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COMPACT NON-NUCLEAR OPERATOR PROBLEM
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The problem (+) of the existence of compact non-nuclear operator reads: Given
two infinite dimensional Banach spaces $X$, $Y$ does there always exist a compact
non-nuclear operator $f : X \to Y$? By Pisier’s space we understand every infinite-di
mensional Banach space $X$ such that $X \otimes \varepsilon X = X \otimes \mathcal{A} X$ and such that $X$ and
$X^*$ are of cotype 2. The problem (+) was studied by several authors and is solved
in the negative by the following

**Theorem.** Let $X$ be a separable Pisier’s space and $Y = X^*$ its dual space. Then
every compact operator $f : X \to Y$ is nuclear.

The proof is based on Lemma 1 and on an approximation result below.

**Lemma 1.** Every approximable operator $f : X \to Y$ is nuclear. (Approximable in
the sense of Pietsch.)

**Lemma 2.** Let $f : E \to F$ be a compact operator factorable through a space
$G$ and let the space $G$ has the approximation property and separable dual $G^*$.
Then $f$ is approximable (i.e. there are finite-dimensional operators $f_n$ such that
$\|f_n - f\| \to 0$).

**Corollary.** Every compact 2-absolutely summing operator $f : E \to F$ on the
separable Banach space $E$ is approximable.