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ON THE EXISTENCE OF PERIODIC SOLUTIONS FOR NONCONVEX DIFFERENTIAL INCLUSIONS

Dimitrios Kravvaritis and Nikolaos S. Papageorgiou

Abstract. Using a Nagumo type tangential condition and a recent theorem on the existence of directionally continuous selector for a lower semicontinuous multifunctions, we establish the existence of periodic trajectories for nonconvex differential inclusions.

1. Introduction

In this paper we examine the following multivalued boundary value problem in $\mathbb{R}^N$:

$$\left\{ \begin{array}{ll} \dot{x}(t) \in F(t, x(t)) \quad \text{a.e. on} \quad T = [0, b] \\ x(0) = x(b) \end{array} \right\} \tag{1}$$

with $F : T \times \mathbb{R}^N \to 2^{\mathbb{R}^N} \backslash \{\emptyset\}$ being a multivalued vector field (an orientor field) which has closed but not necessarily convex values. By a solution of (1) we mean an absolutely continuous function $x : T \to \mathbb{R}^N$ satisfying (1) above. Recall that by Lebesgue's theorem the function $x(\cdot)$ is almost everywhere differentiable and $\dot{x} \in L^1(T, \mathbb{R}^N)$. Earlier works on periodic solutions of differential inclusions considered only systems with convex-valued orientor fields. We refer to Aubin-Cellina [2] (theorem 4, p. 237), Haddad-Lasry [4], Macki-Nistri-Zecca [7] and Papageorgiou [12] for further details. Here using a recent selection theorem of Bressan [3], we obtain a periodic solution for nonconvex differential inclusions.

2. Preliminaries

Throughout this paper we will be using the following notations:

$$P_{f(c)}(\mathbb{R}^N) = \{A \subseteq \mathbb{R}^N : \quad \text{nonempty, closed, (convex)}\}$$

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and

$$P_{k(c)}(\mathbb{R}^N) = \{A \subseteq \mathbb{R}^N : \quad \text{nonempty, compact, (convex)}\}\,.$$

Let K be a nonempty subset of $\mathbb{R}^n$ and $x \in \bar{K}$. The contingent cone $T_K(x)$ is defined by

$$T_K(x) = \left\{ v \in \mathbb{R}^N : \varliminf_{\lambda\ \ 0} \frac{d(x+\lambda v, K)}{\lambda} = 0 \right\},$$

where $d(x+\lambda v, K) = \inf\{\|x+\lambda v - z\| : z \in K\}$. If K is convex and int $K \neq 0$, then int $T_K(x) \neq \emptyset$ and in fact $x \mapsto$ int $T_K(x)$ has an open graph (see Aubin-Cellina [2], proposition 4, p. 221). Given a multifunction $K : T = [0,b] \to P_{fc}(\mathbb{R}^N)$ and $(t,x) \in GrK = \{(s,z) \in T \times \mathbb{R}^N : z \in K(s)\}$, we denote by $DK(t,x)$, the multivalued map from $\mathbb{R}$ into $\mathbb{R}^N$, whose graph is the contingent cone $T_{GrK}(t,x)$. So $v \in DK(t,x)(r)$ if and only if $(r,v) \in T_{GrK}(t,x)$ and the multivalued map $DK(t,x)$ from $\mathbb{R}$ into $\mathbb{R}^N$ is called the "contingent derivative" of K at $(t,x) \in GrK$.

On $P_f(\mathbb{R}^N)$ we can define a (generalized) metric, known in the literature as the Hausdorff metric, as follows:

$$h(A,B) = \max\left[\sup_{a\ \ A} d(a,B),\ \sup_{b\ \ B} d(b,A)\right],$$

where $d(a,B) = \inf\{\|a-b\| : b \in B\}$ and $d(b,A) = \inf\{\|b-a\| : a \in A\}$. Then $(P_f(\mathbb{R}^N), h)$ is a complete metric space and $(P_k(\mathbb{R}^N), h)$ is a closed and separable subspace of it.

