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ANNOUNCEMENTS OF NEW RESULTS

DOUBLE LAYER POTENTIALS ON BOUNDARIES WITH CORNERS AND EDGES
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We say that a bounded open set \( D \subset \mathbb{R}^3 \) is rectangular if each point \( z \) in the boundary \( \partial D \) of \( D \) has a neighbourhood \( U \subset \partial D \) homeomorphic with \( \mathbb{R}^2 \) such that \( U \) is contained in the three planes passing through \( z \) parallel to the coordinate planes. If \( z \in \partial D \) is not situated on an edge then \( n(z) \) denotes the unit exterior normal to \( D \) at \( z \); otherwise \( n(z) = 0 \) (= the zero vector in \( \mathbb{R}^3 \)). \( C(\partial D) \) stands for the space of all continuous functions on \( \partial D \) and \( H_2 \) denotes the surface measure on \( \partial D \). For each \( f \in C(\partial D) \) the double layer potential
\[
W_f(x) = (4\pi)^{-1} \int_{\partial D} f(z)n(z)(z-x)/|z-x|^3 \, dH_2(z)
\]
is a harmonic function of the variable \( x \in \mathbb{R}^3 \setminus \partial D \) admitting a continuous extension from \( D \) to \( \partial D \); \( \lim_{x \to z} W_f(x), \ z \in \partial D \).

We denote by \( \| \cdot \| \) the usual maximum norm, by \( I \) the identity operator, by \( Q \) the space of all compact linear operators on \( C(\partial D) \). As shown in [1] J. Král and W. Wendland: Some examples concerning applicability of the Fredholm-Radon method in potential theory, Aplikace matematiky 31(1986).

It may happen for simple rectangular sets that
\[
\inf \{ \| W_1 - \alpha I \| ; \ T \in Q \} \| \alpha \| \geq 1
\]
for each value of the parameter \( \alpha \neq 0 \). The ideas described in [1] together with some geometrical considerations permit to establish the following result.

Theorem. For each rectangular set \( D \) there is a norm \( p \) inducing the topology of uniform convergence on \( C(\partial D) \) such that
\[
2 \inf \{ \| p(W_1 - \frac{1}{2} I - T) ; \ T \in Q \| ; \ T \in Q \} \leq 1.
\]

This result has applications in connection with potential-theoretic boundary value problems; it implies, in particular, that for each rectangular set \( D \) with a connected complement the corresponding operator \( W_1 \) is invertible on \( C(\partial D) \) (cf. [1]).