Abstracts of CSc. theses in mathematics

Commentationes Mathematicae Universitatis Carolinae, Vol. 33 (1992), No. 1, 183--188

Persistent URL: http://dml.cz/dmlcz/118486

ABSTRACTS OF CSc. THESES IN MATHEMATICS

(Candidatus Scientiarum)
defended recently at Charles University, Prague

SURFACES WITH CONSTANT CURVATURES IN $\mathcal{S}^4(1) \subset E^5$

BAJÁKOVÁ Pavla, Department of Mathematics, Faculty of Electrotechnics, Hilleho 6, 602 00 Brno, Czechoslovakia
(January 16, 1991; supervisor K. Svoboda)

The thesis consists of three parts. The first one contains a preparatory material for parts 2 and 3. Two-dimensional surfaces $m = m(x, y)$ imbedded in the sphere $\mathcal{S}^4(1)$ with center S in the Euclidean space E^5 are investigated. For every point m of the surface there are the corresponding tangent plane $T(m) = T_m\{v_1; v_2\}$, the normal plane $N(m) = N_m\{v_3; v_4\}$ and it is assumed that $S = m+v_5$. The indicatrix of the normal curvature of the surface at a point m is an element of $N(m)$ and it defines four invariants of the surface. The author investigates the surfaces with constant curvatures.

In the second part of the thesis the center of the indicatrix of the normal curvature is supposed to differ from point m of the surface. Surfaces in $\mathcal{S}^4(1)$ with the property that the indicatrix in any point m is an ellipse, a circle, an interval or a point are used. There are only two cases of surfaces with the described properties: the two-dimensional forms in four-dimensional space, where the indicatrix is a segment and the two-dimensional sphere in three-dimensional space, where the indicatrix is a point.

In the third part, surface indicatrices which have the center in the corresponding point of the surface and satisfy the condition of the second part are studied. The indicatrix satisfies the assumptions from the second part of the thesis. If the indicatrix is a circle, we get the Veronese surface. If the indicatrix is a segment or a point, surfaces analogous to those described in part one exist.

THE INEQUALITIES OF SPACE CURVES AND SPACE N-GONS

PECH Pavel, Department of Mathematics, Faculty of Education, Jeronýmova 10, České Budějovice, Czechoslovakia
(January 16, 1991; supervisor Z. Nádeník)

In the first part of the thesis the inequality between sides and diagonals of space n-gon in R^N is established. This inequality is generalized to the integral inequality. It is shown that these inequalities imply the well-known discrete and integral Wirtinger's inequalities. According to the Nádeník sharpening of a continuous case of the Wirtinger's inequality, the discrete case is sharpened. By means of this sharpening the isoperimetric inequality for plane equilateral n-gons is sharpened.

The second part deals with the Petr's Theorem. This theorem is generalized to space n-gons. The construction is presented, which to an arbitrary space n-gon assigns its center of gravity or the affine-regular plane n-gon.

CARDINAL CHARACTERISTICS OF THE REAL LINE AND BOOLEAN-VALUED MODELS

REPICKÝ Miroslav, Mathematical Institute, Slovak Academy of Sciences, Košice, Czechoslovakia
(January 31, 1991; supervisor L. Bukovský)

The author studied some cardinal characteristics concerning the measure and category and cardinal characteristics of porosity of sets of reals. At the same time he investigated properties of forcing notions which make possible to control corresponding cardinal characteristics of measure and category in finite support iterations. The author also examines properties of measure and category in generalized Cohen's forcing $C(J)$ and in generalized Silver's forcing $S(J)$ which depend on combinatorial properties of the ideal J on the set of all natural numbers.

REPRESENTATIONS OF GRAPHS IN EUCLIDEAN SPACE

ŠIŇAJOVÁ Edita, Department of Mathematics, FJFI, Czech Technical University, Trojanova 13, 120 00 Praha 2, Czechoslovakia
(January 31, 1991; supervisor B. Balcar)

The thesis deals with representations of graphs in Euclidean space. In the first chapter it studies two generalizations of threshold graphs. A graph $G = (V, E)$ is threshold if there exist a positive threshold $t \in R_+$ and positive labeling $V : x \to \bar{x}$ such that $xy \in E \Leftrightarrow \bar{x}\bar{y} \geq t$. Passing from R_+ to R, the generalized threshold graphs are obtained. These graphs are characterized by means of forbidden subgraphs.

Further, it investigates two definitions of threshold hypergraphs introduced by M.Ch. Glumbic. It is proved that these two definitions which are the same for graphs are different for hypergraphs.

In the second chapter, four different dimensions of graph are considered: scalar product dimension, spherical dimension, distance dimension and orthogonal dimension.

Scalar product dimension is determined for C_n, $\bar{C}_n$, nK_n, K^{nn} and P_n. Two theorems give upper bounds for graphs without small cycles. It is also proved that the scalar product dimension is never greater than the threshold dimension.

It is proved that the spherical dimension is closely related to the distance dimension which gives a lower bound for the spherical dimension for graphs with small radius and large independent set. For some graphs are determined or bounded their spherical their spherical dimension.

