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A NOTE ON ALGEBRAIC CATEGORIES

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This note is a sequel to [4] but it can be read independently. Both main results (Corollary 2 and the Example) were motivated by Reiterman [3].

For the precise set-theoretic foundation we will need two universes $\mathscr{U}_1$ and $\mathscr{U}_2$ such that $\mathscr{U}_1 \subseteq \mathscr{U}_2$ and $\mathscr{U}_1 \in \mathscr{U}_2$. $\mathscr{U}_1$-sets will be called sets, $\mathscr{U}_1$-classes classes, $\mathscr{U}_2$-sets metaclasses and $\mathscr{U}_2$-classes superclasses. There are four corresponding levels of categories: small categories, categories, metacategories and supercategories. We will notationally not distinguish the corresponding levels of functors. Let us emphasize that one can neglect these set-theoretic difficulties because all what we assert about categories may be proved in the Gödel—Bernays set theory.

A *concrete (meta)category* over a category $\mathscr{X}$ is a couple $(\mathscr{A}, U)$ where $\mathscr{A}$ is a (meta)category and $U : \mathscr{A} \to \mathscr{X}$ a faithful functor. A *concrete functor* $H : (\mathscr{A}, U) \to (\mathscr{B}, V)$ between concrete metacategories is a functor $H : \mathscr{A} \to \mathscr{B}$ such that $V . H = U$. Denote by $\mathscr{C}_{\mathscr{X}}$ a supercategory of concrete metacategories and concrete functors over $\mathscr{X}$.

Linton [1] has shown how a concrete metacategory $(\mathscr{A}, U)$ over $\mathscr{X}$ gives rise to a concrete metacategory U-Alg of U-algebras over $\mathscr{X}$. A *U-algebra* $\mathfrak{A}$ consists of an object $X \in \mathscr{X}$ and of mappings $\varphi^{\mathfrak{A}} : X^n \to X^k$, where φ carries over natural transformations $U^n \to U^k$ with $n, k \in \mathscr{X}$, and these data satisfy

$$(U^f)^{\mathfrak{A}} = X^f \tag{1}$$

for any morphism $f : k \to n$ of $\mathscr{X}$ and

$$(\varphi . \psi)^{\mathfrak{A}} = \varphi^{\mathfrak{A}} . \psi^{\mathfrak{A}} \tag{2}$$

for any natural transformations $\psi : U^m \to U^n$ and $\varphi : U^n \to U^k$. Concerning the notation, if $n \in \mathscr{X}$ then $U^n : \mathscr{A} \to Set$ is the composition $\mathscr{X}(n, -) . U$, $U^f : U^k \to U^n$ has components $(U^f)_A = (UA)^f$ for $A \in \mathscr{A}$, X^n is the set $\mathscr{X}(n, X)$ and X^f is the mapping $\mathscr{X}(f, X) : X^n \to X^k$. Similarly h^n is the mapping $\mathscr{X}(n, h) : X^n \to Y^n$ for any morphism $h : X \to Y$ of $\mathscr{X}$.

Homomorphisms $h : \mathfrak{A} \to \mathfrak{B}$ of U-algebras are defined obviously, i.e. as morphisms $h : |\mathfrak{A}| \to |\mathfrak{B}|$ between underlying $\mathscr{X}$-objects of $\mathfrak{A}$ and $\mathfrak{B}$ such that $h^k . \varphi^{\mathfrak{A}} = \varphi^{\mathfrak{B}} . h^n$ for any $\varphi : U^n \to U^k$.

The metaclass of all natural transformations $U^n \to U^k$ will be denoted by $\tau_{\mathscr{A}}$. Emphasize that φ_A, for $A \in \mathscr{A}$, will always denote the A-th component of a natural transformation $\varphi \in \tau_{\mathscr{A}}$.

Put $T(\mathscr{A}) = U\text{-}Alg$ and let $T(H) : T(\mathscr{A}) \to T(\mathscr{B})$, for a concrete functor $H : \mathscr{A} \to \mathscr{B}$, is given as follows

$$\varphi^{T(H)(\mathfrak{A})} = (\varphi H)^{\mathfrak{A}}$$

for any $\mathfrak{A} \in T(\mathscr{A})$ and any $\varphi \in \tau_{\mathscr{B}}$. It is evident that $T : \mathscr{C}_{\mathfrak{X}} \to \mathscr{C}_{\mathfrak{X}}$ is a functor.

