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Commutative nonstationary stochastic fields


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these Terms of use.
The present paper is devoted to further development of commutative nonstationary field themes; the first studies in this area were performed by K. Kirchev and V. Zolotarev [4, 5].

In this paper a more complicated variant of commutative field with nonstationary rank 2, carrying into more general situation for correlation function is studied. A condition of consistency (see (7) below) for commutative field is placed in the basis of the method proposed in [4, 5] and developed in this paper. The following semigroup structures of correlation theory for disturbances and semigroups are used in this case: $T_t(\varepsilon) = \exp(itA_\varepsilon)$, $A_\varepsilon = A_1 + \varepsilon A_2$, $|\varepsilon| \ll 1$.

1. In this section we will present the main preliminary information [4, 5].

Let us consider a two-dimensional curve $T_t = \exp(it_1A_1 + it_2A_2)$ in Hilbert space $H$. From now on we will assume that the system of linear bounded operators $\{A_1, A_2\}$ is a commutative one, $[A_1, A_2] = 0$, and there hold true:

1) $(A_2)_I H \subset (A_1)_I H$;
2) $(A_1)_I \geq 0$;
3) $(A_1)_I [\overline{(A_1)_I H}]$ is restrictedly invertible.

As it is known [7], the system $\{A_1, A_2\}$ can be included in the commutative colligation

$$\Delta = (A_1, A_2, H, \Phi, E, \sigma_1, \sigma_2, \gamma, \tilde{\gamma})$$

Where: $E$ is Hilbert space; $\Phi : H \to E$; $\sigma_1, \sigma_2, \gamma, \tilde{\gamma}$ are selfadjoint operators in $E$ and also the next colligation relationships are valid:

1) $A_k - A_k^* = i\Phi^*\sigma_k\Phi$ \quad ($k = 1, 2$);
2) $\gamma \Phi = \sigma_1 \Phi A_2^* - \sigma_2 \Phi A_1^*$;
3) $\tilde{\gamma} = \gamma + i(\sigma_1 \Phi \Phi^* \sigma_2 - \sigma_2 \Phi \Phi^* \sigma_1)$

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From now on we will study only the case of finite-dimensional space $E$. From the assumptions (1) follows that we can conclude that $\sigma_1 = I_E$; i.e., $A_1$ is dissipative. This means that the semigroup $T_t$ when $t_2 = 0$ is contractive. It is evident that when $\varepsilon$ is small enough then the operator $A_\varepsilon = A_1 + \varepsilon A_2$ is also dissipative and the semigroup $T_t(\varepsilon) = \exp(itA_\varepsilon); (t_1 = t, t_2 = \varepsilon t)$ is contractive. We will study CF and ICF of semigroup of contractions $T_t(\varepsilon)$ as a function of the variables $t$ and $\varepsilon$. Similarly to [2] it is easy to prove that there exists the limit

$$ S \cdot \lim_{t \to \infty} T_t^\ast(\varepsilon) T_t(\varepsilon) = K_\varepsilon $$

and also $0 \leq K_\varepsilon \leq I$.

**Proposition 1.** For every $x$ and $s$ from $R$ there holds true

$$ e^{isA_x^\ast} K_\varepsilon e^{isA_x} = K_\varepsilon, \quad \text{where} \quad A_x = A_1 + xA_2. $$

**Proof.** It is evident that

$$ e^{-isA_x^\ast} K_\varepsilon e^{isA_x} = S \cdot \lim_{t \to \infty} e^{-i(sA_x^\ast + tA_x^\ast)} e^{itA_x + sA_x} $$

$$ = S \cdot \lim_{t \to \infty} e^{i\frac{t(s-x)}{t+s} A_x^\ast} e^{-i(t+s)A_x^\ast} e^{i(t+s)A_x} e^{i\frac{(t-x)}{t+s} A_2} = K_\varepsilon $$

because of strong continuity of semigroup $e^{i\delta A_2}$ which tends to $I$ when $\delta \to 0$. □