Let Y, Z be Hausdorff topological spaces and $G : Y \to 2^Z \backslash \{\emptyset\}$. We say that $G(\cdot)$ is lower semicontinuous (l.s.c.), if for every $U \subseteq Z$ open, $G\ \ (U) = \{y \in Y : G(y) \cap U \neq \emptyset\}$ is open in Y.

Our hypotheses on the data of (1) are the following:

<u>$H(F)$</u>: $F : T \times \mathbb{R}^N \to P_f(\mathbb{R}^N)$ is a lower semicontinuous (l.s.c.) multifunction such that $|F(t,x)| = \sup\{\|v\| : v \in F(t,x)\} \leq M$, with $M > 0$.

Also there is a multifunction $K(\cdot)$ satisfying the following two hypotheses $H(K)$ and H_T:

<u>$H(K)$</u>: $K : T \to P_{kc}(\mathbb{R}^N)$ is a Hausdorff Lipschitz multifunction (i.e. $h(K(t\ ), K(t)) \leq y|t\ - t|$ with $y > 0$), $K(b) \subseteq K(0)$ and
(i) $(t,x) \mapsto DK(t,x)(1)$ is l.s.c. on $GrK = \{(s,z) \in T \times \mathbb{R}^N : z \in K(s)\}$,
(ii) for all $(t,x) \in GrK$, int $DK(t,x)(1) \neq \emptyset$.

Remark. This hypothesis is automatically satisfied if $K(t) = K \in P_{kc}(\mathbb{R}^N)$ for all $t \in T$ and int $K \neq \emptyset$ (see for example Aubin-Cellina [2], theorem 1 and proposition 4, pp. 220-221). Also note that the lower semicontinuity of $(t,x) \mapsto DK(t,x)(1)$ on GrK, implies that for all $(t,x) \in GrK$, $DK(t,x)(1) \in P_{fc}(\mathbb{R}^N)$ (see Aubin [1]). Following Aubin [1], we call a multifunction $K(\cdot)$ "sleek", if $(t,x) \in GrK \mapsto$

$Gr(DK(t,x))$ is l.s.c. Then for such a multifunction $(t,x) \in GrK \mapsto DK(t,x)\,(1)$ is l.s.c.

$\underline{H_T}$: for all $(t,x) \in GrK$, we have $F(t,x) \cap \operatorname{int} DK(t,x)\,(1) \neq \emptyset$.

Let $C_M = \{(t,x) \in \mathbb{R} \times \mathbb{R}^N : \|x\| \leq Mt\}$. This is a closed, convex and pointed cone. Let $h : T \times \mathbb{R} \quad \mathbb{R}^N$ be a map such that $\|h(t,x)\| \leq M$. We say that $h(\cdot\,,\,\cdot)$ is C_M-continuous, if $(t_n, x_n) \in (t,x) + C_M$, $(t_n, x_n) \to (t,x)$ in $T \times \mathbb{R}^N$, imply that $h(t_n, x_n) \to h(t,x)$. For such a map we can define its set-valued Filippov regularization $G : T \times \mathbb{R}^N \to P_{kc}(\mathbb{R}^N)$ as follows:

$$G(t,x) = \bigcap_{\varepsilon > 0} \overline{\operatorname{conv}} \{h(s,y) : |s-t| < \varepsilon, \quad \|y-x\| < \varepsilon\}\,.$$

From Aubin-Cellina [2], p. 101, we know that $(t,x) \mapsto G(t,x)$ is upper semi-continuous (i.e. for all $U \subseteq \mathbb{R}^N$ open, $G^+(U) = \{(t,x) \in T \times \mathbb{R}^N : G(t,x) \subseteq U\}$ is open) and clearly for all $(t,x) \in T \times \mathbb{R}^N$, we have $h(t,x) \in G(t,x)$. Then we consider the following two Cauchy problems:

$$\left\{ \begin{array}{l} \dot{x}(t) \in G(t, x(t)) \text{ a.e.} \\ x(0) = z \in \mathbb{R}^N \end{array} \right\} \tag{2}$$

and

$$\left\{ \begin{array}{l} \dot{x}(t) = h(t, x(t)) \text{ a.e.} \\ x(0) = z \in \mathbb{R}^N \end{array} \right\} \tag{3}$$

