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The Mal'tsev operation on countably compact spaces

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Abstract. Let X be a countably compact topological space and $f : X^3 \to X$ be a continuous mapping such that the identity $f(x,y,y) = f(y,y,x) = x$ holds. Then βX is Dugundji. It follows that compact retracts of topological groups are dyadic.

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We say that a topological space X is Mal'tsev if there exists a continuous mapping $f : X^3 \to X$ (called a Mal'tsev operation [M]) such that the identity $f(x,y,y) = f(y,y,x) = x$ holds for all $x, y \in X$. Every topological group G is a Mal'tsev space, since the mapping $(x,y,z) \to xy^{-1}z$ is a Mal'tsev operation on G. M.G.Tkačenko proved in 1981 that compactly–generated topological groups have the Suslin property [T_1]. This result was extended by the author in [U_1]: every σ–compact Mal'tsev space has the Suslin property. In the present paper we show that compact Mal'tsev spaces are Dugundji (Theorem 1). Moreover, if X is a countably compact Mal'tsev space, then βX is Dugundji (Theorem 2). Clearly any retract of a Mal'tsev space is Mal'tsev. It follows that compact retracts of topological groups are Dugundji. This is an extension of the Ivanovskij – Kuzminov theorem (its proof can be found in [U_3]) which says that compact groups are dyadic.

Let X be a subspace of Y. A topological space Z is said to be injective with respect to the pair (X, Y) iff every continuous function $f : X \to Z$ has a continuous extension $\bar{f} : Y \to Z$. For a compact space X the following are equivalent [H]: 1) if Y is a zerodimensional compact space and Z is closed in Y, then X is injective with respect to the pair (Z, Y); 2) if X is a subspace of a compact space Y, then every compact convex subset of a locally convex topological vector space is injective with respect to the pair (X, Y). A compact space X is called Dugundji [P] if one of these conditions holds. Let X be Dugundji. There exist a zero–dimensional compact space Y and an onto mapping $f : Y \to X$. We may assume that Y is a subspace of a Cantor cube 2^m. Since X is injective with respect to $(Y, 2^m)$, there is an extension $f : 2^m \to X$ of f. This shows that Dugundji spaces are dyadic [H].

An equivalence relation R on a space X is open if the quotient mapping $X \to X/R$ is open.

Ščepin's theorem S1 [Š]. *A compact space X is Dugundji iff there exists a family Φ of equivalence relations on X such that:*

1) *for every $R \in \Phi$ the quotient space X/R is metrizable*
2) *Φ is closed under countable intersections*

3) *all* $R \in \Phi$ *are open*
4) Φ *separates points of* X *(that means the intersection* $\bigcap\Phi$ *equals the diagonal of* X^2*).*

Let $S = \{X_\alpha, p^\beta_\alpha, A\}$ be an inverse system of compact metric spaces X_α and onto bonding mappings $p^\beta_\alpha : X_\beta \to X_\alpha$. We call S a Ščepin system if for every countable subset $B \subseteq A$ there exists a least upper bound $\alpha = \sup B \in A$ and the family $\{p^\alpha_\beta : \beta \in B\}$ separates points of X_α. Theorem S1 means that a compact space X is Dugundji iff it is the inverse limit of a Ščepin system $S = \{X_\alpha, p^\beta_\alpha, A\}$ such that the projections $p_\alpha : X \to X_\alpha$ are open (or, equivalently, the bonding mappings p^β_α are open).

Ščepin's theorem S2 [Š]. *Let* $S = \{X_\alpha, p^\beta_\alpha, A\}$ *and* $T = \{Y_\alpha, q^\beta_\alpha, A\}$ *be two Ščepin systems (the directed index set* A *is the same for* S *and* T*),* $X = \lim S, Y = \lim T$*. Let* f *be a mapping of* X *to* Y*. Let* A_f *be the set of all* $\alpha \in A$ *with the following property: there exists* $f_\alpha : X_\alpha \to Y_\alpha$ *such that* $f_\alpha \circ p_\alpha = q_\alpha \circ f$ *(here* $p_\alpha : X \to X_\alpha$ *and* $q_\alpha : Y \to Y_\alpha$ *are the projections). Then* A_f *is cofinal in* A*.*

Let $f : X^3 \to X$ be a Mal'tsev operation on a space X. An equivalence relation R on X is called a congruence (or an f-congruence) if $f(x_1, x_2, x_3)$ is R-equivalent to $f(y_1, y_2, y_3)$ whenever x_i is R-equivalent to y_i, $i = 1, 2, 3$.

