Shu Hao Sun

Errata to the paper “On paracompact locales and metric locales”

Commentationes Mathematicae Universitatis Carolinae, Vol. 30 (1989), No. 4, 817--818

Persistent URL: http://dml.cz/dmlcz/106807

ERRATA TO THE PAPER "ON PARACOMPACT LOCALES AND METRIC LOCALES"

by Sun Shu-Hao

in Comment. Math. Univ. Carolinae **30** (1989), 101-107

By mistake of the editorial procedure, the symbol $\leqslant$ was printed as $\leq$, which makes the paper difficult to understand. The following are the correct formulations:

p.101, lines 10,9 from the bottom:

Lemma 1. *Let $\{x_i : i \in J\}$ be locally finite and let $x_i \lessdot y_i$ for all $i \in J$. Then we have $\bigvee\{x_i : i \in J\} \lessdot \bigvee\{y_i : i \in J\}$, where $b \lessdot a$ denotes $\neg b \vee a = 1$.*

p.102, lines 1-3 from the top:

That is, $c \leq \neg \bigvee x_i \vee \bigvee y_i$ and since C was a cover, so we have $\bigvee x_i \lessdot \bigvee y_i$. ∎

Recall that a locale L is said to be regular if for each $a \in L$, we have $a = \bigvee\{b \in L : b \lessdot a\}$.

p.102, line 16 from the bottom:

$$D = \{d \in L : (\exists a \in A)(d \lessdot a)\}.$$

p.102, line 13 from the bottom:

Hence for each $b \in B$, there is an $i(b) \in J = \bigcup J_n$, say $i(b) \in J_m$, such that $b \lessdot a_{i(b)}$.

p.102, line 10 from the bottom:

Then we have $e_{n,i} \lessdot a_i$ for each $i \in J$ and each n by Lemma 1, and $E_{n,m} \subseteq E_{n,m+1}$.

p.103, line 7 from the bottom:

since $\bigvee\{e_{k,i} : k \leq n'\} \lessdot a_i$ for each i that is $z \leq \bigvee a_{f(i),i}$, hence $z \leq \bigvee\{w_i : i \in$

p.104, Proof of Theorem 2:

PROOF : Let A be a regular paracompact locale and let $B = \{b_r : r \in J\}$ be a co-discrete system. Then there is a cover C such that for each $c \in C$, $c \leq b_r$ for all but at most one element $r \in J$. By regularity, we see that

$$D = \{d \in A : (\exists c \in C)(d \lessdot c)\}$$

is a cover of A. By paracompactness, D has a locally finite refinement Z which covers A. For each $z \in Z$ we can assign a $c(z) \in C$ such that $z \lessdot c(z)$. Write

$$z_c = \bigvee\{z \in Z : z \lessdot c(z) = c\}.$$

By Lemma 1, we see that $z_c \lessdot c$ and that $Z_0 = \{z_c : c \in C\}$ is also locally finite and a cover of A.

For each $r \in J$, we write

$$z_r = \bigvee\{z_c \in Z_0 : c \leq b_r\}.$$

Again by Lemma 1, we have $z_r \lessdot b_r$. Now it remains to show that

$$\tilde{B} = \{z_r : r \in J\}$$

is co-discrete. In fact, for each $z_c \in Z_0$, where $c \in C$, if $z_c \nleq z_{r_0} = \bigvee\{z_{c'} \in Z_0 : c' \leq b_{r_0}\}$; then $c \nleq b_{r_0}$. Thus $c \leq b_r$ for all $r \neq r_0$; hence $z_c \leq z_r = \bigvee\{z_{c'} \in Z_0 : c' \leq b_r\}$ for all $r \neq r_0$.

Furthermore, $\neg\tilde{B} = \{\neg z_r : r \in J\}$ is discrete and $\neg z_r \vee b_r = 1$. ∎

Using this occasion, we also correct a mistake in the formulation of Theorem 5 (p. 106):

Theorem 5. *For each Boolean locale L, L is c.c.c. iff L is Lindelöf.*

The editors apologize for causing this unpleasant situation.