To make the presentation relatively self-contained, we state here some known results that we will need in the sequel. We start with a useful description of the elements of the contingent derivative $DK(t,x)$ which can be found in Aubin-Cellina [2], p. 191:

$$v \in DK(t,x)(u) \quad \text{if and only if} \quad \varliminf_{h \quad 0} d\Big(v, \frac{K(t+hu) - x}{h}\Big) = 0 \quad .$$

Since we will be dealing with nonconvex multifunctions, we will need a "continuous" selection theorem for such set-valued maps. This was done by Bressan [3] who introduced the notion of directional (or $K-$) continuity already introduced above for the cone $K = C_M$. Let us give here the general definition:

Definition. Let $K \subseteq \mathbb{R}^N$ be a cone and Y a metric space. A function $f : \mathbb{R}^N \to Y$ is said to be K-continuous at $\bar{x} \in \mathbb{R}^N$ if for every $\varepsilon > 0$ there is a $\delta > 0$ such that $d(f(x), f(\bar{x})) < \varepsilon$ for every $x \in B(\bar{x}, \delta) \cap (\bar{x} + K)$ with $B(\bar{x}, \delta) = \{z \in \mathbb{R}^N : \|z - \bar{x}\| < \delta\}$ (equivalently $f(x_n) \to f(\bar{x})$ for every $x_n \to \bar{x}$ with $x_n - \bar{x} \in K$, $n \geq 1$).

Using this notion Bressan [3] proved the following selection theorem:

Theorem A. **If** *Y is a complete metric space and $F : \mathbb{R}^N \to P_f(Y)$ is a l.s.c. multifunction,* **then** *for every cone $K \subseteq \mathbb{R}^N$, $F(\cdot)$ admits a K-continuous selector.*

Again the nonconvexity of our setting, requires a fixed-point theorem for non-convex-valued multifunctions. This can be found in Lasry-Robert [6]. First a definition (see [6] definition 5).

Definition. Let X, Y be metric spaces and $\Gamma : X \to 2^Y \setminus \{\emptyset\}$. We say that $\Gamma(\cdot)$ is "pseudo-acyclic" if there is a metric space Z, an u.s.c. multifunction $L : X \to X \to P_k(Z)$ with acyclic values and $r : Z \to Y$ continuous such that $\Gamma = r \circ L$.

Using this notion we can have the following fixed-point theorem (see [6] theorem 8).

Theorem B. **If** *X is a metrizable locally convex vector space, $C \subseteq X$ is nonempty, convex and $\Gamma : C \to 2^C \setminus \{\emptyset\}$ is pseudo-acyclic such that $\overline{\Gamma(C)}$ is compact,* **then** *there is $x \in C$ such that $x \in \Gamma(x)$.*

3. Auxiliary results

In this section we prove four lemmata, which will be needed in the proof our main theorem (section 4) and which are also of independent interest.

Our first auxiliary result relates the solutions sets of (2) and (3).

Lemma 3.1. **If** *$x : T \to \mathbb{R}^N$ is an absolutely continuous function solving* (2), then *$x(\cdot)$ is also a solution of* (3).

