Ivan Havel; Petr Liebl
Embedding the polytomic tree into the $n$-cube

Časopis pro pěstování matematiky, Vol. 98 (1973), No. 3, 307--314

Persistent URL: [http://dml.cz/dmlcz/117800](http://dml.cz/dmlcz/117800)

Terms of use:
© Institute of Mathematics AS CR, 1973

Institute of Mathematics of the Academy of Sciences of the Czech Republic provides access to
digitized documents strictly for personal use. Each copy of any part of this document must
contain these Terms of use.

This paper has been digitized, optimized for electronic delivery and stamped
with digital signature within the project DML-CZ: The Czech Digital
Mathematics Library [http://project.dml.cz](http://project.dml.cz)
EMBEDDING THE POLYTOMIC TREE INTO THE n-CUBE

IVAN HAVEL, PETR LIEBL, Praha
(Received October 13, 1972)

In the whole paper a "graph" is a nondirected, possibly infinite graph without loops and multiple edges, expressed as an ordered pair $\mathcal{G} = \langle V, E \rangle$, where $V$ is the set of vertices and $E$ is the set of edges, a subset of $V^{(2)}$, the set of all unordered pairs of elements of $V$. $\mathcal{G}' = \langle V', E' \rangle$ is said to be the subgraph of $\mathcal{G} = \langle V, E \rangle$ induced by $V'$ iff $V' \subseteq V$, $E' = E \cap V'^{(2)}$. $\mathcal{G}' = \langle V', E' \rangle$ is said to be a partial subgraph of $\mathcal{G} = \langle V, E \rangle$ iff $V' \subseteq V$, $E' \subseteq E \cap V'^{(2)}$. (Cf [3].) By $\mathcal{C}$ we denote the post-office function.

Definition 1. Let $S$ be a set, by $2^S$ denote as usual the set of all subsets of $S$. Put $E(S) = \{(A, B) | A \subseteq S, B \subseteq S, \text{card } (A - B) = 1\}$. $(A - B)$ denotes here the symmetric difference of $A$ and $B$. By the $S$-cube we understand the graph $X(S) = \langle 2^S, E(S) \rangle$.

Definition 2. By $\mathcal{R}(S)$ denote the class of all graphs isomorphic to some partial subgraph of $X(S)$. If $S = \{1, 2, \ldots, n\}$, write $\mathcal{R}(S) = \mathcal{R}_n$. Put $\mathcal{R} = \{\mathcal{G} \mid \exists S, \mathcal{G} \in \mathcal{R}(S)\}$. By $\mathcal{R}$ denote the class of all graphs $\mathcal{G}$ such that for any finite partial subgraph $\mathcal{G}'$ of $\mathcal{G}$, $\mathcal{G}' \in \mathcal{R}$.

Trivially, if $\mathcal{G} \in \mathcal{R}(S)$ and $\mathcal{G}'$ is a partial subgraph of $\mathcal{G}$, then $\mathcal{G}' \in \mathcal{R}(S)$.

Definition 3. Let $\mathcal{G} = \langle V, E \rangle$ be a graph, $F$ a set. Assume there exists a mapping $\psi : E \to F$ such that

(i) if $(e_1, e_2, \ldots, e_r)$ is the sequence of edges of a finite open path in $\mathcal{G}$, then there is an element of $F$ that appears an odd number of times in the sequence $(\psi(e_1), \psi(e_2), \ldots, \psi(e_r))$.

(ii) if $(f_1, f_2, \ldots, f_s)$ is the sequence of edges of a finite closed path in $\mathcal{G}$, then all the elements of $F$ appear an even number (possibly null) of times in the sequence $(\psi(f_1), \psi(f_2), \ldots, \psi(f_s))$. 

307
Then we call \( \psi \) a \( C \)-valuation of \( \mathcal{G} \). Let \( n \) be a natural number. If \( \text{card}(\psi(E)) \leq n \), we call \( \psi \) a \( C_n \)-valuation of \( \mathcal{G} \).

**Definition 4.** By \( \mathcal{K} \) denote the class of all graphs \( \mathcal{G} \) such that there exists a \( C \)-valuation of \( \mathcal{G} \), by \( \mathcal{E} \) denote the class of all graphs \( \mathcal{G} \) such that for any finite partial subgraph \( \mathcal{G}' \) of \( \mathcal{G} \), \( \mathcal{G}' \in \mathcal{E} \). Let \( n \) be a natural number. By \( \mathcal{E}_n \) denote the class of all graphs \( \mathcal{G} \) such that there exists a \( C_n \)-valuation of \( \mathcal{G} \).