Mal'tsev's theorem [M]. *If* f *is a Mal'tsev operation on a space* X*, then every* f*-congruence* R *on* X *is open.*

PROOF : Let U be open in X and $V = \{y \in X : (x, y) \in R \text{ for some } x \in U\}$. We have to show that V is open. Let $y \in V$. Choose $x \in U$ so that $(x, y) \in R$. If z is close enough to y, then $f(x, y, z)$ is in U, since $f(x, y, y) = x$ is in U. On the other hand, $f(x, y, z)$ is R-equivalent to $f(x, x, z) = z$. Hence V is a neighbourhood of y. ∎

Theorem 1. *Every compact Mal'tsev space is Dugundji.*

PROOF : Let $f : X^3 \to X$ be a Mal'tsev operation on a compact space X. Let Φ be the family of all f-congruences R on X such that the quotient space X/R is metrizable. Clearly Φ is closed under countable intersections, so conditions 1) and 2) of Ščepin's theorem S1 are satisfied. Mal'tsev's theorem shows that condition 3) holds, too. It remains to prove that Φ separates points of X. Let $S = \{X_\alpha, p^\beta_\alpha, A\}$ be a Ščepin system whose limit is X. Then X^3 is the limit of Ščepin's system $S^3 = \{(X_\alpha)^3, (p^\beta_\alpha)^3, A\}$. Let $p_\alpha : X \to X_\alpha$ be the projections of S. Call $\alpha \in A$ nice if there exists $f_\alpha : X^3_\alpha \to X_\alpha$ such that $f_\alpha \circ (p_\alpha)^3 = p_\alpha \circ f$. The relation $R_\alpha = \{(x, y) \in X^2 : p_\alpha(x) = p_\alpha(y)\}$ is an f-congruence whenever α is nice. Theorem S2 implies that the set A_f of all nice α is cofinal in A. Hence the subfamily $\{R_\alpha : \alpha \in A_f\}$ of Φ separates points of X. ∎

If βX is Dugundji, then X is pseudocompact. We give a characterization of spaces X for which βX is Dugundji.

Proposition 1. *Let X be pseudocompact. Then βX is Dugundji if and only if there exists a family Φ of equivalence relations on X such that:*

1) *for every $R \in \Phi$ the quotient space X/R is submetrizable*
2) *Φ is closed under countable intersections*
3) *every $R \in \Phi$ is open*
4) *the topology of X is determined by the quotient mappings $f_R : X \to X/R$, $R \in \Phi$.*

We need some lemmas. Recall that a mapping $p : Y \to Z$ is d–open if one of the equivalent conditions holds:

1) $\mathrm{cl}\, p^{-1}(U) = p^{-1}(\mathrm{cl}\, U)$ for every U open in Z
2) $p(V) \subseteq \mathrm{int}\,\mathrm{cl}\, p(V)$ for every V open in Y.

Lemma 1. *If X is pseudocompact and M is metric, then every d-open mapping $f : X \to M$ is open.*

PROOF : Let U be open in X and $x \in U$. Choose an open set V so that $x \in V \subseteq \mathrm{cl}\, V \subseteq U$. Since $\mathrm{cl}\, V$ is pseudocompact, $f(\mathrm{cl}\, V)$ is closed in M. It follows that $\mathrm{cl}\, f(V) = f(\mathrm{cl}\, V)$, hence $f(x) \in \mathrm{int}\,\mathrm{cl}\, f(V) = \mathrm{int}\, f(\mathrm{cl}\, V) \subseteq \mathrm{int}\, f(U)$. This means that $f(U)$ is open. ■