**Corollary 1.** In this way for every small enough $\varepsilon(\varepsilon << 1)$ we can assert that $A_1^\ast K_\varepsilon = K_\varepsilon A_1$, $A_2^\ast K_\varepsilon = K_\varepsilon A_2$. From the existence of the limit $K_\varepsilon$, (5), there follows that for the correlation function $K_\varepsilon(t, s) = \langle T_t(\varepsilon) \Psi, T_s(\varepsilon) \Psi \rangle$ the next formula is valid

$$ K_\varepsilon(t, s) = V_\varepsilon(t - s) + \int_0^\infty W_\varepsilon(t + \tau, s + \tau) d\tau $$

where, as usual, ICF is defined by the formula

$$ W_\varepsilon(t, s) = -(\partial t + \partial s)K_\varepsilon(t, s) $$

and

$$ V_\varepsilon(t - s) = \langle K_\varepsilon e^{i(t-s)A_\varepsilon} \Psi, \Psi \rangle. $$

Let us now notice the ICF, $W_\varepsilon(t, s)$, which, obviously, has the form

$$ W_\varepsilon(t, s) = 2\langle (A_\varepsilon)_I T_t(\varepsilon) \Psi, T_s(\varepsilon) \Psi \rangle = \langle \sigma_\varepsilon \Phi \Psi_\varepsilon(t, \cdot), \Phi \Psi_\varepsilon(s, \cdot) \rangle, $$

here $\Psi_\varepsilon(t, \cdot) = T_t(\varepsilon) \Psi$, $\sigma_\varepsilon = \sigma_1 + \varepsilon \sigma_2$, $(A_\varepsilon)_I = (2i^{-1})(A_\varepsilon - A_x^\ast).$
Proposition 2. The function \( f(\varepsilon, t) = \Phi \Psi_\varepsilon(t) \) is a solution of the equation (7):

\[
[\sigma_2 i \sigma_t - i t^{-1} \sigma_\varepsilon - \tilde{\gamma}] f(\varepsilon, t) = 0.
\]

Proof. As far as \( \tilde{\gamma} \Phi = \sigma_\varepsilon \Phi A_2 - \sigma_2 \Phi A_\varepsilon \), so

\[
\tilde{\gamma} f = \sigma_\varepsilon \Phi A_2 \Psi - \sigma_2 \Phi A_\varepsilon \Psi = \sigma_\varepsilon \Phi A_2 e^{itA_\varepsilon} \Psi - \sigma_2 \Phi A_\varepsilon e^{itA_\varepsilon} \Psi
\]

\[
= \sigma_\varepsilon (it)^{-1} \Phi \sigma \varepsilon e^{itA_\varepsilon} \Psi - \sigma_2 (i^{-1}) \Phi \sigma_\varepsilon e^{itA_\varepsilon} \Psi = \sigma_2 i \sigma_1 f - i t^{-1} \sigma_\varepsilon \sigma_\varepsilon f,
\]

which proves the assertion. \( \square \)

Therefore, knowing the function \( f(t) \) (when \( \varepsilon = 0 \)) we can compute also the function \( f(\varepsilon, t) \) as a solution of the next Cauchy problem:

\[
\left\{
\begin{array}{l}
    i \sigma_\varepsilon f(\varepsilon, t) = t \sigma_\varepsilon [\sigma_2 i \sigma_t - \tilde{\gamma}] f(\varepsilon, t) \\
    f(\varepsilon, t)|_{\varepsilon = 0} = f(t)
\end{array}
\right.
\]

2. Let \( \dim E = 2 \) and the operators \( \sigma_1, \sigma_2, \tilde{\gamma} \) are of the form

\[
\sigma_1 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \quad \sigma_2 = \begin{pmatrix} 0 & m \\ \overline{m} & \sigma \end{pmatrix}, \quad \tilde{\gamma} = \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix}
\]

where \( \alpha, \beta \in \mathbb{R}; \ m \in \mathbb{C} \).

In the Hilbert space \( L^2_{(0,\ell)}(E^2, dx) \), we consider a model operator system

\[
A_1 f_x = i \int_x^\ell f_\zeta d\zeta
\]

\[
A_2 f_x = f_x \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} + i \int_x^\ell f_x \begin{pmatrix} 0 & m \\ \overline{m} & 0 \end{pmatrix} d\zeta,
\]

where \( f_x = (f^1(x), f^2(x)) \in L^2_{(0,\ell)}(E^2, dx) \).