The setting

$$\varphi^{\eta_{\mathscr{A}}(A)} = \varphi_A,$$

where $A \in \mathscr{A}$ and $\varphi \in \tau_{\mathscr{A}}$, gives rise to a concrete functor $\eta_{\mathscr{A}} : \mathscr{A} \to T(\mathscr{A})$. The verification that $\eta_{\mathscr{A}}(A)$ is a U-algebra is easy. The functor $\eta_{\mathscr{A}}$ is called the comparison functor of $\mathscr{A}$.

Since $\varphi^{T(H)\cdot\eta_{\mathscr{A}}(A)} = (\varphi H)^{\eta_{\mathscr{A}}(A)} = (\varphi H)_A = \varphi_{H(A)} = \varphi^{\eta_{\mathscr{B}}(HA)}$ for any $A \in \mathscr{A}$ and $\varphi \in \tau_{\mathscr{B}}$, $\eta_{\mathscr{A}} : 1 \to T$ is a natural transformation (1 denotes the identity functor on $\mathscr{C}_{\mathfrak{X}}$).

It is easy to see that the assignment

$$\bar{\varphi}_{\mathfrak{A}} = \varphi^{\mathfrak{A}}$$

where $\mathfrak{A} \in T(\mathscr{A})$ gives a natural transformation $\bar{\varphi} \in \tau_{T(\mathscr{A})}$ for any $\varphi \in \tau_A$. Assign to any algebra $\mathfrak{A} \in T^2(\mathscr{A})$ an algebra $\mu_{\mathscr{A}}(\mathfrak{A}) \in T(\mathscr{A})$ by means of

$$\varphi^{\mu_{\mathscr{A}}(\mathfrak{A})} = \bar{\varphi}^{\mathfrak{A}}$$

for any $\varphi \in \tau_{\mathscr{A}}$. Since $h^k . \varphi^{\mu_{\mathscr{A}}(\mathfrak{A})} = h^k . \bar{\varphi}^{\mathfrak{A}} = \bar{\varphi}^{\mathfrak{B}} . h^n = \varphi^{\mu_{\mathscr{A}}(\mathfrak{B})} . h^n$ for any homomorphism $h : \mathfrak{A} \to \mathfrak{B}$ of algebras $\mathfrak{A}, \mathfrak{B} \in T^2(\mathscr{A})$, $\mu_{\mathscr{A}} : T^2(\mathscr{A}) \to T(\mathscr{A})$ is a functor. Since $(\bar{\varphi} . T(H))_{\mathfrak{A}} = \bar{\varphi}_{T(H)(\mathfrak{A})} = \varphi^{T(H)(\mathfrak{A})} = (\varphi H)^{\mathfrak{A}} = (\overline{\varphi H})_{\mathfrak{A}}$ for any $H : \mathscr{A} \to \mathscr{B}$, $\varphi \in \tau_{\mathscr{B}}$ and $\mathfrak{A} \in T(\mathscr{A})$, we get $\varphi^{T(H)(\mu_{\mathscr{A}}(\mathfrak{A}))} = (\varphi H)^{\mu_{\mathscr{A}}(\mathfrak{A})} = \overline{\varphi H}^{\mathfrak{A}} = (\bar{\varphi} T(H))^{\mathfrak{A}} = \bar{\varphi}^{T^2(H)(\mathfrak{A})} = \varphi^{\mu_{\mathscr{B}}\cdot(T^2(H)(\mathfrak{A}))}$ for any $H : \mathscr{A} \to \mathscr{B}$, $\varphi \in \tau_{\mathscr{B}}$ and $\mathfrak{A} \in T^2(\mathscr{A})$. Hence $\mu_{\mathscr{A}} : T^2 \to T$ is a natural transformation.

