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Structure resolvability

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Abstract. We introduce the general notion of structure resolvability and structure irresolvability, generalizing the usual concepts of resolvability and irresolvability.

Keywords: resolvability, irresolvability, structure resolvability, Borel resolvability

Classification: Primary 54A35, 03E35; Secondary 54A25

1. Introduction

In 1943 E. Hewitt [1] introduced the notion of resolvable space. *A topological space is said to be* **resolvable** *if it has two disjoint dense subsets. A topological space is said to be* **irresolvable** *if it is not resolvable.* We introduce the general notion of **structure resolvability** and **structure irresolvability**.

Let X be a set and $\mathcal{S}$ be a family of subsets of X. The pair $(X, \mathcal{S})$ is called **a structure** (this differs a bit from the notion of structure in mathematical logic and algebra). A subset $A \subset X$ is called "dense" in (with respect to) $\mathcal{S}$ if $A \cap S \neq \emptyset$ for each $S \in \mathcal{S}$. We say that the structure $(X, \mathcal{S})$ is resolvable if X contains two disjoint subsets A_1, A_2 which are "dense" in $\mathcal{S}$, i.e. $A_i \cap S \neq \emptyset$ for every $S \in \mathcal{S}$ and $i = 1, 2$. The structure $(X, \mathcal{S})$ is said to be irresolvable if it is not resolvable, i.e. there are no disjoint dense subsets with respect to $\mathcal{S}$.

The notion of topological resolvability (irresolvability) introduced by Hewitt becomes a particular case of the notion of resolvable (irresolvable) structure. To see this we take the family of all nonempty open subsets of the given topological space as $\mathcal{S}$.

Structure resolvability affirms in some sense the simplicity of the structure itself. Indeed if the structure $\mathcal{S}$ on the set X is irresolvable, then we can consider that it is a rather complicated one: for any partition of X into two disjoint parts there exists some element S of the structure that is hidden (i.e., is contained in) in one of these parts.

We recall here two fundamental results of the theory of resolvable (irresolvable) topological spaces. The first is an old one and the second is recent.

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1.1 El'kin's criterion of topological irresolvability ([2]). *The topology τ on X is irresolvable if and only if τ contains a base of some (set-theoretic) ultrafilter on X.*

1.2 Illanes' proposition for topological resolvability ([3]). *If a space X is m-resolvable for each natural number m, then it is $\aleph_0$-resolvable.*

We recall that the topology τ on X is called k-**resolvable**, where k is a cardinal number, if X contains k disjoint dense subsets. The topology τ is called **maximally resolvable** if it is $\Delta(\tau)$-resolvable, where $\Delta(\tau) = \min\{|V| : V \in \tau, V \neq \emptyset\}$. This cardinal $\Delta(\tau)$ is called the **dispersion character** of the topology τ of the space X.

The well studied notion of topological resolvability (irresolvability) allows us to ask the following questions about a given structure $\mathcal{S}$.

1.3 Question. Does El'kin's criterion of irresolvability hold for $\mathcal{S}$?
1.4 Question. Does Illanes' proposition hold for $\mathcal{S}$?
1.5 Question. Is $\mathcal{S}$ maximally resolvable?

2. Examples of resolvable or irresolvable structures

2.1 Topological spaces. Many beautiful results have been obtained in this field. We mention only some recent results:

W.W. Comfort (see [4]) proved resolvability of any Tychonoff countably compact space without isolated points;

W.W. Comfort and J. van Mill [5] proved resolvability of any infinite Hausdorff topological abelian group with only finitely many elements of order 2;

V.I. Malykhin [6] proved maximal resolvability of any Hausdorff totally bounded topological group.

2.2 Various measure spaces. Is the family of all Lebesgue measurable subsets of the real line resolvable? It is evident, that this does not hold. But what about the family of all subsets having strictly positive measure? Yes, because every such subset contains some closed subset of cardinality $\mathfrak{C}$, and it is not difficult to see that an arbitrary family of cardinality $\mathfrak{C}$ consisting of closed subsets of cardinality $\mathfrak{C}$ is resolvable — we can apply the Generalized Bernstein-Kuratovskii's Lemma [7, p. 514].

2.3 Algebraic groups. Let G be a group with the unit e and $\mathcal{S} = \{A \setminus \{e\} : A$ is an infinite subgroup of $G\}$, i.e. $\mathcal{S}$ is the family of all infinite subgroups with deleted unit. Is $\mathcal{S}$ resolvable? The answer is evident for many groups. Nevertheless, we have

2.3.1 Proposition. *Let Ω be a countably infinite Boolean group. Then the family $\mathcal{S}$ mentioned above is not resolvable.*

Indeed, for any partition $\Omega = \Omega_1 \cup \Omega_2$ there exists an infinite subgroup B such that $B \setminus \{e\}$ is contained in one of the two parts Ω_1, Ω_2. See details in [8]. □

3. Resolvability and irresolvability of one structure by means of elements of another one

The introduced notion of resolvable structure has an essential inadequacy: the dense subsets available in the case of resolvable structure can be extremely nonconstructive. As a rule, their existence is proved with the aid of the axiom of choice. The desire of having dense subsets of rather simple good nature leads to the following notion.