Lemma 2. *Let X be pseudocompact. If R is an open equivalence relation on X such that the quotient space X/R is submetrizable, then X/R is metrizable (and hence compact).*

PROOF : Let $f : X \to X/R$ be the quotient mapping, $g : X/R \to K$ be a one–to–one mapping of X/R onto a compact metric space K, and $h = g \circ f$. We have to show that g is a homeomorphism, or, equivalently, that h is open. In virtue of Lemma 1 it suffices to show that h is d–open. Let U be open in K and $F = \mathrm{cl}\, h^{-1}U$. Since f is open, $F = \mathrm{cl}\, f^{-1}g^{-1}(U) = f^{-1}\,\mathrm{cl}\, g^{-1}U = h^{-1}E$ for some $E \subseteq K$. Now F is pseudocompact, being a regular closed set in X, so $E = h(F)$ is closed in K. It follows that $E = \mathrm{cl}\, U$ and $\mathrm{cl}\, h^{-1}U = h^{-1}\,\mathrm{cl}\, U$. This means that h is d–open. ■

Lemma 3. *Let X be G_δ-dense in Y (= every non-empty G_δ-subset of Y meets X). If f maps Y to a metric space M and the restriction $f|X : X \to M$ is open, then $f : Y \to M$ is open.*

PROOF : If U is open in Y, then $f(U) = f(U \cap X)$. ■

Lemma 4. *If Y is Dugundji–compact and X is a dense subspace of Y, then the following are equivalent:*

1) $Y = \beta X$
2) *X is pseudocompact*
3) *X is G_δ-dense in Y.*

PROOF : Dugundji spaces are perfectly κ–normal (= closures of open sets are zero–sets), and G_δ–dense subspaces of perfectly κ–normal spaces are C–embedded [T2]. This gives 3)⇒1). If βX is dyadic, then X is pseudocompact [EP]. Since Dugundji spaces are dyadic, 1)⇒2) follows; and 2)⇒3) is obvious. ■

PROOF of Proposition 1: The "only if" part follows from theorem S1 and Lemma 1. Conversely, let Φ be such a family as described in Proposition 1. The spaces $X/R, R \in \Phi$, are compact and constitute a Ščepin system S. Let $Y = \lim S$. Then X can be regarded as a G_δ–dense subspace of Y. The open mappings $f_R : X \to X/R$ are restrictions of the projections of S, so Lemma 3 shows that the projections of S are open. Theorem S1 implies that Y is Dugundji, and Lemma 4 shows that $Y = \beta X$. ■

Suppose G is a subgroup of the product of metrizable groups, H is a closed subgroup of G. Let $X = G/H$ be the quotient space. If X is pseudocompact, then βX is Dugundji. This follows from Proposition 1, combined with Lemma 7 in [U3]. Theorem 1 in [U3] also can be extended to the pseudocompact case: if G is a subgroup of the product of groups with a countable base and G acts transitively on a pseudocompact space X, then βX is Dugundji. If a pseudocompact space X is a retract of a topological group, then βX is Dugundji [U4].

Conjecture 1. If X is pseudocompact and Mal'tsev, then βX is Dugundji.

Conjecture 2. The product of any two pseudocompact Mal'tsev spaces is pseudocompact.

Proposition 2. *Conjectures 1 and 2 are equivalent.*

PROOF : Suppose $\{X_\alpha : \alpha \in A\}$ is a family of pseudocompact spaces such that βX_α is Dugundji for every $\alpha \in A$. Then $Q = \Pi \beta X_\alpha$ is Dugundji and $P = \Pi X_\alpha$ is G_δ–dense in Q. Lemma 4 implies that P is pseudocompact. Hence if Conjecture 1 is true, then the product of any family of pseudocompact Mal'tsev spaces is pseudocompact. Conversely, let $f : X^3 \to X$ be a Mal'tsev operation on a pseudocompact space X. If Conjecture 2 is true, then X^3 is pseudocompact. By virtue of Glicksberg's theorem [G], [E, problem 3.12.20], $\beta(X^3) = (\beta X)^3$, hence f has an extension $\bar{f} : (\beta X)^3 \to \beta X$. Clearly $\bar{f}$ is a Mal'tsev operation on βX, so Theorem 1 shows that βX is Dugundji. ■

We shall prove that Conjecture 1 is true for countably compact spaces (Theorem 2).

