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Toshihide Ibaraki; Svatopluk Poljak  
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### THREE-LINKING IN EULERIAN DIGRAPHS

T.Ibaraki(Kyoto University, Faculty of Engineering, Dept. of Applied Math. and Physics) S.Poljak (Charles University, Faculty of Mathematics and Physics, Dept. of Applied Math., received 30.12.1988)

Let  $G$  be an Eulerian digraph and  $a, b$  and  $c$  an ordered triple of its vertices. We say that an instance  $(G; a, b, c)$  is feasible, if there are three edge disjoint paths  $P_{ab}, P_{bc}$  and  $P_{ca}$ , where  $P_{xy}$  denotes a directed path from  $x$  to  $y$ .

We say that an instance  $(G; a, b, c)$  is minimal infeasible, if it is infeasible, and after contraction of any edge whose both ends are not simultaneously in  $\{a, b, c\}$  we get a graph  $G'$  such that  $(G'; a, b, c)$  is feasible.

**Theorem 1.** Let  $(G; a, b, c)$  be a minimal infeasible instance. Then  $G$  has the following properties.

- (i)  $G$  is planar 2-connected. Vertices  $a, b$  and  $c$  have degree 2, and all other vertices have degree 4.
- (ii) Every face of  $G$  is a directed cycle, or equivalently, the edges incident to a vertex are alternatively oriented out and in.
- (iii) Vertices  $a, b$  and  $c$  lie on the outer face which goes through them in the order  $c, b$  and  $a$ .

Conversely, any instance  $(G; a, b, c)$  satisfying (i), (ii) and (iii) is infeasible.

**Theorem 2.** There is a polynomial time algorithm to decide whether an instance  $(G; a, b, c)$  is feasible or infeasible.

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<sup>0</sup>The results was obtained when the second author was visiting the Kyoto University, Faculty of Engineering, Department of Applied Mathematics and Physics.