In the last section is solved a problem considered in T.D. Parsons and T. Pisanski: Vector Representation of Graphs, to appear in Disc. Math. It is proved that $\mathrm{d}(G, R) = \mathrm{d}(G, Q)$ for each graph, where $\mathrm{d}(G, R)$, resp $\mathrm{d}(G, Q)$ are orthogonal dimensions for which the representations of vertices are considered in R^n, resp. in Q^n (Q is the set of rational numbers).

STOCHASTIC APPROXIMATION WITH DELAYED OBSERVATIONS

FROLÍKOVÁ Ivana, Department of Statistics, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 00 Praha 8, Czechoslovakia
(March 26, 1991; supervisor V. Dupač)

Robbins–Monro type stochastic approximation is investigated, modifying the classic Robbins–Monro procedure to be usable for problems with delayed observations. In order to diminish time losses due to delays of observations two modifications are treated:

1) Allocating the experiments into K parallel series (collaborating or noncollaborating) is the first one. In each series a Robbins–Monro process is constructed, an approximation of the zero point of the regression function is made using the current approximations of all series. Next experiment is done in one of the open series at the point of the last approximation in this series, if there is no open series, no experiment is made and a time loss thus occurs.

2) Second approach to the problem is to construct a single approximation sequence. The next experiment is made at the same point as the preceding one if no result of any observations has become known during the time interval between. It is made at the preceding point corrected by the average of all observations whose results have become known during that time interval, if there are such observations.

Convergence properties and the asymptotical distribution of estimators are investigated for both methods.

FINE TOPOLOGY AND UNIQUE EXTENSION IN COMPLEX PLANE

PYRIH Pavel, Department of Math. Analysis, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 00 Praha 8, Czechoslovakia
(May 15, 1991; supervisor J. Lukeš)

In the thesis, the problem of unique extension of a function of complex variable is solved using classical results from the potential theory, especially from the fine topology in the plane. Finely holomorphic functions are treated with the use of logarithmic capacity and the Wiener criterion.

A new quasi-analytical class of functions $C\{n^n \cdot (\ln n)^n\}$, which is strongly greater than $\{n^n\}$, represents one of the results of this thesis.

In the book of Landkof: Foundations of Modern Potential Theory, Nauka, Moscow, 1966 (in Russian) there is a proof of the assertion that logarithmic capacity is strongly subadditive. This is not true, it is proved that there exist compact sets A and B such that

$$cap\ A + cap\ B\ <\ cap\ (A \cup B)$$

In the first chapter basic results concerning fine topology and potential theory are described. In the second chapter capacity and the Choquet theory of capacities are introduced. In the third chapter there are introduced finely holomorphic functions and important theorems. In the fourth chapter monogenic functions of E. Borel (1917) and their relations to finely holomorphic functions are treated. The fifth chapter is devoted to analytic extensions of complex functions. There are several

methods which can be used in examples. In the sixth chapter finely holomorphic functions are used to obtain some results about logarithmic capacity.

MATHEMATICAL AND PHYSICAL THEORY OF MULTIPOLAR VISCO-ELASTICITY

RŮŽIČKA Michael, Mathematical Institute, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 00 Praha 8, Czechoslovakia
(June 13, 1991; supervisor J. Nečas)

In the thesis, the physical and mathematical theory of multipolar materials is presented.

The constitutive laws of such materials depend on the deformation gradients up to the order L, on the gradients of velocity up to the order K and on the temperature and its gradient. The author developed a thermodynamic theory of such constitutive functions satisfying the second law of thermodynamics, the balances of energy and mass and the principle of material frame indifference. He investigates the restrictions on the constitutive equations arising from the principle of material frame indifference and from the Clausius–Duhem-inequality.

The mathematical theory of three-dimensional nonlinear elastodynamics is up to now not very satisfactory. The presented work is one of the first attempts to handle this problem under acceptable physical assumptions. The author gives an appropriate mathematical theory for the special case of k-polar, $k \geq 3$, incompressible, homogeneous, isotropic materials. Especially he proves the existence and uniqueness of weak solutions for the dynamics and the thermodynamics of the initial-boundary value problem in two- and three-dimensional bounded domains without any restriction on the size of the initial data.

TOPOLOGICAL PROPERTIES OF MONOTONE OPERATORS, ACCRETIVE OPERATORS AND METRIC PROJECTIONS

VESELÝ Libor, Department of Math. Analysis, Faculty of Mathematics and Physics, Charles University, Sokolovská 83, 186 00 Praha 8, Czechoslovakia
(July 1, 1991; supervisor J. Lukeš)

In the dissertation, some topological properties of monotone operators, accretive operators and metric projections are considered and new related results are given. The classes of monotone and accretive operators coincide and contain all metric projections if the space is Hilbert.

In Chapter I, the basic properties of monotone operators in Hilbert spaces are collected. Next chapters are devoted to examining these and analogous properties of monotone operators, accretive operators and metric projections in normed or Banach spaces.

In Chapter II the author gives a short proof of a general theorem on local boundedness of monotone operators, he investigates the "smallness" of the set of points at which a monotone operator on a separable normed space is multivalued, and proves a connectedness property of maximal monotone operators on Hilbert spaces.

