Archivum Mathematicum

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Archivum Mathematicum, Vol. 17 (1981), No. 1, 3--5

Persistent URL: `http://dml.cz/dmlcz/107084`

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ARCH. MATH. 1, SCRIPTA FAC. SCI. NAT. UJEP BRUNENSIS
XVII: 3—6, 1981

A NOTE ON METRICS AND TOLERANCES

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(Received May 6, 1979)

1. **Introduction.** Some results on Metrics and Tolerances have been given by Chajda and Zelinka [2]. This note is intended to bring out the significance of connectedness of metric space in some of their results. For simplicity we state the results for a metric space and leave their extensions to a pseudometric space (Willard [4]) or to a quasimetric space (Pervin [3]) to the reader.

We introduce a new type of connectedness in an integer-valued metric. Results on connectedness in an integer-valued metric and its relation with discrete dynamical systems will be announced separately.

2. **Definitions** ([2]). A reflexive and symmetric binary relation T on a non-empty set A is said to be a tolerance relation (or tolerance for brevity) on A. Moreover the ordered pair (A, T) is called a tolerance space. Denoting the identity relation I by T^0 we define $T^{n+1} = T \, . \, T^n$ inductively for any positive integer n.

A tolerance space (A, T) is said to be T-connected (or simply connected), if given x, y in A, there exists a non-negative integer p such that $xT^p y$.

Let (A, e) be a metric space and ε a positive real number. Then $T_{e(\varepsilon)}$ denotes the relation (on A) defined by $xT_{e(\varepsilon)}y$ if and only if $e(x, y) \leqq \varepsilon$.

Let (A, T) be a connected tolerance space. Let δ_T denote the integer-valued function on $A \times A$ defined by $\delta_T(x, y) =$ least non-negative integer p such that $xT^p y$.

For the relation $T_{e(\varepsilon)}$ and function δ_T, we have the following

3. **Proposition.** Let (A, e) be a connected metric space and ε a positive real number. Then $(A, T_{e(\varepsilon)})$ is a connected tolerance space. Further if $0 < \varepsilon \leqq 1$, then $\delta_T(x, y) \geqq e(x, y)$ for all x, y in A.

4. **Remark.** Above proposition does not hold without the hypothesis of connectedness on the metric space (A, e). To see this, let $A = \{x\colon 1 \leqq |x| \leqq 2\}$ be a subset of reals with usual metric. Let $\varepsilon = \frac{1}{2}$. Then the tolerance space $(A, T_{e(\varepsilon)})$ is, clearly, not connected.

5. **Definition.** Let (A, d) be an integer-valued metric. Then (A, d) is said to be connected if there do not exist non-empty disjoint sets G and H, $G \subset A$, $H \subset A$ such that $G \cup H = A$ and $\min\{d(x, y): x \in G, y \in H\} > 1$.

6. **Theorem.** *Let (A, d) be an integer-valued metric. The following are equivalent:*

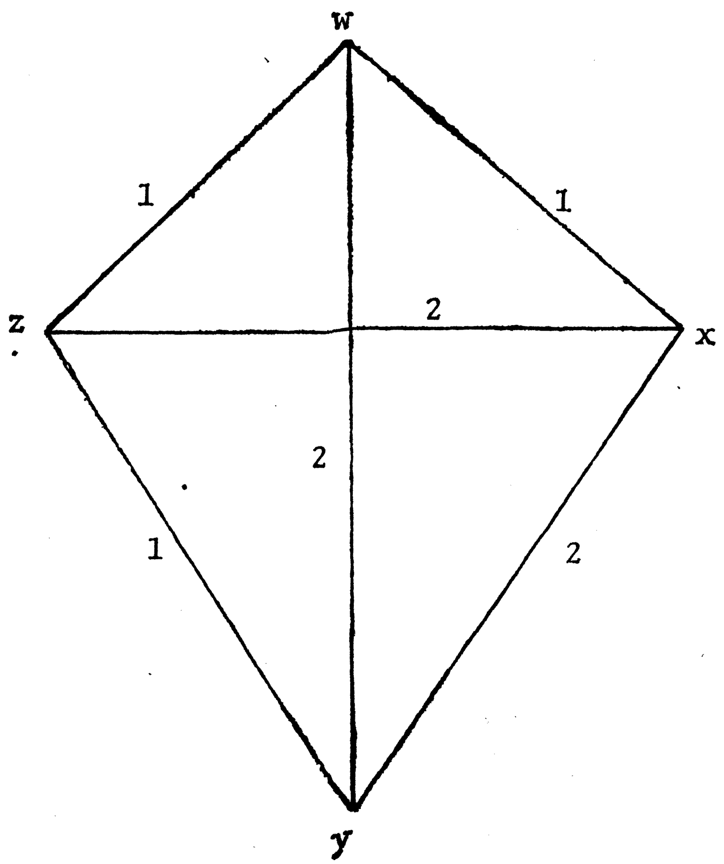
(i) *(A, d) is connected.*

(ii) *For every non-empty proper subset G of A,* $\min\{d(x, y): x \in G, y \notin G\} = 1$.

(iii) *For every proper non-empty subset G of A, $d(x, y) = 1$ for some $x \in G$ and some $y \notin G$.*

(iv) *Given any pair x, y of distinct points in A, there exist points $x = x_1, x_2, x_3, \ldots, \ldots, x_p = y$* such that $d(x_i, x_{i+1}) = 1, i = 1, 2, \ldots, p - 1$.

7. **Theorem.** *Let (A, d) be an integer-valued metric. Let (A, d) be connected (in the sense of definition 5). Define a relation T on A by xTy iff $d(x, y) = 1$. If $\delta_T(x, y)$ is defined as above* (§ 2) *corresponding to the relation T for all x, y in A, then (A, δ_T) is an integer-valued metric. Moreover $\delta_T(x, y) \geqq d(x, y)$ for all x, y in A.*



8. **Remark.** In the above theorem, $\delta_T(x, y) = d(x, y)$ does not, in general, hold. To see this let $A = \{x, y, z, w\}$. Let $d(x, y) = d(x, z) = d(y, w) = 2$, $d(x, w) = d(w, z) = d(z, y) = 1$ and $d(y, y) = d(z, z) = d(w, w) = d(x, x) = 0$. (See Figure 1.) Then (A, d) is a metric space. Moreover $\delta_T(x, y) = 3$ whereas $d(x, y) = 2$.

Acknowledgement. The author is grateful to the referee for his constructive criticism of the earlier version of this paper.

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