Rezničenko's theorem R1. *Suppose X_1, X_2, X_3 are countably compact and have the Suslin property, M has a countable base, $f : X_1 \times X_2 \times X_3 \to M$ is separately continuous (i.e., if x_1, x_2, x_3 are fixed, $x_i \in X_i$, then the functions $f(x_1, x_2, \cdot), f(x_1, \cdot, x_3)$, $f(\cdot, x_2, x_3)$ are continuous on X_3, X_2, X_1, respectively). Then there exist compact metric spaces Y_1, Y_2, Y_3, continuous onto mappings $p_i : X_i \to Y_i, i = 1, 2, 3$, and a separately continuous function $g : Y_1 \times Y_2 \times Y_3 \to M$ such that $f = g \circ (p_1 \times p_2 \times p_3)$.*

Rezničenko's theorem R2. *Suppose X_1 and X_2 are pseudocompact, M has a countable base, $f : X_1 \times X_2 \to M$ is (jointly) continuous. Then there exist Eberlein-compact spaces Y_1, Y_2, continuous onto mappings $p_i : X_i \to Y_i$, and a separately continuous function $g : Y_1 \times Y_2 \to M$ such that $f = g \circ (p_1 \times p_2)$. If X_1 and X_2 have the Suslin property, then Y_1 and Y_2 are metrizable.*

Recall that a compact space X is Eberlein iff X embeds in the function space $C_p(Y)$ for some compact Y.

Proposition 3. *Suppose X is a Tikhonov countably compact space having the Suslin property. If there exists a separately continuous Mal'tsev operation $f : X^3 \to X$, then βX is Dugundji.*

PROOF : Let $\Psi = \{R : R \text{ is an equivalence relation on } X \text{ and } X/R \text{ is submetrizable}\}$ and $\Phi = \{R \in \Psi : R \text{ is an } f\text{–congruence}\}$. Mal'tsev's theorem shows that every $R \in \Phi$ is open. Clearly Φ is closed under countable intersections. To check the condition 4) of Proposition 1, it suffices to prove that for every $T \in \Psi$ there exists $R \in \Phi$ such that $R \subseteq T$. Let $T \in \Psi$. Using theorem R1, construct a sequence $T_0 = T, T_1, T_2 \ldots$ such that every T_n is in Ψ and the conditions $(x_i, y_i) \in T_{n+1}, i = 1,2,3$, imply that $f(x_1, x_2, x_3)$ and $f(y_1, y_2, y_3)$ are T_n–equivalent, $n = 0, 1, \ldots$ Then $R \cap T_n$ is a congruence, so $R \in \Phi$. It follows that Φ satisfies the conditions of Proposition 1. Hence βX is Dugundji. ∎

Definition 1. A space X has the property (T) iff for every family $\{x_\alpha : \alpha < \omega_1\}$ of points of X and every family $\{\gamma_\alpha : \alpha < \omega_1\}$ of open covers of X there exist $\alpha, \beta < \omega_1$ such that $\alpha \neq \beta$ and $St(x_\alpha, \gamma_\beta)$ meets $St(x_\beta, \gamma_\alpha)$ (where $St(x, \gamma) = \cup\{U \in \gamma : U \ni x\}$).

This property was considered by M.Tkačenko [**T1**] who proved that every compact space has the property (T). Moreover, every Lindelöf Σ–space has the property (T) [**U1**],[**U2**]. A space X is a Σ–space [**N**] iff there are two covers F and C of X such that F is σ–locally finite , C consists of countably compact sets and for every $K \in C$ and every neighbourhood U of K there exists $E \in F$ such that $K \subseteq E \subseteq U$. Now we prove that countably compact spaces have the property (T).

