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ABSTRACTS OF CSc. THESES IN MATHEMATICS (Candidatus Scientiarum) defended recently at Charles University, Prague

PERFECT CODES IN GRAPHS AND THEIR CARTESIAN POWERS

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The theory of error–correcting codes belongs to popular applications of combinatorics. A special attention has been devoted to perfect codes which have been shown to exist quite rarely [**4;15**]. The classical notion of a perfect code in Hamming or Lee metrics has been generalized by Biggs [**3**] to perfect codes in graphs (given a graph $G = (V, E)$, a t–perfect code in G is a subset $C \subset V$ such that every vertex of G is at distance at most t from exactly one code–vertex of C). However, almost exclusively distance–regular graphs were considered [**7;14**] because of the high symmetry of those graphs enabling to prove a strong necessary condition for the existence of perfect codes [**3**]. Perfect codes in general graphs have been studied in [5,8], and from the domination theory point of view in [**6**].

The aim of the thesis was to study questions related to the existence of perfect codes in general graphs from different points of view. It is proved in chapter 2 that deciding whether a given graph contains a t–perfect code is NP–complete for every $t \geq 1$ (cf.[**9**]), but it is polynomially solvable in graphs of bounded tree–width (in particular in trees). (For $t = 1$, both these results have been proved independently by Fellows [**6**]. A stronger result has been afterwards achieved by Křivánek and the author [**11**] – the decision problem remains NP–complete when restricted to k–regular graphs (for every $k \geq 3$) or to planar 3–regular graphs. Recently, solving a problem of Fellows [**6**], the author has proved the following result related to perfect codes – for every $n \geq 4$, given an $(n-1)$–regular graph G, it is NP–complete to decide whether the vertex set of G may be partitioned into n 1–perfect codes, i.e. whether G admits a K_n–cover (cf. [**1,3**])).

Two–graphs, i.e. equivalence classes under Seidel's switching are considered in chapter 3. In particular, 2–graphs all graphs of which contain 1–perfect codes, are characterized (note that the characterization yields a polynomial decision algorithm). In chapter 4, the probability of the existence of a 1–perfect code in the random graph $G_{n,p}$ is studied. The treshold functions for which a 1–perfect code ceases and begins to occur in $G_{n,p}$, are determined (cf.[**12**]. It is also proved in [**12**] that the second power of a random graph is far less probable to contain a 1–perfect code – if p is such that $G_{n,p}$ is almost surely nonempty then $G^2_{n,p}$ contains no 1–perfect code a.s.).

Perfect codes in cartesian powers of graphs (i.e. perfect codes over structured alphabets) are dealt with in the second part of the thesis. An older result of the author saying that there are no nontrivial 1–perfect codes over complete bipartite graphs except $K_{1,1} = K_2$ (cf. [8]) is mentioned here and extended to regular complete k-partite graphs. To the most interesting results of this part, the following two belong: 1) If C is a 1-perfect code in the second power of a graph G with n vertices then card $C \geq n$. It is proved here (cf. [10])that G^2 contains a 1-perfect code of this minimum possible cardinality if and only if G is a self-complementary graph. 2) The isolated vertex and the path of length 3 are the only two trees such that their second powers contain 1-perfect codes.

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ABSTRACT ANALYSIS OF KOROVKIN APPROXIMATION THEORY

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The dissertation is devoted to abstract analysis of Korovkin approximation theory. In the part presenting the results, the (abstract) sufficient conditions are established that a separable Banach lattice E contains the Korovkin set for E. They are obtained by the help of the "order universality" of $C[0,1]$ (the Krejn's embedding theorem). Further, the qualities of the injective linear mapping are discussed, which maps the separable Banach lattice into $C[0,1]$ and the elements from its cone (but only the elements from the cone) map on the positive elements of $C[0,1]$. It is proved