**Remark 1.** If \( \mathcal{G} \in \mathcal{K} \) is finite, then for some \( n \), \( \mathcal{G} \in \mathcal{E}_n \). Further, \( \mathcal{E}_n \subset \mathcal{K} \subset \mathcal{E} \).

Theorem 1 in [2] asserts that

(a) \( \mathcal{E}_n \subset \mathcal{E}_m \)
(b) \( \mathcal{G} \in \mathcal{E}_n \) connected \( \Rightarrow \mathcal{G} \in \mathcal{E}_m \)
(c) \( \mathcal{E} = \mathcal{K} \).

**Remark 2.** Let \( \mathcal{F} \) be an arbitrary tree. Then condition (ii) of Def. 3 is empty and moreover, putting \( F = E \), \( \psi \) the identity map, we have \( \mathcal{F} \in \mathcal{E} \) and hence \( \mathcal{F} \in \mathcal{E}_n \). Also, \( \mathcal{F} \in \mathcal{E}_n \Rightarrow \mathcal{F} \in \mathcal{E}_n \).

In what remains, we shall be concerned with trees only, and with the problem to find to a tree \( \mathcal{F} \) the smallest \( n \) such that \( \mathcal{F} \in \mathcal{E}_n \). We shall denote this \( n \) by \( \text{dim}(\mathcal{F}) \).
To study trees the vertices of which have their degree bounded from above by a given number, we introduce three infinite classes of trees, closely related to each other. \( \mathcal{T}_1(k) \), the "polytomic tree", is a straightforward generalization of the dichotomic tree \( \mathcal{D}_1 \) of [1]. \( \mathcal{T}_1(k) \) may be considered to be a star of \( k \) rays, each endpoint of a ray being again the center of a new \( k \)-star, and this procedure repeated \( l \) times. So, there are vertices of "level" 1 to \((l + 1)\), where the (single) vertex of level 1 has degree \( k \), the vertices of the outermost level \((l + 1)\) have degree 1 and the remaining vertices have degree \((k + 1)\). \( \mathcal{T}_1(k) \) and \( \mathcal{T}_2(k) \) arise from \( \mathcal{T}_1(k) \) if it is completed in such a way that all its vertices have either degree 1 or degree \((k + 1)\).

**Definition 5.** Let \( k \geq 2 \) and \( l \geq 1 \) be natural numbers. Define

\[
\mathcal{T}_1(k) = \langle V_1(k), E_1(k) \rangle, \quad \mathcal{T}_1(k) = \langle \mathcal{T}_1(k), E_1(k) \rangle, \quad \mathcal{F}_1(k) = \langle \mathcal{F}_1(k), \mathcal{F}_1(k) \rangle
\]

as follows:

Put

\[
V_1(k) = \{ v_j \mid 1 \leq i \leq l + 1, 1 \leq j \leq k^{i-1} \}
\]

\[
V_1(k) = \{ v_j \mid 1 \leq i \leq l + 1 \} \cup \{ v_j \mid -l \leq i \leq -1, 1 \leq j \leq k^{\mid i \mid -1} \}
\]

\[
\mathcal{F}_1(k) = \{ v_j \mid 1 \leq i \leq l + 1, 1 \leq j \leq k^{\mid i \mid -1} \}
\]

Further, for \( v_j \in V_1(k), v_{j'} \in V_1(k), (v_j, v_{j'}) \in E_1(k) \Leftrightarrow (|i| = |i'|-1) \land (j' = j/j/k^2 \lor (i = 1) \land (i' = -1)). \) Denote \((v_j^{(1)}, v_j^{(-1)})\) by \( E_j^{(0)} \) and further \( (v_j, v_{j'}) \in E_1(k) \) by \( E_j^{(0)}, \) if \(|i| < |i'|\). \( \mathcal{T}_1(k) \) resp. \( \mathcal{F}_1(k) \) are defined as the subgraphs of \( \mathcal{T}_1(k) \) induced by \( \mathcal{T}_1(k) \) resp. \( V_1(k) \).