Proposition 4. *If X is a Σ–space in which every closed discrete subset is countable, then X has the property (T).*

PROOF : Let $\{x_\alpha : \alpha < \omega_1\}$ be a family of points of X and $\{\gamma_\alpha : \alpha < \omega_1\}$ be a family of open covers of X. For $\alpha < \omega_1$ put $X_\alpha = \{x_\beta : \beta < \alpha\}$ and $\lambda_\alpha = \{X_\alpha \cap St(x_\alpha, \gamma_\beta) : \beta < \alpha\}$. For $E \subseteq X$ let $S(E)$ be the set of all $\alpha < \omega_1$ such that $x_\alpha \in E$ and the family $\lambda_\alpha | E = \{L \cap E : L \in \lambda_\alpha\}$ does not have the finite intersection property. We assert that for every $E \subseteq X$ the set $S(E)$ is non–stationary (i.e. some closed unbounded subset of ω_1 does not meet $S(E)$). For every finite set $A \subseteq \omega_1$ fix an open cover γ_A of X which refines γ_α for every $\alpha \in A$. If $\alpha \in S(E)$, there is a finite set $A = f(\alpha) \subseteq \alpha$ such that $X_\alpha \cap E \cap St(x_\alpha, \gamma_A) = \emptyset$. Suppose $S(E)$ is stationary. Fodor's Lemma [**K**] implies there is an uncountable subset $B \subseteq S(E)$ and a finite set $A \subseteq \omega_1$ such that $f(\alpha) = A$ for every $\alpha \in B$. If $\alpha, \beta \in B$ and $\beta < \alpha$, then $x_\beta \in X_\alpha \cap E \subseteq X \setminus St(x_\alpha, \gamma_A)$. It follows that the set $\{x_\alpha : \alpha \in B\}$ is closed discrete in X, since every element of γ_A contains at most one point of this set. This contradiction proves that $S(E)$ is non–stationary.

Let F and C be two covers of X witnessing the Σ–property. Then F is countable. Since the union of countably many non–stationary subsets of ω_1 is non–stationary [**K**], there exists $\alpha \in \omega_1 \setminus \{0\}$ such that $\alpha \notin S(E)$ for every $E \in F$. Let K be an element of C containing x_α. If $E \in F$ and $K \subseteq E$, then the family $\lambda_\alpha | E$ has finite intersection property. It follows that the family $\mu = \{K \cap \overline{L} : L \in \lambda_\alpha\}$ also has the finite intersection property. Since K is countably compact and μ is countable, there exists a point $z \in \bigcap\mu$. Then $z \in \overline{X_\alpha}$ and $z \in \overline{St(x_\alpha, \gamma_\beta)}$ for every $\beta < \alpha$. Let U be an element of γ_α containing z. Since $z \in \overline{X_\alpha}$, U meets X_α, so there is $\beta < \alpha$ such that $x_\beta \in U$. Since $z \in \overline{St(x_\alpha, \gamma_\beta)}$ and $z \in U \subseteq St(x_\beta, \gamma_\alpha)$, the intersection $St(x_\alpha, \gamma_\beta) \cap St(x_\beta, \gamma_\alpha)$ is non–empty. The proof is complete. ∎

Proposition 5. *If a Mal'tsev space X has the property (T) then X has the Suslin property.*

PROOF : Let $\{O_\alpha : \alpha < \omega_1\}$ be a family of non–empty open sets in X. We have to show that this family is not disjoint. For every $\alpha < \omega_1$ pick $x_\alpha \in O_\alpha$. Let $f : X^3 \to X$ be a Mal'tsev operation. Call a set $U \subseteq X$ α–small if $f(x_\alpha, y, z) \in O_\alpha$ and $f(y, z, x_\alpha) \in O_\alpha$ whenever $y, z \in U$. Since $f(x_\alpha, y, y) = f(y, y, x_\alpha) = x_\alpha \in O_\alpha$ for every $y \in X$, the collection γ_α of all α–small open sets covers X. The property (T) implies there are distinct $\alpha, \beta < \omega_1$ and a point $z \in X$ such that $z \in St(x_\alpha, \gamma_\beta) \cap St(x_\beta, \gamma_\alpha)$. Then $f(x_\alpha, z, x_\beta) \in O_\alpha \cap O_\beta \neq \emptyset$. ∎

