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THREE-LINKING IN EULERIAN DIGRAPHS

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Let G be an Eulerian digraph and a, b and c an ordered triple of its vertices. We say that an instance $(G; a, b, c)$ is feasible, if there are three edge disjoint paths P_{ab}, P_{bc} and P_{ca}, where P_{xy} denotes a directed path from x to y.

We say that an instance $(G; a, b, c)$ is minimal infeasible, if it is infeasible, and after contraction of any edge whose both ends are not simultaneously in $\{a, b, c\}$ we get a graph G' such that $(G'; a, b, c)$ is feasible.

Theorem 1. Let $(G; a, b, c)$ be a minimal infeasible instance. Then G has the following properties.

(i) G is planar 2–connected. Vertices a, b and c have degree 2, and all other vertices have degree 4.

(ii) Every face of G is a directed cycle, or equivalently, the edges incident to a vertex are alternatively oriented out and in.

(iii) Vertices a, b and c lie on the outer face which goes through them in the order c, b and a.

Conversely, any instance $(G; a, b, c)$ satisfying (i), (ii) and (iii) is infeasible.

Theorem 2. There is a polynomial time algorithm to decide whether an instance $(G; a, b, c)$ is feasible or infeasible.

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