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Errata to the paper “On paracompact locales and metric locales”

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ERRATA TO THE PAPER  
 "ON PARACOMPACT LOCALES AND METRIC LOCALES"  
 by Sun Shu-Hao  
 in Comment. Math. Univ. Carolinae 30 (1989), 101-107

By mistake of the editorial procedure, the symbol  $\ll$  was printed as  $\leq$ , which makes the paper difficult to understand. The following are the correct formulations:

**p.101**, lines 10,9 from the bottom:

**Lemma 1.** *Let  $\{x_i : i \in J\}$  be locally finite and let  $x_i \ll y_i$  for all  $i \in J$ . Then we have  $\bigvee \{x_i : i \in J\} \ll \bigvee \{y_i : i \in J\}$ , where  $b \ll a$  denotes  $\neg b \vee a = 1$ .*

**p.102**, lines 1-3 from the top:

That is,  $c \leq \neg \bigvee x_i \vee \bigvee y_i$ , and since  $C$  was a cover, so we have  $\bigvee x_i \ll \bigvee y_i$ . ■

Recall that a locale  $L$  is said to be regular if for each  $a \in L$ , we have  $a = \bigvee \{b \in L : b \ll a\}$ .

**p.102**, line 16 from the bottom:

$$D = \{d \in L : (\exists a \in A)(d \ll a)\}.$$

**p.102**, line 13 from the bottom:

Hence for each  $b \in B$ , there is an  $i(b) \in J = \bigcup J_n$ , say  $i(b) \in J_m$ , such that  $b \ll a_{i(b)}$ .

**p.102**, line 10 from the bottom:

Then we have  $e_{n,i} \ll a_i$  for each  $i \in J$  and each  $n$  by Lemma 1, and  $E_{n,m} \subseteq E_{n,m+1}$ .

**p.103**, line 7 from the bottom:

since  $\bigvee \{e_{k,i} : k \leq n'\} \ll a_i$  for each  $i$  that is  $z \leq \bigvee a_{f(i),i}$ , hence  $z \leq \bigvee \{w_i : i \in$

**p.104**, Proof of Theorem 2:

**PROOF :** Let  $A$  be a regular paracompact locale and let  $B = \{b_r : r \in J\}$  be a co-discrete system. Then there is a cover  $C$  such that for each  $c \in C$ ,  $c \leq b_r$  for all but at most one element  $r \in J$ . By regularity, we see that

$$D = \{d \in A : (\exists c \in C)(d \ll c)\}$$

is a cover of  $A$ . By paracompactness,  $D$  has a locally finite refinement  $Z$  which covers  $A$ . For each  $z \in Z$  we can assign a  $c(z) \in C$  such that  $z \ll c(z)$ . Write

$$z_c = \bigvee \{z \in Z : z \ll c(z) = c\}.$$

By Lemma 1, we see that  $z_c \ll c$  and that  $Z_0 = \{z_c : c \in C\}$  is also locally finite and a cover of  $A$ .

For each  $r \in J$ , we write

$$z_r = \bigvee \{z_c \in Z_0 : c \leq b_r\}.$$

Again by Lemma 1, we have  $z_r \leq b_r$ . Now it remains to show that

$$\tilde{B} = \{z_r : r \in J\}$$

is co-discrete. In fact, for each  $z_c \in Z_0$ , where  $c \in C$ , if  $z_c \not\leq z_{r_0} = \bigvee \{z_{c'} \in Z_0 : c' \leq b_{r_0}\}$ ; then  $c \not\leq b_{r_0}$ . Thus  $c \leq b_r$  for all  $r \neq r_0$ ; hence  $z_c \leq z_r = \bigvee \{z_{c'} \in Z_0 : c' \leq b_r\}$  for all  $r \neq r_0$ .

Furthermore,  $\neg \tilde{B} = \{\neg z_r : r \in J\}$  is discrete and  $\neg z_r \vee b_r = 1$ . ■

Using this occasion, we also correct a mistake in the formulation of Theorem 5 (p. 106):

**Theorem 5.** *For each Boolean locale  $L$ ,  $L$  is c.c.c. iff  $L$  is Lindelöf.*

The editors apologize for causing this unpleasant situation.