Fig. 1a, b, c shows \( \mathcal{T}_1(k) \), \( \mathcal{T}_2(k) \) and \( \mathcal{T}_3(k) \).

As is seen, \( \mathcal{T}_1(k) \) consists of two trees \( \mathcal{T}_1(k) \) with their "roots" joined by a new edge whereas \( \mathcal{T}_1(k) \) arises in a similar manner from one \( \mathcal{T}_1(k) \) and one \( \mathcal{T}_1(k) \) (for \( l \geq 2 \)). As for the number of vertices, card \( V_1(k) = 2(k^{l+1} - 1) \) (for \( k \)). card \( V_2(k) = (k^{l+1} + k^l - 2)/(k - 1) \) and card \( V_3(k) = (k^{l+1} - 1)/(k - 1) \). In [1], \( \mathcal{T}_1(2) \) is denoted by \( \mathcal{D}_1 \). Theorem 3 of [1] asserts that for \( l \geq 2 \), \( \dim \mathcal{T}_1(2) = l + 2 \) (dim \( \mathcal{T}_1(2) = 2 \) being trivial). Another partial result of the general problem of \( \dim \mathcal{T}_1(k) \) is supplied by the following theorem. But first a

**Remark 3.** \( \mathcal{F}_1(k) \in \mathcal{R}_n \Rightarrow \mathcal{T}_1(k) \in \mathcal{R}_n \Rightarrow \mathcal{T}_1(k) \in \mathcal{R}_n \Rightarrow \mathcal{F}_1(k) \in \mathcal{R}_n \). The first two implications being trivial, consider for the third the two constituent \( \mathcal{T}_1(k) \) of \( \mathcal{F}_1(k) \) as having a \( C_n \)-valuation with the same \( F \) and the joining edge being assigned a new element \( f_{n+1} \).

**Theorem 1.**

\[
\dim (\mathcal{F}_2(2p)) = \dim (\mathcal{T}_2(2p)) = \dim (\mathcal{T}_2(2p)) = 3p + 1,
\]
\[
\dim (\mathcal{F}_2(2p+1)) = \dim (\mathcal{T}_2(2p+1)) = 3p + 3,
\]
\[
\dim (\mathcal{T}_2(2p+1)) = 3p + 2.
\]
Proof. In view of Remark 3, it is sufficient to prove
\[ f(2p) \in \mathcal{X}_{3p+1}, \quad \mathcal{F}_{2p+1}^{(2p+1)} \in \mathcal{X}_{3p+2}, \quad \mathcal{F}_2^{(2p)} \notin \mathcal{X}_{3p}, \quad \mathcal{F}_{2p+1}^{(2p+1)} \notin \mathcal{X}_{3p+1}, \]
\[ b \mathcal{F}_{2p+1}^{(2p+1)} \notin \mathcal{X}_{3p+2}. \]

1. To construct a \( C_{3p+1} \)-valuation \( \psi \) of \( \mathcal{F}_2^{(2p)} \), put
\[ F = \{a_{p+1}, a_{p+2}, \ldots, a_{2p}, a_1, a_2, \ldots, a_{2p+1}\}. \]
Further define
\[ (*) \]
\[ \psi(e_i^{(0)}) = a_{2p+1}, \]
\[ \psi(e_j^{(1)}) = a_j \quad (1 \leq j \leq 2p), \]
\[ \psi(e_j^{(-1)}) = a_j'' \quad (1 \leq j \leq 2p), \]
where we write for short
\[ a_t'' = a_t \quad (1 \leq t \leq p), \quad a_t'' = a_t' \quad (p + 1 \leq t \leq 2p), \quad a_{2p+1}' = a_{2p+1}. \]

Instead of proceeding by defining explicitly \( \psi(e_j^{(2)}) \) and \( \psi(e_j^{(-2)}) \), observe that the edges \( e_j^{(2)} \) and \( e_j^{(-2)} \) are classified naturally into groups of \( 2p \) by the \( j \) of the \( e_j^{(1)} \) they are adjacent to:
\[ G_j^{(1)} = \{e_j^{(2)} \mid 2p(j - 1) + 1 \leq t \leq 2pj\}, \quad 1 \leq j \leq 2p, \]
\[ G_j^{(-1)} = \{e_j^{(-2)} \mid 2p(j - 1) + 1 \leq t \leq 2pj\}, \quad 1 \leq j \leq 2p. \]

