,\mathcal{L}) \longrightarrow (X_i,\mathcal{L}(\lambda_i)))$ be an L-source. We define

$$\lambda : X \longrightarrow P(F(X))$$

by stating that a filter $\mathcal{F} \in F(X)$ belongs to $\lambda(x)$ for some $x \in X$ if and only if there is a filter $\mathcal{G} \subset \mathcal{F}$ with a countable base satisfying:

(a) $\mathcal{G} \in \gamma(\mathcal{L})(x)$

(b) $f_i(\mathcal{G}) \in \lambda_i(f_i(x))$ for all i.

We have to show:

(i) $\mathcal{L}(\lambda) = \mathcal{L}$.

(ii) Given a first countable filter convergence space (Y,μ) and a continuous mapping $g:(Y,\mathcal{L}(\mu)) \longrightarrow (X,\mathcal{L})$ such that $f_i \circ g:(Y,\mu) \longrightarrow (X_i,\lambda_i)$ is continuous for all i, then $g:(Y,\mu) \longrightarrow (X,\lambda)$ is continuous.

<u>(i)</u>: For all $x \in X$ we have $\lambda(x) \subset \gamma(\mathcal{L})(x)$, giving

$$\mathcal{L}(\lambda)(x) \subset \mathcal{L}(\gamma(\mathcal{L}))(x) = \mathcal{L}(x).$$

If, on the other hand, $\xi \in \mathcal{L}(x)$, then $\xi \in \mathcal{L}(\gamma(\mathcal{L}))(x)$ and so $\mathcal{F}(\xi) \in \gamma(\mathcal{L})(x)$. Furthermore $f_i \circ \xi \in \mathcal{L}(\lambda_i)(f_i(x))$ for all i and so $f_i(\mathcal{F}(\xi)) \in \lambda_i(f_i(x))$ for all i. Both together imply $\mathcal{F}(\xi) \in \lambda(x)$ and so $\xi \in \mathcal{L}(\lambda)(x)$.

<u>(ii)</u>: By Theorem 2 we know that $g:(Y,\mu) \longrightarrow (X,\gamma(\mathcal{L}))$ is continuous. Now take any $y \in Y$ and any $\mathcal{F} \in \mu(y)$. Since μ is first countable there is a filter $\mathcal{G} \subset \mathcal{F}$ with a countable base such that $\mathcal{G} \in \mu(y)$. Then $g(\mathcal{G}) \in \gamma(\mathcal{L})(y)$ and $f_i(g(\mathcal{G})) \in \lambda_i(f_i(g(y)))$ for all i, implying that $g(\mathcal{G}) \in \lambda(g(y))$ as desired.

<u>Remark</u>. Since L^o is topological, it has a right- and a left adjoint. Here we are able to give an explicit description of it: The right adjoint is G^o, the restriction of G to <u>S-Conv</u>^{o} and the left adjoint is F^o, the restriction of F' to <u>S-Conv</u>^{o}, the domain-codomain restriction of F.

Corollary 1. The category S-Conv^0 is simultaneously a reflective and a coreflective modification of S-Conv, i.e. the embedding functor $E':$ S-Conv$^0 \longrightarrow$ S-Conv has a concrete right adjoint R' and a concrete left adjoint C'.

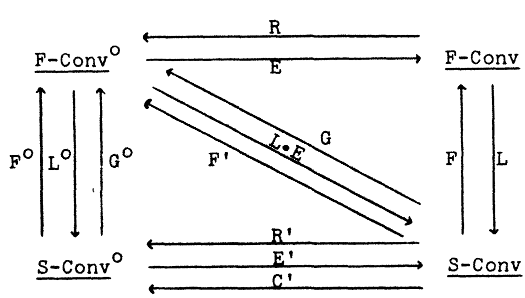
Proof. By the above Remark, F^0 is a left adjoint of L^0 and $L \circ E$ is by Theorem 2 a left adjoint of G and so $E' = L \circ E \circ F^0$ is a left adjoint of $R' := L^0 \circ G$. On the other hand, again by the Remark and Theorem 2, we know that L^0 is a left adjoint of G^0 and F' is a left adjoint of $L \circ E$ and so $C' := L^0 \circ F'$ is a left adjoint of $E' = L \circ E \circ G^0$.

We call a filter convergence structure λ on a set X compatible with a sequential convergence structure $\mathcal{L}$ on X if $\mathcal{L}(\lambda) = \mathcal{L}$.

Corollary 2. For every $(X, \mathcal{L})$ in S-Conv^0 the set of all first countable convergence structures on X which are compatible with $\mathcal{L}$ forms a complete lattice (in the natural order) with largest element $\varphi(\mathcal{L})$ and smallest element $\gamma(\mathcal{L})$.

Proof. Since the fibres of a topological functor form a complete lattice (see e.g. [8], p.283), we get the first part of the assertion. The second follows from Proposition 3.

For an easy survey we place the introduced functors in one diagram:



R e f e r e n c e s

[1] E. BINZ: Continuous convergence on C(X), Lecture Notes in Mathematics 469, Springer Verlag, Berlin-Heidelberg-New York, 1975.

[2] H.-P. BUTZMANN and R. BEATTIE: Some relations between sequential and filter convergence, to appear in: Proceedings of the conference on convergence, Bechyně, ČSSR 1984.

[3] R. FRIČ: On convergence spaces and groups, Math.Slovaca 27 (1977), 323-330.

[4] R. FRIČ and D.C. KENT: On c-embedded sequential convergence spaces, in: Convergence structures and applications to analysis, Abh.Akad.Wiss.DDR,Abt.Math.-Naturwiss.-Technik Nr.4N (1979), 33-36.

[5] R. FRIČ and V. KOUTNÍK: Sequential structures, in: Convergence structures and applications to analysis, Abh.Akad. Wiss,DDR,Abt.Math.-Naturwiss.-Technik Nr.4N (1979), 37-56.

[6] R. FRIČ and V. KOUTNÍK: Recent development in sequential convergence, to appear in: Proc.Intern.School on Convergence Structures and Appl.II, Schwerin 1983 (Abh.Akad. Wiss.DDR), in print.

[7] W. GÄHLER: Grundstrukturen der Analysis Band I, II, Berlin-Basel 1977 and 1978.

[8] H. HERRLICH: Categorical Topology 1971-1981, in: General Topology and its Relations to Modern Analysis and Algebra V, Proc.Fifth Prague Topological Sympos. 1983, Heldermann Verlag Berlin 1982, 279-383.

[9] H. HERRLICH: Universal Topology, in: Categorical Topology, Proc.Conf.Toledo,Ohio,USA 1983, Heldermann Verlag Berlin 1984, 223-281.

[10] H. HERRLICH and M. HUŠEK: Galois connections, Preprint.

[11] H. HERRLICH and G. STRECKER: Category Theory, 2nd ed., Heldermann Verlag Berlin 1979.

[12] H. POPPE: Compactness in general function spaces, VEB Deutscher Verlag der Wiss. Berlin 1974.

R. Beattie: Dept. of Math., Mount Allison Univ., Sackville, N.B. EOA 3CO, Canada

H.-P. Butzmann: Fakultät für Mathematik und Inf.,Universität Mannheim, A 5, 6800 Mannheim, B R D

H. Herrlich: Fachbereich Mathematik/Inf., Universität Bremen, 2800 Bremen, B R D

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