Further, we consider a contractive semigroup

\[
f(t, x) = T_t(\varepsilon) f_x = e^{itA_\varepsilon} f_x
\]

with \( 1 - \varepsilon|m|^2 > 0 \), i.e., \( \varepsilon \ll |m|^{-1} \). Then \( f(t, x) \) is a solution of the Cauchy problem

\[
\left\{
\begin{array}{l}
    \frac{d}{dx} f(t, x) = f_x(t, x) i\varepsilon \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} - \int_x^\ell f(t, \zeta) d\zeta \begin{pmatrix} 1 & \varepsilon m \\ \varepsilon \overline{m} & 1 \end{pmatrix} \\
    f(0, x) = f_x
\end{array}
\right.
\]

We introduce in the consideration a vector-function \( F(t, x) \) such that

\[
F(t, x) = f(t, x) \exp \left\{ -i t \varepsilon \begin{pmatrix} \alpha & \beta \\ \bar{\beta} & \bar{\alpha} \end{pmatrix} \right\}
\]
Then the Cauchy problem (12) for \( F(t,x) \) takes the form

\[
\begin{align*}
F_t(t,x) &= -\int_x^t F(t,\zeta) \, d\zeta b_t \\
F(0,t) &= f(x)
\end{align*}
\]  

where the matrix \( b_t \) has the form

\[
b_t = e^{i\varepsilon t \alpha \beta} \left( \frac{1}{\varepsilon m} \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \right) e^{-i\varepsilon t \alpha \beta} = e^{i\varepsilon t \alpha \beta} U \left( \begin{pmatrix} 1 + \varepsilon m & 0 \\
0 & 1 - \varepsilon m \end{pmatrix} \right) U^* e^{-i\varepsilon t \alpha \beta}
\]

and the unitary matrix \( U \) has the form

\[
U = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\
1 & -1 \end{pmatrix}.
\]

We calculate the expression

\[
e^{i\varepsilon t \alpha \beta} U = e^{i\varepsilon t \alpha \beta} e^{i\varepsilon t \alpha \beta} U = \frac{e^{i\varepsilon t \alpha \beta}}{\sqrt{2}} \begin{pmatrix} \cos \varepsilon \beta & i \sin \varepsilon \beta \\
i \sin \varepsilon \beta & \cos \varepsilon \beta \end{pmatrix} \begin{pmatrix} 1 & 1 \\
1 & -1 \end{pmatrix} = \frac{e^{i\varepsilon t \alpha \beta}}{\sqrt{2}} \begin{pmatrix} e^{i\varepsilon t \beta} & e^{-i\varepsilon t \beta} \\
e^{i\varepsilon t \beta} & -e^{-i\varepsilon t \beta} \end{pmatrix}
\]

Therefore,

\[
b_t = \frac{1}{2} \begin{pmatrix} e^{i\varepsilon t \beta} & e^{-i\varepsilon t \beta} \\
e^{i\varepsilon t \beta} & -e^{-i\varepsilon t \beta} \end{pmatrix} \begin{pmatrix} 1 + \varepsilon \beta & 0 \\
0 & 1 - \varepsilon \beta \end{pmatrix} \begin{pmatrix} e^{-i\varepsilon t \beta} & e^{-i\varepsilon t} \\
e^{i\varepsilon t \beta} & -e^{-i\varepsilon t} \end{pmatrix} = \begin{pmatrix} 1 & \varepsilon m \\
\varepsilon m & 1 \end{pmatrix}
\]

Thus we obtain \( b_t \) is independent on \( t \).

Finally, let us determine an explicit form of \( F(t,x) \) (13).

For this purpose we represent \( F(t,x) \) in the form

\[
(15) \quad F(t,x) = \exp \left\{ t \int_x^t \cdot d\zeta \right\} \cdot \exp \left\{ -t\varepsilon m \int_x^t \cdot d\zeta \begin{pmatrix} 0 & 1 \\
1 & 1 \end{pmatrix} \right\} f(x)
\]

First we calculate

\[
\exp \left( -t \int_x^t \cdot d\zeta \right) f = \sum_{n=0}^{\infty} \frac{(-t)^n}{n!} \left( \int_x^t \cdot d\zeta \right)^n f(x)
\]

\[
= f(x) - \frac{t}{n!} \int_x^t f_{\zeta} d\zeta + \frac{t^2}{2!} \int_x^t (\zeta - x) f_{\zeta} d\zeta - \frac{t^3}{3!} \int_x^t (\zeta - x)^2 f_{\zeta} d\zeta + \cdots
\]
Finally, we have

\[ \exp \left( -t \int_x^\ell \cdot d\zeta \right) = - \int_x^\ell f'_\zeta y_0 \left( 2\sqrt{t(\zeta - x)} \right) d\zeta \]

where \( J_0(z) = \sum_{k=0}^{\infty} \frac{(-1)^k z^{2k}}{k!2^k} \) the Bessel function of zero order.