Obviously a permutation of the valuation \( \psi \) inside one group is immaterial. So, we define merely a set of \( 2p \) values for each group putting
\[ (**) \]
\[ \psi(G_j^{(1)}) = \{a_t \mid j + 1 \leq t \leq \min((j + p), (2p + 1))\} \cup \]
\[ \cup \{a_t \mid 1 \leq t \leq j - p - 1\} \cup \{a_t' \mid p + 1 \leq t \leq 2p\}, \]
\[ \psi(G_j^{(-1)}) = \{a_t'' \mid j + 1 \leq t \leq \min((j + p), (2p + 1))\} \cup \]
\[ \cup \{a_t'' \mid 1 \leq t \leq j - p - 1\} \cup \{a_t' \mid p + 1 \leq t \leq 2p\}. \]

(One such valuation \( \psi \) is shown for \( p = 2 \) on Fig. 2, where for transparency we write 1 for \( a_1 \), 3' for \( a'_3 \) etc.) (Observe that considering the valuation induced by \( \psi \) on \( b \mathcal{F}_2^{(2p)} \) and looking at \( e_i^{(0)} \) as "\( e_i^{(1)} \)" and at \( e_i^{(-1)} \mid 1 \leq j \leq 2p \) as "\( G_{2p+1}^{(1)} \), \( \psi \) on them meets the rules (*) and (**) .)

Let us now show that \( \psi \) so defined is a \( C \)-valuation. For paths of odd length the condition (i) of Def. 3 holds trivially, so we concern ourselves only with paths of length 2 or 4 in \( \mathcal{F}_2^{(2p)} \). The paths of length 2 being well valuated by inspection, assume there is a path \( x \) of length 4 such that two elements of \( F \), say \( x \) and \( y \), appear on it twice each. The center of any path of length 4 in \( \mathcal{F}_2^{(2p)} \) is either in \( v_1^{(1)} \) or in \( v_1^{(-1)} \). Assume for \( x \) the former happens. Hence \( x \) and \( y \) must be both unprimed \( a \)'s, say \( a_r \) and \( a_s \).
So it must simultaneously be $a_J \in G_s^{(1)}$, $a_r \in G_r^{(1)}$, with possible $r = k + 1$ or $s = k + 1$. That however is impossible by definition of $\psi(G_t^{(1)})$. What concerns the case that the center of $p$ is in $\nu_1^{(-1)}$, observe the symmetry in $\psi$ which permits us to repeat the former argument with interchange of $a_J$ and $a_r$ $(p + 1 \leq j \leq 2p)$. Q.E.D.

2. To construct a $C_{3p+2}$-valuation of $\mathcal{F}_2^{(2p+1)}$, consider the valuation used for $\mathcal{F}_2^{(2p)}$, specifically that induced on $\mathcal{F}_2^{(2p+1)}$. $\mathcal{F}_2^{(2p+1)}$ arises from $\mathcal{F}_2^{(2p)}$ by adding one $e_j^{(2)}$ in each $G_r^{(1)}$. The desired $C_{3p+2}$-valuation is simply obtained by modifying $\psi$ in the way that to each mentioned new $e_j^{(2)}$ the new value $a_{2p+1}$ is assigned. Obviously this does not spoil the property (i) of Def. 3. Q.E.D.

3. We proceed now to show that $\mathcal{F}_2^{(2p+1)} \notin \mathcal{X}_{3p+2}$. Assume the contrary. Consider $\mathcal{F}_2^{(2p+1)}$ as a partial subgraph of $\mathcal{X}_{3p+2}$. Without loss of generality assume $\nu_1^{(1)}$ is in the vertex 0 of $\mathcal{X}_{3p+2}$, and the 2$p + 2$ neighbours of $\nu_1^{(1)}$ in $\mathcal{F}_2^{(2p+1)}$ are in the vertices $\{j\}$ for $1 \leq j \leq 2p + 2$ of $\mathcal{X}_{3p+2}$. It is now necessary to place the $(2p + 1)(2p + 2) = 4p^2 + 6p + 2$ vertices of degree 1 of the $\mathcal{F}_2^{(2p+1)}$ into the $(3p + 2) - \left(\frac{p}{2}\right) = 4p^2 + 5p + 1$ vertices $\{i,j\}$ of $\mathcal{X}_{3p+2}$ with $1 \leq i \leq 3p + 2$, $1 \leq j \leq 3p + 2$, $i \neq j$, such that not both $i$ and $j$ are $>2p + 2$. As this is not possible by reason of numbers, the proof is complete.