Theorem. *(T, η, μ) is a monad in $\mathscr{C}_{\mathfrak{X}}$.*

Proof: Since $\varphi^{(\mu\cdot\eta T)(\mathfrak{A})} = \bar{\varphi}^{\eta T(\mathscr{A})(\mathfrak{A})} = \bar{\varphi}_{\mathfrak{A}} = \varphi_{\mathfrak{A}}$ for any $\mathfrak{A} \in T(\mathscr{A})$ and $\varphi \in \tau_A$, we get that $\mu . \eta T = 1$. Since $(\bar{\varphi} . \eta_{\mathscr{A}})_{\mathfrak{A}} = \bar{\varphi}_{\eta_{\mathscr{A}}(\mathfrak{A})} = \varphi^{\eta_{\mathscr{A}}(\mathfrak{A})} = \varphi_{\mathfrak{A}}$ for any $\mathfrak{A} \in \mathscr{A}$ and $\varphi \in \tau_{\mathscr{A}}$, it holds $\varphi^{\mathfrak{A}} = (\bar{\varphi}\eta_{\mathscr{A}})^{\mathfrak{A}} = \bar{\varphi}^{T(\eta_{\mathscr{A}})(\mathfrak{A})} = \varphi^{(\mu\cdot T(\eta))_{\mathscr{A}}(\mathfrak{A})}$ for any $\mathfrak{A} \in T(\mathscr{A})$ and $\varphi \in \tau_A$. Hence $\mu . T(\eta) = 1$. Finally, since $(\bar{\varphi}\mu_{\mathscr{A}})_{\mathfrak{A}} = \bar{\varphi}_{\mu_{\mathscr{A}}(\mathfrak{A})} = \varphi^{\mu_{\mathscr{A}}(\mathfrak{A})} = \bar{\varphi}^{\mathfrak{A}} = \bar{\bar{\varphi}}_{\mathfrak{A}}$ for any $\mathfrak{A} \in T^2(\mathscr{A})$ and $\varphi \in \tau_{\mathscr{A}}$, it holds $(\varphi^{(\mu\cdot T(\mu))_{\mathscr{A}}(\mathfrak{A})} = \bar{\varphi}^{T(\mu_{\mathscr{A}})(\mathfrak{A})} = (\bar{\varphi}\mu_{\mathscr{A}})^{\mathfrak{A}} = \bar{\bar{\varphi}}^{\mathfrak{A}} = \bar{\varphi}^{\mu T(\mathscr{A})(\mathfrak{A})} = \varphi^{(\mu\cdot\mu T)_{\mathscr{A}}(\mathfrak{A})}$ for any $\mathfrak{A} \in T^3(\mathscr{A})$ and $\varphi \in \tau_{\mathscr{A}}$. Therefore $\mu . T(\mu) = \mu . \mu T$ holds.

A concrete (meta)category $(\mathscr{A}, U)$ over $\mathfrak{X}$ will be called *canonically algebraic* if the comparison functor $\eta_{\mathscr{A}}$ is an isomorphism.

Corollary 1: *Let $\mathcal{A}$ be a concrete category over $\mathcal{X}$ such that $T(\mathcal{A})$ is canonically algebraic and $\eta_{\mathcal{A}}$ is a coretraction. Then $\mathcal{A}$ is canonically algebraic, too.*

Proof: Since $T(\mathcal{A})$ is canonically algebraic, $\eta_{T(\mathcal{A})}$ is an isomorphism. Hence $\eta_{T(\mathcal{A})} = T(\eta_{\mathcal{A}})$ because $\mu_{\mathcal{A}} . \eta_{T(\mathcal{A})} = \mu_{\mathcal{A}} . T(\eta_{\mathcal{A}}) = 1$ implies that $\eta_{T(\mathcal{A})} = \mu_{\mathcal{A}}^{-1} = T(\eta_{\mathcal{A}})$.

$\eta_{\mathcal{A}}$ being a coretraction means that there is a functor $H : T(\mathcal{A}) \to \mathcal{A}$ such that $H . \eta_{\mathcal{A}} = 1$. Since $\eta_{\mathcal{A}} . H(\mathfrak{A}) = T(H) . \eta_{T(\mathcal{A})}(\mathfrak{A}) = T(H) . T(\eta_{\mathcal{A}})(\mathfrak{A}) = \mathfrak{A}$, it holds that $\eta_{\mathcal{A}} . H = 1$. Hence $\eta_{\mathcal{A}}$ is an isomorphism and $\mathcal{A}$ is canonically algebraic.