Propositions 4 and 5 imply

Proposition 6. *If X is a Mal'tsev Σ–space in which every closed discrete subset is countable, then X has the Suslin property.*

Let us say that a space X is G_δ–cellular if for every family λ of G_δ–sets in X there exists a countable subfamily $\mu \subseteq \lambda$ whose union is dense in the union of λ. It can be shown that under the assumptions of Proposition 6, X is G_δ–cellular. For Mal'tsev Lindelöf Σ–spaces this fact is proved in [U2]. In particular, σ–compact topological groups are G_δ–cellular.

Propositions 3 and 6 imply the main result of the paper:

Theorem 2. *If X is a countably compact Mal'tsev space, then βX is Dugundji.*

Now we introduce a new class of spaces which is contained in the class of Mal'tsev spaces. For $a \in X$ define subsets X_α and X^a of X^2 as follows: $X_a\{(a, x) : x \in X\}$ and $X^a = \{(x, a) : x \in X\}$. Let $\Delta_X = \{(x, x) : xX\}$ be the diagonal of X^2.

Definition 2. A space X has a rectifiable diagonal iff there is a homeomorphism $f : X^2 \to X^2$ of X^2 onto itself such that:

1) $f(X_x) = X_x$ for every $x \in X$;
2) $f(\Delta_X) = X^a$ for some $a \in X$.

Every topological group has a rectifiable diagonal, and every space with a rectifiable diagonal is homogeneous.

Proposition 7 (M.M.Čoban). *A space X has a rectifiable diagonal if and only if there exist mappings $p : X^2 \to X$ and $q : X^2 \to X$ satisfying the identities $p(x, q(x, y)) = q(x, p(x, y)) = y$ and $p(x, x) = p(y, y)$.*

PROOF : If p and q are as above, let $f(x, y) = (x, p(x, y))$. Then $f^{-1}(x, y) = (x, q(x, y))$, so f is a homeomorphism of X^2 onto itself. If $p(x, x) = a$ for all $x \in X$, then $f(\Delta_X) = X^\alpha$. Conversely, if f is as in definition 2, the same formulas $f(x, y) = (x, p(x, y))$ and $f^{-1}(x, y) = (x, q(x, y))$ define p and q. ∎

Corollary. *Every space with a rectifiable diagonal is Mal'tsev.*

PROOF : If p and q are as in Proposition 7, then $(x, y, z) \to q(x, p(y, z))$ is a Mal'tsev operation. ∎

Theorem 3. *Let X be a pseudocompact space with a rectifiable diagonal. If X has the Suslin property, then βX is Dugundji.*

PROOF : Let p and q be as in Proposition 7. We say that an equivalence relation R on X is a congruence if the following conditions holds: if x_i is R–equivalent to $y_i, i = 1, 2$, then $p(x_1, x_2)$ is R–equivalent to $p(y_1, y_2)$ and $q(x_1, x_2)$ is R–equivalent to $q(y_1, y_2)$. Let $\Phi = \{R : R$ is a congruence and X/R is submetrizable$\}$. Then Φ satisfies the conditions of Proposition 1. If R is a congruence, then R is also an m–congruence, where $m(x, y, z) = q(x, p(y, z))$ is a Mal'tsev operation, so the Mal'tsev theorem shows that R is open. It remains to show that the quotient mappings $X \to X/R, R \in \Phi$, determine the topology of X. The argument is essentially the same as in the proof of Proposition 3, only Rezničenko's theorem R2 should be used instead of R1. ∎

Questions. Is it true that every pseudocompact space with a rectifiable diagonal has the Suslin property? Is it true that every space with a rectifiable diagonal is a retract of a topological group?

O.V.Sipačeva has recently proved that every compact Mal'tsev space is a retract of a topological group. It can be shown that the same is true for countably compact Mal'tsev spaces.

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