4. To complete the proof of the whole theorem, we have to show $\mathcal{F}_2^{(2p)} \notin \mathcal{X}_{3p}$, $\mathcal{F}_2^{(2p+1)} \notin \mathcal{X}_{3p+1}$. To that purpose we show that from $\mathcal{F}_2^{(k)} \in \mathcal{X}_n$, follows $2n \geq 3k + 1$. Indeed, if $\mathcal{F}_2^{(k)}$ is a partial subgraph of $\mathcal{X}_n$, there are certain $k^2$ vertices of $\mathcal{F}_2^{(k)}$ to be placed into \(\binom{n}{2} - \binom{n-k}{2}\) vertices of $\mathcal{X}_n$, hence $k^2 \leq \binom{n}{2} - \binom{n-k}{2}$ and the desired inequality follows.
To be able to derive statements about much wider classes of trees than $\mathcal{T}_k$, $\mathcal{T}_k^1$, $\mathcal{T}_k^2$, we observe that $\mathcal{T}_k^1$ and $\mathcal{T}_k^2$ are in a sense the most general trees with given diameter and given maximum degree of the vertices. Strictly speaking, the following holds:

**Lemma 1.** Let the maximum degree of the vertices of the tree $\mathcal{T}$ be $k + 1$. If the diameter of $\mathcal{T}$ equals $2l$ resp. $(2l + 1)$, then $\mathcal{T}$ is a partial subgraph of $\mathcal{T}_k^1$ resp. $\mathcal{T}_k^2$.

Proof is obvious.

**Corollary 1.** Suppose the maximum degree of the vertices of the tree $\mathcal{T}$ is $d \geq 1$ and the diameter of $\mathcal{T}$ is $\leq 5$. If $d = 2a$ then $\dim \mathcal{T} \leq 3a$, if $d = 2a + 1$ then $\dim \mathcal{T} \leq 3a + 1$. There is, on the other hand, to any $d \geq 1$ a tree $\mathcal{T}$ with maximum degree of the vertices equal $d$ and diameter $\leq 4$ such that $\dim \mathcal{T} = 3a$ for $d = 2a$ resp. $\dim \mathcal{T} = 3a + 1$ for $d = 2a + 1$.

Proof. The inequalities follow, for $d \geq 3$, from L 1 and Th 1. On the other hand observe that $\mathcal{T}_k^1$ has diameter 4 and maximal degree of its vertices ($k + 1$). The cases $d = 1$ and $d = 2$ are trivial.

For $\mathcal{T}_1^{(2)}$ and $\mathcal{T}_1^{(k)}$ the results obtained are exact. For $k > 2$, $l > 2$ we are only able to give bounds for $\dim \mathcal{T}_1^{(k)}$. From one side, we only succeeded in finding trivial bounds:

**Remark 4.** $\dim \mathcal{T}_1^{(k)} = k$. The proof of this rests on the following $C_{kr}$-valuation of $\mathcal{T}_1^{(k)}$. For the edges of each level of $\mathcal{T}_1^{(k)}$, $k$ different elements of $F$ are reserved and distributed in such a way that adjacent edges are assigned different values. In fact, an insubstantially better bound is obtained by using Th 1. for the first two levels, and applying a slightly finer reasoning to the remaining ones. For $k > 2$, $l > 2$ it holds that $\dim \mathcal{T}_1^{(k)} \leq 3/2k + 1 + (l - 2)(k - 1)$.