Proof. From Lusin's theorem, we know that we can find $T_n \subseteq T$, $n \geq 1$ disjoint measurable sets such that $\dot{x}|_{T_n}$ is continuous , $\dot{x}(t) \in G(t, x(t))$ on T_n and $\lambda\Big(T \setminus \bigcup_{n \quad 1} T_n\Big) = 0$ with $\lambda(\cdot)$ being Lebesgue measure on T. Also invoking Lebesgue's density theorem (see for example Oxtoby [10], p. 17), we can find sets $N_n \subseteq T_n$ $n \geq 1$, with $\lambda(N_n) = 0$, such that every point in $T_n \setminus N_n$ is a density point of T_n. Next let $t \in T_n \setminus N_n$. Then we can find $t_k \in T_n \setminus N_n$ $t_k > t$ $k \geq 1$ and t_k t. Therefore $\dot{x}(t_k) \to \dot{x}(t)$. Note that because $\|h(t, x)\| \leq M$ we have $|G(t, x)| = \sup\{\|v\| : v \in G(t, x)\} \leq M$ and so $\|x(t) - x(s)\| \leq M|t - s|$ for all t, $s \in T$. Let $\varepsilon > 0$. Then we have $\dot{x}(t_k) \in G(t_k, x(t_k)) \subseteq h(t_k, x(t_k)) + \frac{\varepsilon}{2} B_1$, where $B_1 = \{z \in \mathbb{R}^N : \|z\| < 1\}$. Also since $h(t, x)$ is C_M-continuous, we can find $k_0(\varepsilon) \geq 1$ such that for $k \geq k_0(\varepsilon)$, we will have

$$
\begin{aligned}
&\|h(t_k, x(t_k)) - h(t, x(t))\| < \frac{\varepsilon}{2} \\
\Rightarrow &h(t_k, x(t_k)) \in h(t, x(t)) + \frac{\varepsilon}{2} B_1 \,.
\end{aligned}
$$

So for $k \geq k_0(\varepsilon)$, we have

$$
\begin{aligned}
&\dot{x}(t_k) \in h(t, x(t)) + \varepsilon\, B_1 \\
\Rightarrow\ &\dot{x}(t) \in h(t, x(t)) + \varepsilon\, B_1 \,.
\end{aligned}
$$

Let $\varepsilon \quad 0$. We finally get that for all $t \in \bigcup_{n \quad 1} (T_n \backslash N_n) = \hat{T}, \lambda(T - \hat{T}) = 0$, $\dot{x}(t) = h(t, x(t))$, $x(0) = z$, i.e. $x(\cdot)$ solves (3). □

The second auxiliary result, proves an invariance property for a class of differential inclusions. In what follows $p_{K(t)}(\cdot)$ denotes the metric projection on the set $K(t)$, i, e. $p_{K(t)}(x) = v$, where v is the unique vector in $K(t)$ such that $\|v - x\| = \min\{\|v^1 - x\| : v^1 \in K(t)\}$.

Lemma 3.2. If *$G : T \times \mathbb{R}^N \to P_{kc}(\mathbb{R}^N)$ is a multifunction such that $|G(t,x)| = \sup\{\|v\| : v \in G(t,x)\} \leq M$, hypothesis $H(K)$ holds, $x : T \to \mathbb{R}^N$ is an absolutely continuous function such that $\dot{x}(t) \in G(t, p_{K(t)}(x(t)))$ a.e., with $x(0) \in K(0)$ and for all $(t,x) \in GrK$, we have $G(t,x) \subseteq DK(t,x)(1)$,* **then** *for all $t \in T$ $x(t) \in K(t)$.*

Proof. Let $\varphi(t) = d(x(t), K(t))$. Using hypothesis $H(K)$, we can easily check that $\varphi(\cdot)$ is an absolutely continuous function. Since $\varphi(0) = 0$ (because $x(0) \in K(0)$), if we show that $\dot{\varphi}(t) \leq 0$ a.e., then we are done. Let $t \in T$ be a point at which both $\dot{x}(\cdot)$ and $\dot{\varphi}(\cdot)$ exist. Then we have

$$\begin{aligned}
\frac{\varphi(t+h) - \varphi(t)}{h} &= \frac{d(x(t+h), K(t+h)) - d(x(t), K(t))}{h} \\
&= \frac{d(x(t) + h\dot{x}(t) + o(h), K(t+h)) - d(x(t), K(t))}{h} \\
&\leq \frac{\|o(h)\|}{h} + \frac{d(x(t) + h\dot{x}(t), K(t+h)) - d(x(t), K(t))}{h}.
\end{aligned}$$