Now we calculate

\[
\exp \left\{ -t\varepsilon m \int_x^\ell \cdot d\zeta \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right) \right\} f = \sum_{n=0}^{\infty} \frac{(-t\varepsilon m)^n}{n!} \left( \int_x^\ell \cdot d\zeta \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right) \right)^n f \\
= f(x) \frac{t^2 \varepsilon^2 m^2}{2!} \int_x^\ell (\zeta - x) f_\zeta d\zeta + \frac{t^4 \varepsilon^4 m^4}{4!} \int_x^\ell (\zeta - x)^3 f_\zeta d\zeta + \cdots \\
- \left\{ \frac{t\varepsilon m}{1!} \int_x^\ell f_\zeta d\zeta + \frac{t^3 \varepsilon^3 m^3}{3!} \int_x^\ell (\zeta - x)^2 f_\zeta d\zeta + \cdots \right\} \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right)
\]

Selecting as earlier \( f(x) \) from a dense set in \( L^2_{(0,\ell)}(E^2, dx) \) such that \( f'(x) \) exists and is in \( L^2_{(0,\ell)}(E^2, dx) \) with \( f(\ell) = 0 \), we obtain after integration by parts

\[
\exp \left\{ -t\varepsilon m \int_x^\ell \cdot d\zeta \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right) \right\} f = - \int_x^\ell f'_\zeta \sum_{n=0}^{\infty} \frac{(t\varepsilon m)^2k(\zeta - x)^{2k}}{(2k)!2^k} d\zeta \\
+ \int_x^\ell f_\zeta \sum_{n=0}^{\infty} \frac{(t\varepsilon m)^{2k+1}(\zeta - x)^{2k+1}}{(2k+1)!2^{k+1}} d\zeta \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right).
\]

Further we make use of

\[ J_0(2\sqrt{x}) + I_0(2\sqrt{x}) = 2 \sum_{k=0}^{\infty} \frac{x^{2k}}{2k!2k!} \]

where \( I_0(x) \) is the modified Bessel function of zero order:

\[ I_0(x) = J_0(ix). \]

Finally, we have

\[
\exp \left\{ -t\varepsilon m \int_x^\ell \cdot d\zeta \left( \begin{array}{c} 0 \\ 1 \\ 0 \end{array} \right) \right\} f = -\frac{1}{2} \int_x^\ell f'_\zeta d\zeta \\
\left( J_0(2\sqrt{t\varepsilon m}(\zeta - x)) + I_0(2\sqrt{t\varepsilon m}(\zeta - x)); J_0(2\sqrt{t\varepsilon m}(\zeta - x)) - I_0(2\sqrt{t\varepsilon m}(\zeta - x)) \right) \\
\left( J_0(2\sqrt{t\varepsilon m}(\zeta - x)) - I_0(2\sqrt{t\varepsilon m}(\zeta - x)); J_0(2\sqrt{t\varepsilon m}(\zeta - x)) + I_0(2\sqrt{t\varepsilon m}(\zeta - x)) \right)
\]
A last, using expressions (16) and (17), we determine the form of function $F(t, x)$ (15).

To this end, it is necessary to calculate the following integrals:

\[
I_1 = \int_x^\ell J_0(2\sqrt{t(\zeta - x)}) \frac{d}{d\zeta} \int_\zeta^\ell J_0(2\sqrt{\varepsilon m(\eta - \zeta)}) \, d\eta
\]

\[
I_2 = \int_x^\ell J_0(2\sqrt{t(\zeta - x)}) \frac{d}{d\zeta} \int_\zeta^\ell J_0(2\sqrt{\varepsilon m(\eta - \zeta)}) \, d\eta
\]

(18)

