

# Commentationes Mathematicae Universitatis Carolinae

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Compact non-nuclear operator problem

*Commentationes Mathematicae Universitatis Carolinae*, Vol. 30 (1989), No. 4,  
819

Persistent URL: <http://dml.cz/dmlcz/106808>

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## ANNOUNCEMENTS OF NEW RESULTS

(of authors having an address in Czechoslovakia)

### COMPACT NON-NUCLEAR OPERATOR PROBLEM

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Submitted to *Mathematische Annalen*

The problem (+) of the existence of compact non-nuclear operator reads: Given two infinite dimensional Banach spaces  $X, Y$  does there always exist a compact non-nuclear operator  $f : X \rightarrow Y$ ? By Pisier's space we understand every infinite-dimensional Banach space  $X$  such that  $X \otimes_{\epsilon} X = X \otimes_{\mathcal{A}} X$  and such that  $X$  and  $X^*$  are of cotype 2. The problem (+) was studied by several authors and is solved in the negative by the following

**Theorem.** Let  $X$  be a separable Pisier's space and  $Y = X^*$  its dual space. Then every compact operator  $f : X \rightarrow Y$  is nuclear.

The proof is based on Lemma 1 and on an approximation result below.

**Lemma 1.** Every approximable operator  $f : X \rightarrow Y$  is nuclear. (Approximable in the sense of Pietsch.)

**Lemma 2.** Let  $f : E \rightarrow F$  be a compact operator factorable through a space  $G$  and let the space  $G$  has the approximation property and separable dual  $G^*$ . Then  $f$  is approximable (i.e. there are finite-dimensional operators  $f_n$  such that  $\|f_n - f\| \rightarrow 0$ ).

**Corollary.** Every compact 2-absolutely summing operator  $f : E \rightarrow F$  on the separable Banach space  $E$  is approximable.