Observe that

$$\begin{aligned}
&\frac{d(x(t) + h\dot{x}(t), K(t+h)) - d(x(t), K(t))}{h} \\
&\leq \frac{\|x(t) - p_{K(t)}(x(t))\|}{h} + \frac{d(p_{K(t)}(x(t)) + h\dot{x}(t), K(t+h))}{h} - \frac{d(x(t), K(t))}{h} \\
&= \frac{d(p_{K(t)}(x(t)) + h\dot{x}(t), K(t+h))}{h} \\
&= d\Big(\dot{x}(t), \frac{K(t+h) - p_{K(t)}(x(t))}{h}\Big)
\end{aligned}$$

Since by hypothesis $\dot{x}(t) \in G(t, p_{K(t)}(x(t))) \subseteq DK(t, p_{K(t)}(x(t)))(1)$ a.e., from section 2 we have

$$\begin{aligned}
&\underline{\lim}_{h \quad 0} d\Big(\dot{x}(t), \frac{K(t+h) - p_{K(t)}(x(t))}{h}\Big) = 0 \\
&\Rightarrow \lim_{h \quad 0} \frac{\varphi(t+h) - \varphi(t)}{h} = \dot{\varphi}(t) \leq 0 \qquad \text{a.e.}
\end{aligned}$$

Thus finally we have $\varphi(t) = 0$ for all $t \in T$, hence $x(t) \in K(t)$ for all $t \in T$. □

Our third auxiliary result establishes a useful property of lower semicontinuous multifunctions.

Lemma 3.3. **If** *Z is a Hausdorff topological space, $F : Z \to 2^{\mathbb{R}^N} \backslash \{\emptyset\}$ is a lower semicontinuous multifunction with convex values and $B(x, \hat{r}) = \{y \in \mathbb{R}^N : \|y - x\| < \hat{r}\} \subseteq F(z_0)$,* **then** *for every $r \in (0, \hat{r})$ there exists U an open neighborhood of z_0 such that $B(x, r) \subseteq F(z)$ for all $z \in U$.*

Proof. Let $r \in (0, \hat{r})$, $0 < r < r\ < \hat{r}$ and $0 < \varepsilon < r\ - r$. We have $B(x, r\ ) \subseteq F(z_0)$. Let $\theta(z) = h(B(x, r\ ),\ F(z)) = \sup\{d(v, F(z)) : v \in B(x, r\ )\}$. From theorem 5, p. 52 of Aubin-Cellina [2], we known that $\theta(\cdot)$ is u.s.c. and $\theta(z_0) = 0$. So we can find U an open neighborhood of z_0 such that $\theta(z) = h(B(x, r\ ), F(z)) < \varepsilon$ for all $z \in U$. Since by hypothesis $F(\cdot)$ has convex values, from the lemma of Moreau [9], we have for all $z \in U$

$$
\begin{aligned}
d(x, \mathbb{R}^N \backslash B(x, r\ )) - d(x, \mathbb{R}^N \backslash F(z)) &\leq h(B(x, r\ ), F(z)) = \theta(z) < \varepsilon \\
&\Rightarrow d(x, \mathbb{R}^N \backslash B(x, r\ )) - \varepsilon < d(x, \mathbb{R}^N \backslash F(z)) \\
&\Rightarrow r < r\ - \varepsilon < d(x, \mathbb{R}^N \backslash F(z)) \\
&\Rightarrow B(x, r) \subseteq F(z) \quad \text{for all} \quad z \in U\ .
\end{aligned}
$$

□

Remark. Clearly we can not have $r = \hat{r}$. Just let $Z = \mathbb{R}^N$ and let $F(z) = B(z, \hat{r})$. Remark that our lemma 3.3 improves lemma 3.1 of Papageorgiou [11].

Our final auxiliary result, gives us new conditions under which the intersection of two multifunctions can be lower semicontinuous . Another result in this direction can be found in Papageorgiou [13].