To simplify the first of them (the second is calculated simplify) we make use of integration by parts:

\[
I_1 = \int_x^\ell f'_\zeta(2\sqrt{\varepsilon m(\zeta - x)}) \, d\zeta
\]

\[
- \int_x^\ell d\zeta J_{-1}(2\sqrt{t(\zeta - x)}) \frac{\sqrt{t}}{\sqrt{\zeta - x}} \int_\zeta^\ell f'_\eta J_0(2\sqrt{\varepsilon m(\eta - \zeta)}) \, d\eta
\]

\[
= \int_x^\ell f'_\zeta(2\sqrt{\varepsilon m(\zeta - x)}) \, d\zeta
\]

\[
- \sqrt{t} \int_x^\ell f'_\eta \, d\eta \int_x^\ell J_{-1}(2\sqrt{t(\zeta - x)}) J_0(2\sqrt{\varepsilon m(\eta - \zeta)}) \frac{d\zeta}{\sqrt{\zeta - x}}
\]

Using the following formula [1]

\[
\int_0^\tau \sqrt{\tau - 1}J_{-1}(\alpha \sqrt{\tau}) J_0(\beta \sqrt{\tau - \tau}) = 2\alpha^{-1} J_0(t \sqrt{\alpha^2 + \beta^2})
\]

we obtain

\[
\int_x^\eta J_{-1}(2\sqrt{t(\zeta - x)}) J_0(2\sqrt{\varepsilon m(\eta - x)}) \frac{d\zeta}{\sqrt{\zeta - x}} = \frac{1}{\sqrt{t}} J_0((\eta - x)2\sqrt{t + \varepsilon m})
\]

Thus

(19) \[ I_1 = \int_x^\ell f'_\zeta \left\{ J_0(2\sqrt{\varepsilon m(\zeta - x)}) - J_0((\zeta - x)2\sqrt{t + \varepsilon m}) \right\} \, d\zeta . \]

In a similar manner we obtain

\[
I_2 = \int_x^\ell f'_\zeta \left\{ I_0(2\sqrt{\varepsilon m(\zeta - x)}) - J_0((\zeta - x)2\sqrt{t - \varepsilon m}) \right\} \, d\zeta .
\]
Taking into account the expressions (16), (17) and (19), we obtain that the components of vector-function \( F(t, x) = (F^1(t, x); F^2(t, x)) \) (13) are as follows:

(20)

\[
F^1(t, x) = \frac{1}{2} \int_{x}^{\ell} \left\{ \partial f^1(\xi) + \partial f^2(\xi) \right\} \left[ J_0(2\sqrt{t(\xi - x)}) - J_0((\xi - x)2\sqrt{t + \varepsilon m}) \right] \, d\xi
\]

\[
F^2(t, x) = \frac{1}{2} \int_{x}^{\ell} \left\{ \partial f^1(\xi) - \partial f^2(\xi) \right\} \left[ J_0(2\sqrt{t(\xi - x)}) - J_0((\xi - x)2\sqrt{t + \varepsilon m}) \right] \, d\xi
\]

Finally it remains to take into account (13):

\[
f(t, x) = F(t, x) \exp \left\{ it \begin{pmatrix} \alpha & \beta \\ \beta & \alpha \end{pmatrix} \right\} = F(t, x) e^{it\alpha} \begin{pmatrix} \cos t\varepsilon \beta & i \sin t\varepsilon \beta \\ i \sin t\varepsilon \beta & \cos t\varepsilon \beta \end{pmatrix}
\]

hence

(21)

\[
f^1(t, x) = e^{it\alpha} [F^1(t, x) \cos t\varepsilon \beta + F^2(t, x)i \sin t\varepsilon \beta]
\]

\[
f^2(t, x) = e^{it\alpha} [F^1(t, x)i \sin t\varepsilon \beta + F^2(t, x) \cos t\varepsilon \beta]
\]

Accounting the asymptotic of Bessel function [1] \( J_0(z) \) when \( z \to \infty (|\arg z| < \pi) \) we will obtain that \( f(t, x) \to 0 \) when \( t \to \infty \). So \( T_t(\varepsilon) \) is asymptotically decaying function and hence \( V_\varepsilon(t - s) = 0 \).