Chapter III concerns accretive operators on Banach spaces. He proves a dense local boundedness principle, gives geometric characterization of spaces in which all

maximal accretive operators have convex, resp. weakly closed values, and states theorems on generic continuity in separable and nonseparable spaces.

In Chapter IV, the metric projections onto arbitrary closed subsets of a normed linear space are considered. The author characterizes the spaces in which metric projections onto various sets (arbitrary closed, two-point sets, subspaces, lines, hyperplanes) are accretive, and he considers some weaker properties, too. He investigates relations between "sunny" properties of a set and properties of its metric projection. He gives a characterization of multivalued mappings of a reflexive Banach space into its closed subspace, which are metric projections for some equivalent norm. He proves that given a reflexive space and an infinitely-dimensional closed subspace which is not a hyperplane, the space in question admits an equivalent norm such that the subspace is Chebyshev and its metric projection is not continuous. He also considers the connectedness structure of the discontinuity set of a metric projection in a Hilbert space.

Ten open problems are stated in Chapter V.

SOME NATURAL GEOMETRIC TRANSFORMATIONS OF THE THIRD ORDER

DOUPOVEC Miroslav, Department of Math. and Descriptive Geometry VUT, Technická 2, 616 69 Brno, Czechoslovakia
(September 11, 1991; supervisor I. Kolář)

In the first part of the thesis the author determines all natural operators $T \to TT^2$ transforming vector fields to the second order tangent bundle and all natural transformations $TT^2 \to TT^2$ over the identity of the functor T^2.

Further he finds all natural transformations $T_1^2T^* \to T^*T_1^2$ and interprets them geometrically.

Finally he determines all natural transformations $TTT^* \to TT^*T$; interprets them geometrically and compares the result with Kobak's recent paper (P. Kobak: Natural liftings of vector fields to tangent bundles of 1-forms, to appear in Čas. pěst. mat.) in which the prolongation of vector fields to the bundle TT^* is studied.

CONNECTIONS IN THE GEOMETRY OF NON-AUTONOMOUS REGULAR HIGHER-ORDER DYNAMICS

VONDRA Alexandr, Katedra matematiky Vojenské akademie v Brně, PS 13, 612 00 Brno, Czechoslovakia
(September 11, 1991; supervisor D. Krupka)

The aim of the work is to give a self-contained survey of the author's new results concerning the role played by connections of various kinds in the geometry of non-autonomous higher-order dynamics on fibred manifolds with one-dimensional base. The main emphasis is laid on looking for various connections whose paths have a dynamical importance either for a given regular lagrangian or more generally for regular locally variational equations. These connections and related dynamical structures are then applied to the more detailed investigations of the corresponding geometry on tangent bundles of jet bundles.

MATRICES IN BOTTLENECK ALGEBRA

CEHLÁROVÁ Katarína, Katedra geometrie a algebry PF UPJŠ, Jesenná 5, 041 54 Košice, Czechoslovakia
(September 24, 1991; supervisor K. Zimmermann)

A quadruple $\mathcal{B} = (, \oplus, \otimes, \leq)$ is called a bottleneck algebra, if $(B, \leq)$ is a linearly ordered set without maximum and minimum and $(\oplus, \otimes) = (\max, \min)$. A matrix A is said to have strongly linearly independent columns, if for some b the system of linear equations $A \otimes x = b$ has a unique solution, a square matrix is said to be strongly regular.

The author characterizes solution sets of systems $A \otimes x = b$, proves necessary and sufficient conditions for strong linear independence and strong regularity both over dense and discrete bottleneck algebra and presents algorithms for testing these properties. She shows how the obtained results can be applied to the bottleneck assignment problem and to the matching problem.

In Chapter 8 eigenvectors are studied and a description of the eigenspace in terms of the upper and lower basic eigenvector is derived together with a method how to find them.

DEVIATIONS FROM ASYMPTOTIC NORMALITY FOR OPTIMAL SOLUTIONS OF STOCHASTIC PROGRAMMING PROBLEMS

TLUSTÝ Pavel, Faculty of Education, Jeronýmova 10, 371 15 České Budějovice, Czechoslovakia
(October 15, 1991; supervisor J. Dupačová)

The true solution $\bar{x}$ of the stochastic program which minimizes $Ef(x, \xi)$ on the convex set X can be approximated by optimal solutions $\bar{c}_N$ that minimize the empirical expectation $\frac{1}{N}\Sigma s(x, \xi_i)$ on the convex set X. If $\bar{x} \notin \operatorname{int} X$ the asymptotic distribution of $\bar{x}_N$ will not be generally normal. The dissertation extends King's approach to the asymptotic analysis that has been built on the new concept of pseudo-derivative of mapping introduced by Rockafellar that characterizes the non-normal asymptotic distribution by means of optimal solutions of a random stochastic quadratic program. Detailed analysis of this quadratic program gives a starting point to simulation studies and a hint for diagnostics of the behavior of optimal solutions. Weaker assumptions introduced in the dissertation give possibility to apply the results under more realistic circumstances, e.g. for on-line data and for multiple optimal solutions of the underlining true program.