Lemma 3.4. **If** *Z is a Hausdorff topological space, $H_1, H_2 : Z \to 2^{\mathbb{R}^N} \backslash \{\emptyset\}$ are lower semicontinuous multifunctions such that $H_2(\cdot)$ has open and convex values and for all $z \in Z$ $H_1(z) \cap H_2(z) \neq \emptyset$,* **then** *$z \mapsto H_1(z) \cap H_2(z) = H(z)$ is l.s.c.*

Proof. We need to show given $V \subseteq \mathbb{R}^N$ open, the set $H\ (V) = \{z \in Z : H(z) \cap V \neq \emptyset\} = \{z \in Z : H_1(z) \cap H_2(z) \cap V \neq \emptyset\}$ is open in Z. Because of the local convexity of $\mathbb{R}^N$ we can always assume V to be convex. Let $z_0 \in H\ (V)$ and let $x \in H(z_0) \cap V$. Since $H_2(z_0) \cap V$ is open, we can find $\hat{r} > 0$ such that $B(x, \hat{r}) \subseteq H_2(z_0) \cap V$. Note that $z \mapsto H_2(z) \cap V$ is lower semicontinuous (see Michael [8], proposition 2.4). So we can apply lemma 3.3 and get for $r < \hat{r}$ a U_1 open neighborhood of z_0 such that $B(x, r) \subseteq H_2(z) \cap V$ for all $z \in U_1$. Since by hypothesis $H_1(\cdot)$ is l.s.c. and $x \in H_1(z_0)$, we can find U_2 another neighborhood of z_0 such that for all $z \in U_2$, we have $H_1(z) \cap B(x, r) \neq \emptyset$. Set $U = U_1 \cap U_2$. Then for $z \in U$, we have

$$
\begin{aligned}
H(z) \cap V = H_1(z) \cap H_2(z) \cap V &\supseteq H_1(z) \cap B(x, r) \neq \emptyset \\
&\Rightarrow z \in H\ (V) \\
&\Rightarrow H\ (V) \quad \text{is open} \\
&\Rightarrow H(\cdot) \quad \text{is l.s.c.}
\end{aligned}
$$

□

4. Main result

In this section we state and prove our main result, concerning the existence of solutions for problem (1).

Theorem 4.1. **If** *hypotheses* $H(F)$, $H(K)$ *and* H_T *hold,* **then** *problem* (1) *admits a solution.*

Proof. Let $H : T \times \mathbb{R}^N \to P_f(\mathbb{R}^N)$ be the multifunction defined by $H(t,x) = \overline{F(t, p_{K(t)}(x)) \cap \operatorname{int} DK(t, p_{K(t)}(x))(1)} \neq \emptyset$ (see hypothesis H_T).

By hypothesis $H(K)$, $(t,z) \mapsto DK(t,z)(1)$ is l.s.c. on GrK and so it is convex-valued (see Aubin [1]). Hence $DK(t, p_{K(t)}(x))(1) = \overline{\operatorname{int} DK(t, p_{K(t)}(x))(1)}$ and from proposition 2.3 of Michael [8], we get that $(t,x) \mapsto \operatorname{int} DK(t, p_{K(t)}(x))(1)$ is l.s.c. Hence applying lemma 3.4, we get that $(t,x) \mapsto F(t, p_{K(t)}(x)) \cap \operatorname{int} DK(t, p_{K(t)}(x))(1)$ is l.s.c. and so once again proposition 2.3 of Michael [8], tells us that $(t,x) \mapsto H(t,x)$ is l.s.c. Apply theorem A, to get $h : T \times \mathbb{R}^N \to \mathbb{R}^N$ a C_M-continuous selector of $H(t,x)$. Note that because of hypothesis $H(F)$, we have $\|h(t,x)\| \leq M$. Let $G(t,x) = \bigcap_{\varepsilon>0} \overline{\operatorname{conv}}\{h(s,y) : |s-t| < \varepsilon,\ \|y-x\| < \varepsilon\}$ (the Filippov regularization of $h(t,x)$). Recall that $(t,x) \mapsto G(t,x)$ is u.s.c. and for $(t,x) \in T \times \mathbb{R}^N$, $h(t,x) \in G(t,x)$. Then consider the following multivalued Cauchy problem

$$\left\{\begin{array}{ll} \dot{x}(t) \in G(t, p_{K(t)}(x(t))) & \text{a.e.} \\ x(0) = z \in K(0) & \end{array}\right\}$$