Corollary 2: *Let $\mathcal{A}$ be a concrete category over $\mathcal{X}$. Then either $\mathcal{A}$ is canonically algebraic, or $T(\mathcal{A})$ is canonically algebraic or no of $T^n(\mathcal{A})$ is canonically algebraic.*

Proof: Since $\mu . \eta T = 1$, $\eta_{T^n(\mathcal{A})}$ is a coretraction for $n \geqq 1$. Hence by the previous corollary whenever $T^n(\mathcal{A})$ is not canonically algebraic then neither $T^{n+1}(\mathcal{A})$ is.

Let us specify what means that a category is canonically algebraic over *Set*. Under a type t we will mean a class of (infinitary) operation symbols. Having a class E of equations of type t we may form the metacategory (t, E)-*Alg* of all t-algebras satisfying all equations from E. Under an *algebraic category* over *Set* we will mean a concrete category isomorphic (as a concrete category) to some (t, E)-*Alg*. Any canonically algebraic category over *Set* is, of course, algebraic over *Set*. The converse is not true (see [4]). The reason is that, for a given algebraic category (t, E)-*Alg*, there may exist a natural transformation $\varphi \in \tau_{(t, E)\text{-}Alg}$ which is not induced by any term of type t and such that the interpretation of φ is not uniquely determined by the interpretations of terms of type t.

We can similarly introduce the concept of an algebraic category over an arbitrary category $\mathcal{X}$ (see [4]). Here, a t-algebra $\mathfrak{A}$ consists of an object $X \in \mathcal{X}$ and of operations $f : X^n \to X^k$ where f carries over operation symbols of type t and $n, k \in \mathcal{X}$. Hence we have again an algebraic category (t, E)-*Alg* over $\mathcal{X}$. Of course, any canonically algebraic category is again algebraic.

Returning once more to a monad $T : \mathcal{C}_{\mathcal{X}} \to \mathcal{C}_{\mathcal{X}}$, one can ask how T-algebras look like. We are going to show that any algebraic category over $\mathcal{X}$ is a T-algebra.

Consider an algebraic category $\mathcal{A} = (t, E)$-*Alg* and denote by U the forgetful functor into $\mathcal{X}$. Any operation symbol $f \in t$ provides a natural transformation φ_f by means of $(\varphi_f)_{\mathfrak{A}} = f^{\mathfrak{A}}$ for any (t, E)-algebra $\mathfrak{A}$. Then the prescription $f^{H(\mathfrak{A})} = (\varphi_f)^{\mathfrak{A}}$, $\mathfrak{A} \in T(\mathcal{A}), f \in t$ gives the functor $H : T(\mathcal{A}) \to \mathcal{A}$ of t-reducts. We show that $(\mathcal{A}, H)$ is a T-algebra.

Clearly $H . \eta_{\mathcal{A}} = 1$. Further for any $\mathfrak{A} \in T(\mathcal{A})$ and $f \in t$ it holds that $(\bar{\varphi}_f)_{\mathfrak{A}} = (\varphi_f)^{\mathfrak{A}} = f^{H(\mathfrak{A})} = (\varphi_f)_{H(\mathfrak{A})} = (\varphi_f H)_{\mathfrak{A}}$. Since $f^{H . \mu_{\mathcal{A}}(\mathfrak{A})} = (\varphi_f)^{\mu_{\mathcal{A}}(\mathfrak{A})} = (\bar{\varphi}_f)^{\mathfrak{A}} = (\varphi_f H)^{\mathfrak{A}} = (\varphi_f)^{T(H)(\mathfrak{A})} = f^{H . T(H)(\mathfrak{A})}$ holds for any $\mathfrak{A} \in T^2(\mathcal{A})$ and any $f \in t$, we get that $H . \mu_{\mathcal{A}}(\mathfrak{A}) = H . T(H)(\mathfrak{A})$ for any $\mathfrak{A} \in T^2(\mathcal{A})$. Hence $H . \mu_{\mathcal{A}} = H . T(H)$.

