Commentationes Mathematicae Universitatis Carolinae

Thomas S. Angell; Ralph Ellis Kleinman; Josef Král

Double layer potentials on boundaries with corners and edges

Commentationes Mathematicae Universitatis Carolinae, Vol. 27 (1986), No. 2, 419

Persistent URL: http://dml.cz/dmlcz/106461

DOUBLE LAYER POTENTIALS ON BOUNDARIES WITH CORNERS AND EDGES

T.S. Angell and R.E. Kleinman (Department of Mathematical Sciences, University of Delaware, Newark, Delaware 19711,U.S.A.) and J. Král (Matematický ústav ČSAV, Žitná 25, 11567 Praha 1, ČSSR),
received 20.1. 1986.

We say that a bounded open set $D \subset R^3$ is rectangular if each point z in the boundary ∂D of D has a neighbourhood $U \subset \partial D$ homeomorphic with R^2 such that U is contained in the three planes passing through z parallel to the coordinate planes. If $z \in \partial D$ is not situated on an edge then n(z) denotes the unit exterior normal to D at z; otherwise n(z)=0 (= the zero vector in R^3). $C(\partial D)$ stands for the space of all continuous functions on ∂D and H_2 denotes the surface measure on ∂D. For each $f \in C(\partial D)$ the double layer potential

$$Wf(x) = (4\pi)^{-1} \int_{\partial D} f(z)n(z)(z-x)/|z-x|^3 \, dH_2(z)$$

is a harmonic function of the variable $x \in R^3 \setminus \partial D$ admitting a continuous extension from D to ∂D: $W_i f(z) = \lim_{\substack{x \to z \\ x \in D}} Wf(x)$, $z \in \partial D$. $W_i : f \mapsto W_i f$ is a bounded linear operator acting on $C(\partial D)$. Let us denote by $\|\cdot\|$ the usual maximum norm, by I the identity operator, by Q the space of all compact linear operators on $C(\partial D)$. As shown in [1] J. Král and W. Wendland: Some examples concerning applicability of the Fredholm-Radon method in potential theory, Aplikace matematiky 31(1986).
it may happen for simple rectangular sets that

$$\inf \{ \|W_i - \alpha I - T\| \; ; \; T \in Q \} / |\alpha| \geq 1$$

for each value of the parameter $\alpha \neq 0$. The ideas described in [1] together with some geometrical considerations permit to establish the following result.

Theorem. For each rectangular set D there is a norm p inducing the topology of uniform convergence on $C(\partial D)$ such that

$$2 \inf \{ p(W_i - \tfrac{1}{2} I - T); \; T \in Q \} < 1 .$$

This result has applications in connection with potential-theoretic boundary value problems; it implies, in particular, that for each rectangular set D with a connected complement the corresponding operator W_i is invertible on $C(\partial D)$ (cf.[1]).