**Theorem 1.** The limit correlation function \( V_\varepsilon(t - s) \) for the stochastic field \( T_t(\varepsilon) = \exp it(A_1 + \varepsilon A_2) \), when \( A_1, A_2 \) have the form (10) \((|\varepsilon| < \frac{1}{m})\) is equal to zero \( V_\varepsilon(t - s) = 0 \). Therefore infinitesimal correlation function \( W_\varepsilon(t, s) \) has the form

(22)

\[
W_\varepsilon(t, s) = (1 + \varepsilon m)\Phi^1(t, \varepsilon)\overline{\Phi^1(s, \varepsilon)} + (1 - \varepsilon m)\Phi^2(t, \varepsilon)\overline{\Phi^2(s, \varepsilon)}
\]

where it is obvious that

\[
\Phi^1(t, \varepsilon) = \frac{1}{2} \int_{0}^{\ell} (f^1(t, x) + f^2(t, x)) \, dx
\]

\[
\Phi^2(t, \varepsilon) = \frac{1}{2} \int_{0}^{\ell} (f^1(t, x) - f^2(t, x)) \, dx
\]

The Cauchy problem (7) for this case and the function \( \Phi(t, \varepsilon) = (\Phi^1(t, \varepsilon), \Phi^2(t, \varepsilon)) \) has the form

(23)

\[
\begin{cases}
\partial \Phi(t, \varepsilon) = t \begin{pmatrix} 1 + \varepsilon m & 0 \\ 0 & \varepsilon m - 1 \end{pmatrix} \partial t \\
+ 2i \begin{pmatrix} \alpha + \beta (1 + \varepsilon m) \\ (\alpha - \beta) (1 - \varepsilon m) \end{pmatrix} \Phi(t, \varepsilon) \\
\Phi(t, 0) = (\Phi^1(t), \Phi^2(t))
\end{cases}
\]
where Φ(t, 0) is determined by a dissipative process with a spectrum at zero and

\[
\begin{align*}
\Phi^1(t) &= \frac{1}{2} \int_0^t (f^1(\zeta) + f^2(\zeta)) J_0(2\sqrt{\zeta}) \, d\zeta \\
\Phi^2(t) &= \frac{1}{2} \int_0^t (f^1(\zeta) - f^2(\zeta)) J_0(2\sqrt{\zeta}) \, d\zeta
\end{align*}
\]

(24)

One can write the equation of Cauchy problem (23) in the following form:

\[
\begin{align*}
\Phi^1_\varepsilon &= (1 + \varepsilon m)(mt\Phi^1_t + 2i(\alpha + \beta)\Phi^1) \\
\Phi^2_\varepsilon &= (\varepsilon m - 1)(mt\Phi^2_t + 2i(\beta - \alpha)\Phi^2)
\end{align*}
\]

(25)

where \(\Phi^k = \partial_\varepsilon \Phi^k\), \(\Phi^k_t = \partial_t \Phi^k\), \((k = 1, 2)\); therefore it is necessary to solve in a general form the equation

\[
\partial_\varepsilon \Phi = (\varepsilon m + a)(mt\partial_t \Phi + ib\Phi),
\]

where \(a, b, m \in \mathbb{R}\). It is easy to see that a general solution of this equation has the form

\[
\Phi(t, \varepsilon) = e^{\frac{i b}{4m}((a+\varepsilon m)^2 - 2 \ln t)} G(2 \ln t + (\varepsilon m + a)^2)
\]

where \(G(x)\) is an arbitrary differentiable function.

Taking into account the initial condition of the problem (23), it is easy to obtain

\[
\begin{align*}
\Phi^1(t, \varepsilon) &= e^{\frac{i(\alpha + \beta)}{2}\varepsilon(\varepsilon m + 4)} \Phi^1(e^{\frac{2t + \varepsilon^2 m^2}{2} + 2\varepsilon m}) \\
\Phi^2(t, \varepsilon) &= e^{\frac{i(\alpha - \beta)}{2}\varepsilon(\varepsilon m - 4)} \Phi^2(e^{\frac{2t + \varepsilon^2 m^2}{2} - 2\varepsilon m})
\end{align*}
\]

(26)

where \(\Phi^k(t)\) has the form of (24).

Thus, the following theorem is proved.

**Theorem 2.** The correlation function of stochastic field \(T_t(\varepsilon)\) for the commutative system of operators \(A_1, A_2\) (10) has the form

\[
K_\varepsilon(t, s) = \int_0^\infty W_\varepsilon(t + \tau, s + \tau) \, d\tau
\]

where \(W_\varepsilon(t, s)\) has the form (22) and \(\Phi^k(t, \varepsilon)\) is represented in the form (26).
References


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