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On estimating the diffusion coefficient

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Consider the diffusion process $\xi_t$ defined on $(\Omega, \mathcal{F}, \mathbb{P})$ by
\[
d\xi_t = a(\xi_t, \phi)dt + b(\xi_t, \phi)d\mathbf{W}_t, \quad \xi_0 = x_0, \quad t \in [0, T],
\]
where $\phi \in \Theta$ is an open subset of real line, $\{\mathbf{W}_t, t \in [0, T]\}$ is a standard Wiener process. Suppose that $a(x, \phi), b(x, \phi)$ are real-valued functions, continuous on $\mathbb{R} \times \Theta$, $b(x, \phi) > 0$ for all $(x, \phi) \in \Theta \times \mathbb{R}$, and such that $a^\prime, a^\prime\prime, b^\prime, b^\prime\prime, b^\prime\prime\prime, b^\prime\prime\prime\prime$ are continuous on $\mathbb{R} \times \Theta$ (here the stroke and the dot denote derivative with respect to $x$ and $\phi$ respectively). Denote $g(x, \phi) = b(x, \phi)/b(x, \phi)$. The chain $\{\xi^n_{k}\}_{k=0}^{n}$ of observations of the process $\xi_t$ at discrete sampling points $0=t_0 < t_1 < \ldots < t_n=T$ is the Markov chain which generates on $(\Omega^n, \mathcal{F}(X_1, \ldots, X_n))$ the probability measure $\mathbb{P}_n^\phi$. Local asymptotic mixed normality (LAMN). The families $\mathcal{P}_n^\phi, \phi \in \Theta^n_{n\uparrow}$ satisfy the LAMN condition in some $\phi_0 \in \Theta$. The minimax theorem. For any sequence $\{\mathbb{P}_n^\phi\}_{n \geq 1}$ of estimators based on $X_k, k=0,1, \ldots, n$, of unknown parameter $\phi_0$ holds
\[
\lim_{n \to \infty} \sup_{\phi \neq \phi_0} \frac{\mathbb{E}(1(\sqrt{n}(\mathbb{P}_n^\phi - \mathbb{P}_n^{\phi_0})))^2}{1 + \mathbb{E}(1(\sqrt{n}(\mathbb{P}_n^\phi - \mathbb{P}_n^{\phi_0})))^2} \leq \frac{1}{\sqrt{2n}} \int_0^\infty (z^2 - 1) dzG(w),
\]
where $\mathbb{P}_n^\phi = \mathbb{P}_n^{\phi_0} + n/\sqrt{n}$, $1(x)$ is a loss function and $G(w)$ is the distribution function of $\mathbb{P}(\phi_0) = \frac{2}{T} \int_0^T g^2(\xi_t, \phi)dt$.

The lower bound is obtained only if
\[
(\mathbb{P}_n^\phi - \mathbb{P}_n^{\phi_0}) \to [\sum_{k=0}^{m-1} g(X_k, \phi_0)(n(\mathbf{W}_k^2 - 1))] \cdot [2 \sum_{k=0}^{m-1} g^2(X_k, \phi_0)]^{-1}
\]
in $\mathbb{P}_n^{\phi_0}$-probability as $n \to \infty$, where $\mathbf{W}_k = W_{k+1} - W_k$.

In particular, if $1(x) = x^2$, then for any $\varepsilon > 0$ and for sufficiently large $T$
\[
\lim_{n \to \infty} \sup_{\phi \neq \phi_0} n^{1/2}(\mathbb{P}_n^\phi - \mathbb{P}_n^{\phi_0})^2 \geq [\mu(g^2)] - \varepsilon,
\]
where $\mu$ is the invariant measure of $\xi$. 

ON A CLASS OF WEAK ASPLUIND SPACES WHICH HAS SOME PERMANENCE PROPERTIES

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Real Banach spaces, $X, Y, \ldots$ are considered. The set of all