**Theorem 2.** $\dim \mathcal{T}_1^{(k)} > kl/e$ where $e = 2, 71 …$

Proof. Assume $\mathcal{T}_1^{(k)}$ to be isomorphic to some partial subgraph of $\mathcal{K}_n$. Then comparing the number of vertices, $2^n \geq \text{card } V_1^{(k)} > k^l$ and hence

$$n > l \log_2 k.$$  

Consider first $2 \leq k \leq 8$. Here we have $e \log_2 k > k$ and hence $n > l \log_2 k > kl/e$ and the desired inequality holds. Assume now $k > 8$. It follows from (1) that

$$n > 3l.$$  

The isomorphism may be assumed such that to the vertex $v_1^{(1)}$ of $\mathcal{T}_1^{(k)}$ the vertex $0$ of $\mathcal{K}_n$ corresponds. Then to the $k^l$ vertices of distance $l$ from $v_1^{(1)}$ in $\mathcal{T}_1^{(k)}$ there must
correspond vertices of $X_n$ whose cardinalities are either $l$ or less than $l$ by an even number, hence

$$k^l < \binom{n}{l} + \binom{n}{l+2} + \binom{n}{l-4} + \ldots$$

where the sum at the right is finite, ending either with $n$ or $1$ depending on the parity of $l$. As

$$\binom{n}{p-2}/\binom{n}{p} \leq \binom{n}{l-2}/\binom{n}{l} = q$$

for $p \leq l$, we may write

$$\binom{n}{l} + \binom{n}{l-2} + \binom{n}{l-4} + \ldots < \binom{n}{l}(1 + q + q^2 + \ldots) = \binom{n}{l}/(1 - q).$$

Using (2) we have, however,

$$q = l(l-1)/((n-l+1)(n-l+2)) < l(l-1)/((2l+1)(2l+2)) < 1/4$$

and this yields together with (3) and (4)

$$k^l < \frac{4}{3}\binom{n}{l}.$$ 

For estimating $\binom{n}{l}$ we use the trivial $n(n-1)\ldots(n-l+1) < n^l$ and Stirling’s formula

$$l! = \sqrt{(2\pi l)}(l/e)^l \exp(\theta_i)$$

where $|\theta_i| < 1/(12l)$ and get from (5)

$$k^l < \frac{4}{3} \exp(-\theta_i)(ne/l)^l(2\pi l)^{-1/2}.$$ 

Finally

$$\binom{ne}{k^l} < \frac{4}{3} \sqrt{(2\pi l)} \exp(\theta_i) = \sqrt{[9/8\pi l \exp(2\theta_i)]} > \sqrt{[9/8\pi l \exp(-1/6)]} > 1.$$ 

Q.E.D.

**Corollary 2.** Suppose the maximum degree of the vertices of the tree $T$ is $d \geq 3$ and the diameter of $T$ is $D > 5$. Then $\dim T \leq \frac{1}{4}(d - 1)D$. On the other hand, given $d \geq 3$ and $D > 5$, there is a tree $T$ with maximum degree of the vertices equal $d$ and of diameter $\leq D$ such that $\dim T > \lceil(D - 1)/2\rceil \cdot (d - 1)/e$.

**Proof.** The first inequality follows from Lemma 1, Remark 4 and Remark 3. The proof of the second statement follows by observing that for the tree $T$ we may take $T^{(k)}$ for $l = \lceil(D - 1)/2\rceil$ and $k = d - 1$. 

313
Compared with Theorem 3 in [1] and Theorem 1 of this paper, the result of Remark 4 and Theorem 2 is much less satisfactory. It would be desirable to narrow the bounds, if not find an equality — which, however, seems difficult. It appears to us that while the lower bound is rather close to the actual value of $\dim \mathcal{F}_i^{(k)}$ there is much space for improvement with the upper bound.

One remark more. It may be noted that we mention $\dim \mathcal{F}_i^{(2)}$ or $\dim \mathcal{F}_i^{(2)}$ nowhere. Trivially, there is an inequality following from Remark 3 and from Theorem 3 of [1], namely $l + 2 \leq \dim \mathcal{F}_i^{(2)} \leq \dim \mathcal{F}_i^{(2)} \leq l + 3$. We have, however, a conjecture, which we were not able to prove and only succeeded in verifying for $l = 2, 3, 4$:

Conjecture. $\dim \mathcal{F}_i^{(2)} = l + 2$.

Added in proof. Meanwhile, L. NEBESKÝ in a paper to appear has proved the Conjecture. Also, F. OLLÉ in his M. Sc. thesis has substantially improved Remark 4, proving $\dim \mathcal{F}_i^{(k)} \leq \frac{1}{4}(kl + 2l + k - 2)$.

References


Authors' address: 115 67 Praha 1, Žitná 25 (Matematický ústav ČSAV v Praze).