But there are more T-algebras than algebraic categories and it is an open question to characterize them.

Linton has shown (see [1]) that any monadic category over $\mathscr{X}$ is canonically algebraic. We are going to show that the converse is not true even in the case $\mathscr{X} = Set$. We will need two auxiliary assertions.

Proposition: *Let (t, E)-Alg be an algebraic category over Set, $\mathfrak{A}_i$, $i \in I$ be a set of its objects and $\varphi \in \tau_{(t,E)\text{-}Alg}$. Then there is a term p of type t such that $\varphi_{\mathfrak{A}_i}$ equals to its interpretation $p^{\mathfrak{A}_i}$ on $\mathfrak{A}_i$ for each $i \in I$.*

Proof: Following theorem 6.5. of [4] there is a chain $E_0 \supseteqq E_1 \supseteqq \dots \supseteqq E_\alpha \supseteqq \dots \supseteqq E$ of classes of equations of type t indexed by all ordinals such that any category (t, E_α)-*Alg* is monadic over *Set* and $(t, E)\text{-}Alg = \bigcup_{\alpha \in Ord} (t, E_\alpha)\text{-}Alg$. Hence there is an ordinal α such that $\mathfrak{A}_i \in (t, E_\alpha)\text{-}Alg$ for any $i \in I$. The natural transformation φ induces an element $\varphi \in \tau_{(t,E_\alpha)\text{-}Alg}$. It is well-known (see e.g. [2], 1.5.5.) that (t, E_α)-*Alg* being monadic implies that φ is induced on (t, E_α)-*Alg* by a term of type t. Hence the result follows.

It is clear that the condition of the Proposition is also sufficient for φ being a natural transformation.

Lemma: *Let (t, E)-Alg be an algebraic category over Set such that any operation symbol from t is unary. Denote by U the forgetful functor. Then any natural transformation $\varphi : U^n \to U$ is a composition $U^n \xrightarrow{\pi_i} U \xrightarrow{\alpha} U$ where α is natural and π_i is the projection given by some $i \in n$.*

Proof: At first, assume that $\varphi_{\mathfrak{A}} : |\mathfrak{A}|^n \to |\mathfrak{A}|$ is constant for any $\mathfrak{A} \in (t, E)\text{-}Alg$. Let $\alpha_{\mathfrak{A}} : |\mathfrak{A}| \to |\mathfrak{A}|$ be constant with the same value as $\varphi_{\mathfrak{A}}$. Then $\alpha : U \to U$ is clearly natural and $\varphi = \alpha \,.\, \pi_i$ for any $i \in n$.

Let there be an algebra $\mathfrak{A}$ such that $\varphi_{\mathfrak{A}}$ is not constant. Assume that $\varphi_{\mathfrak{A}} = f \,.\, (\pi_i)_{\mathfrak{A}} = g \,.\, (\pi_j)_{\mathfrak{A}}$ for $i, j \in n$ and $f, g : |\mathfrak{A}| \to |\mathfrak{A}|$. Consider $a, b \in |\mathfrak{A}|$. There is $c = (c_k)_{k \in n} \in |\mathfrak{A}|^n$ such that $c_i = a$ and $c_j = b$. Since $f(a) = f \,.\, (\pi_i)_{\mathfrak{A}}(c) = g \,.\, (\pi_j)_{\mathfrak{A}}(c) = g(b)$ and $\varphi_{\mathfrak{A}}$ is not constant, we get that $i = j$. Hence $\varphi_{\mathfrak{A}}$ factorizes over at most one π_i. Denote this i, if it exists, by i_0.