Let $S : K(0) \to 2^{C(T,\mathbb{R}^N)} \setminus \{\emptyset\}$ be the solution multifunction for the above problem; i.e. for every $z \in K(0)$, $S(z)$ is the set of solutions of the problem. From Himmelberg-Van Vleck [5], we know that $S(\cdot)$ is an upper semicontinuous multifunction with nonempty, compact and acyclic values. Also from lemma 3.1, we know that for every $x(\cdot) \in S(z)$, we have $\dot{x}(t) = h(t, p_{K(t)}(x(t)))$ a.e., $x(0) = z$ and so lemma 3.2 tells us that $x(t) \in K(t)$ for all $t \in T$ and all $x(\cdot) \in S(z)$. Let $y : C(T,\mathbb{R}^N) \to \mathbb{R}^N$ be defined by $y(x) = x(b)$ (i.e. $y(\cdot)$ is the evaluation at b map, hence is continuous). Set $R = y \circ S : K(0) \to P_k(K(0))$ (recall that by hypothesis $K(b) \subseteq K(0)$). Then $R(\cdot)$ is pseudo-acyclic in the sense of Lasry-Robert [6] (see section 2) and applying theorem B, we get $z \in K(0)$ such that $z \in R(z)$. Let $x \in S(z)$ such that $z = y(x)(b)$. Then from what was said above we have $\dot{x}(t) = h(t, x(t))$ a.e., $x(0) = x(b)$. So $\dot{x}(t) \in F(t, x(t))$ a.e., $x(0) = x(b)$; i.e. $x(\cdot)$ is the desired solution of (1). □

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References

[1] Aubin, J.-P., *A survey of viability theory*, SIAM J. Control Optim. **28** (1990), 749-788.

[2] Aubin, J.-P., Cellina, A., *Differential Inclusions*, Springer, Berlin, 1984.

[3] Bressan, A., *Directionally continuous selections and differential inclusions*, Funkc. Ekvac. **31** (1988), 459-470.

[4] Haddad, G., Lasry, J.-M., *Periodic solutions of functional differential inclusions and fixed points of σ-selectionable correspondes*, J. Math. Anal. Appl. **96** (1983), 295-312.

[5] Himmelberg, C., Van Vleck, F. S., *A note on solutions sets of differential inclusions*, Rocky Mountain J. Math. **12** (1982), 621-625.

[6] Lasry, J.-M., Robert, R., *Degré topologique pour certains couples de fonctions et applications aux equations differentielles multivoques*, C. R. Acad. Sci. Paris **t. 283** (1976), 163-166.

[7] Macki, J., Nistri, P., Zecca, P., *The existence of periodic solutions to nonautonomous differential inclusions*, Proc. Amer. Math. Soc. **104** (1988), 840-844.

[8] Michael, E., *Continuous selections I*, Annals of Math. **63** (1956), 361-381.

[9] Moreau, J.-J., *Intersection of moving convex sets in a normed space*, Math. Scand. **36** (1975), 159-173.

[10] Oxtoby, J., *Measure and Category*, Springer, New York (1971).

[11] Papageorgiou, N. S., *On the attainable set of differential inclusions and control systems*, J. Math. Anal. Appl. **125** (1987), 305-322.

[12] Papageorgiou, N. S., *Viable and periodic trajectories for differential inclusions in Banach spaces*, Kobe Jour. Math. **5** (1988), 29-42.

[13] Papageorgiou, N. S., *On the existence of ψ-minimal viable solutions for a class of differential inclusions*, Archivum Mathematicum (Brno) **27** (1991), 175-182.

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