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CONVERGENCE CRITERION FOR MULTIPARAMETER STOCHASTIC PROCESSES

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Bickel and Wichura [1] extended the tightness criterion from processes on $D(0,1)$ (see Billingsley [2]) to processes on $D(0,1)^k$, $k>1$. However, they imposed an additional condition that the processes should vanish along the lower boundary of $<0,1)^k$. This means that their criterion does not apply to many empirical processes of interest.

We shall provide an improved tightness criterion for processes in $D(0,1)^k$ without the above additional condition.

**Definition:** Let $k \in \mathbb{N}$, $d=1,...,k$, $j=0,...,k-d$, $\varphi: (0,1)^k \rightarrow (0,1)^k$ be a permutation of coordinates and $X=(X(t), t \in (0,1)^k)$ be a random process. Define
\begin{equation}
\Delta X(d,j, \varphi)(A_1^{a_1=b_1}, \ldots, A_k^{a_k=b_k}) = \sum_{a_1=b_1}^{A_1} \cdots \sum_{a_k=b_k}^{A_k} (-1)^{\sum_{i=1}^{k} a_i} X \cdot \varphi(d_1, \ldots, d_{k-d}, 0, \ldots, 0, 1, \ldots, 1) \end{equation}
for every $0 \leq a_i < b_i$, $i=1,...,d$.

We shall prove the following theorem.

**Theorem:** Let $X=(X(t), t \in (0,1)^k)$, $k \in \mathbb{N}$, be a random process right-continuous in every coordinate. Let $\mu_d[A, \varphi]$, $d=1,...,k$, $j=0,...,k-d$ and $\varphi:(0,1)^k \rightarrow (0,1)^k$ being a permutation of coordinates, be a bounded measure with continuous marginals. If there exists $\alpha, \beta > 0$ such that
\begin{equation}
P(\Delta X(d,j, \varphi)(A) > y, |\Delta X(d,j, \varphi)(B)| > y) \leq y^{-\alpha} \mu_d[A, \varphi](A \cup B)^{1+\beta} \end{equation}
holds for every $y > 0$, $d=1,...,k$, $j=0,...,k-d$ and every permutation $\varphi$ and for all
\begin{align*}
A &= A_1^{a_1=b_1} \cdots A_k^{a_k=b_k}, & B &= B_1^{a_1=b_1} \cdots B_k^{a_k=b_k}, \end{align*}
$A \cap B = \emptyset$, $\clo A \cap \clo B = \emptyset$, then there exist an absolute constant $Q > 0$ and a function $R:(0,1)^{k-1} \rightarrow \mathbb{R}$, $\lim_{t \rightarrow 0} R(t) = 0$, such that
\begin{equation}
P(\sup_{t} \min_{v \in (0,1)^{k-1}} |X \cdot \varphi(t,u) - X \cdot \varphi(s,u)|, |X \cdot \varphi(s,u) - X \cdot \varphi(v,u)|) \leq Q y^{-R(t)} \end{equation}
for every $a \in (0,1)^{k-1}$.

If $k=1$ then the criterion (2) reduces to the criterion in Billingsley [2] (see Theorem 15.6) while it is an improvement of the criterion of [1] if $k>1$.

References:

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