Consider a (t, E)-algebra $\mathfrak{B}$. Following the Proposition, there is an n-ary term p of type t such that $\varphi_{\mathfrak{A}} = p^{\mathfrak{A}}$ and $\varphi_{\mathfrak{B}} = p^{\mathfrak{B}}$. But the term p equals to $\pi_i \,.\, q$ where q is the unary term and π_i the projection term. Hence $i = i_0$. Put $\alpha_{\mathfrak{B}} = (q_{\mathfrak{B}})^{\mathfrak{B}}$. For any (t, E)-algebras $\mathfrak{B}, \mathfrak{C}$ and any homomorphism $h : \mathfrak{B} \to \mathfrak{C}$ it holds that $h \,.\, \alpha_{\mathfrak{B}} \,.\, (\pi_{i_0})^{\mathfrak{B}} = h \,.\, (q_{\mathfrak{B}})^{\mathfrak{B}} \,.\, (\pi_{i_0})^{\mathfrak{B}} = h \,.\, p^{\mathfrak{B}} = h \,.\, \varphi_{\mathfrak{B}} = \varphi_\alpha \,.\, h^n = \alpha_\alpha \,.\, (\pi_{i_0})^{\mathfrak{C}} \,.\, h^n = \alpha_{\mathfrak{C}} \,.\, h^n \,.\, (\pi_{i_0})^{\mathfrak{B}}$. Since $(\pi_{i_0})^{\mathfrak{B}}$ is onto, $\alpha : U \to U$ is a natural transformation. Thus $\varphi = \alpha \,.\, \pi_{i_0}$.

Example: Let $\mathscr{X} = Set$ and consider the type t having a class of unary operation symbols $f_0, f_1, f_2, \dots$ indexed by all ordinals. Let E consist of equations

$$f_i^2 = f_i$$

for any ordinal i and

$$f_i \cdot f_j = f_0$$

for any two distinct ordinals i, j.

Let $\mathfrak{A}$ be a (t, E)-algebra. If $(f_i)^{\mathfrak{A}} = (f_j)^{\mathfrak{A}}$ for $i \neq j$ then $(f_i)^{\mathfrak{A}} = (f_i)^{\mathfrak{A}} \cdot (f_i)^{\mathfrak{A}} = (f_i)^{\mathfrak{A}} \cdot (f_j)^{\mathfrak{A}} = (f_0)^{\mathfrak{A}}$. Since $\mathfrak{A}$ can have only a set of different $(f_i)^{\mathfrak{A}}$, there is an ordinal j such that $(f_i)^{\mathfrak{A}} = (f_0)^{\mathfrak{A}}$ for any $i \geqq j$. Hence $\mathscr{A} = (t, E)$-*Alg* has only a class of objects and thus it is a category.

Denote by $U : \mathscr{A} \to Set$ the forgetful functor. Let $\varphi : U \to U$ be a natural transformation and assume that φ differs from the natural transformation $\varphi_{f_0} : U \to U$ induced by f_0. Thus there is an algebra $\mathfrak{A} \in \mathscr{A}$ such that $\varphi_{\mathfrak{A}} \neq (f_0)^{\mathfrak{A}}$. Following the Proposition there is an ordinal i such that $\varphi_{\mathfrak{A}} = (f_i)^{\mathfrak{A}}$. Consider an arbitrary $\mathfrak{B} \in \mathscr{A}$. By the Proposition again, there is j such that $\varphi_{\mathfrak{B}} = (f_j)^{\mathfrak{B}}$ and $\varphi_{\mathfrak{A}} = (f_j)^{\mathfrak{A}}$. Since $(f_i)^{\mathfrak{A}} = (f_j)^{\mathfrak{A}}$ and $(f_i)^{\mathfrak{A}} \neq (f_0)^{\mathfrak{A}}$, we get that $i = j$. Hence $\varphi = \varphi_{f_i}$. We have proved that φ_{f_i}'s exhaust all natural transformations $U \to U$.

Following the Lemma any natural transformation $U^n \to U$ is induced by a term of type t. Hence $\mathscr{A}$ is canonically algebraic.

It remains to verify that $\mathscr{A}$ is not monadic. We will prove it by exhibiting one-generated (t, E)-algebras of arbitrary cardinalities. It suffices to consider, for any ordinal i, a (t, E)-algebra $\mathfrak{A}_i$ on the set i such that $(f_k)^{\mathfrak{A}_i}(k) = k$ if $k < i$ and $(f_k)^{\mathfrak{A}_i}(j) = 0$ otherwise.

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