3.1 Definition. *Let X be a set. Let $\mathcal{S}$ and $\mathcal{B}$ be two families of subsets of X. The structure $\mathcal{S}$ is called $\mathcal{B}$-resolvable if there exist disjoint subsets $B_1, B_2 \in \mathcal{B}$ dense with respect to $\mathcal{S}$, i.e. $B_i \cap S \neq \emptyset$ for every $S \in \mathcal{S}$ and $i = 1, 2$.*

If $\mathcal{B}$ is an algebra, i.e., $X \setminus B \in \mathcal{B}$ if $B \in \mathcal{B}$, then the structure $\mathcal{S}$ is $\mathcal{B}$-resolvable if there exists $B \in \mathcal{B}$ such that $B \cap S \neq \emptyset$, $(X \setminus B) \cap S \neq \emptyset$ for every $S \in \mathcal{S}$.

3.2 Borel or Baire resolvable topology. Let, for example the first structure $\mathcal{S}$ be a topology τ on X and the second $\mathcal{B}$ be the family of all Borel subsets of (X, τ) or the family of all subsets with the Baire property, i.e. having a form $U \Delta N$, where U is an open subset, N is a subset of the first category and Δ is the operation of symmetric difference. In this situation the topological space (X, τ) is called Borel (Baire) resolvable if there exists a Borel (Baire) subset B such that both B and its complement $X \setminus B$ are dense in the space (X, τ). Generally speaking, the classical context of resolvability of topological spaces leads to dissatisfaction with the classical examples of resolvable topological spaces: compact spaces, spaces of countable character (Hewitt [1], M. Katětov [9]): dense subsets in these spaces were extremely nonconstructive. El'kin's criterion of irresolvability of topological spaces mentioned above has also essentially the same nature. The notion of $\mathcal{B}$-resolvable structure $\mathcal{S}$ should partially offset this dissatisfaction.

The notion of resolvable (irresolvable) structure becomes a particular case of $\mathcal{B}$-resolvability, namely, when $\mathcal{B}$ is the family of all subsets of X.

4. Results and problems on $\mathcal{B}$-resolvable structure

We concentrate our attention on $\mathcal{B}$-resolvability of the topological space (X, τ) when $\mathcal{B}$ is the σ-algebra of all Borel subsets of (X, τ) generated by open (equivalently, by closed) subsets of (X, τ) and then we discuss Borel resolvability of the σ-algebra of all subsets with the Baire property, i.e. representable in the form $V \Delta N$, where V is an open subset and N is a nowhere dense subset of X and then we discuss Baire resolvability. It is clear that Borel resolvability is stronger than Baire resolvability and that Baire irresolvability is stronger than Borel irresolvability because every Borel subset has the Baire property. The idea of Borel resolvability was raised by S. Watson during the visit of the second author to Toronto in July–August 93. Both authors would like to mention the stimulating role of S. Watson and express their gratitude to him. This area has been investigated very little. We mention only J. Ceder's article [10] of 1966 in which he considered the question of the resolvability of complete separable metric spaces into uncountably many disjoint dense Borel subsets.

In what follows we will pay more attention to Borel resolvability.

All spaces are assumed to have no isolated points.

4.1 Proposition. *Every T_1-space with σ-discrete π-base is maximally Borel resolvable.*

PROOF: First let us prove that a space Z with σ-disjoint π-base is maximally resolvable, i.e. $\Delta(Z)$-resolvable, in the usual sense. It is enough to prove maximal resolvability of an open subset of cardinality $\Delta(Z)$. Let V be such a subset. It is easy to see that V has a π-base of cardinality not greater than $|V|$. But in this case V is maximally resolvable — we can apply the Generalized Bernstein-Kuratovskii's Lemma [7, p. 514].

Now let us return to Borel resolvability. Let $\mathcal{B} = \sum\{\mathcal{B}_n : n \in \omega\}$ be a π-base of open subsets of the space X and let each family $\mathcal{B}_n$ be discrete. Let $\Delta = \min\{|B| : B \in \mathcal{B}\}$ be the dispersion character of the space X. Let $\{Y_\alpha : \alpha \in \Delta\}$ be a disjoint system of dense subsets. Of course, $|Y_\alpha \cap B| \geq \Delta$ for every $\alpha \in \Delta$ and $B \in \mathcal{B}$. Now we will follow for every $\alpha \in \Delta$ a usual induction process. From each $B \in \mathcal{B}_1$ we will choose a point $a(B) \in B \cap Y_\alpha$. Let us note that the set $M^1 = \{a(B) : B \in \mathcal{B}_1\}$, is discrete, closed and nowhere dense. Hence we may chose one point $a(B)$ from $(B \setminus M^1) \cap Y_\alpha$ for each $B \in \mathcal{B}_2$ and so on for all natural numbers. After finishing this inductive process we obtain sets $\{M^k : k \in \omega\}$. Let $M_\alpha = \cup\{M^k : k \in \omega\}$. Then $M_\alpha \subset Y_\alpha$ and these sets M_α are disjoint dense and F_σ. □

4.2 Corollary. *Every metrizable space is maximally Borel resolvable.*

Let us introduce the notion of the resolvability degree rd of a structure $\mathcal{S}$. We say that $rd(\mathcal{S}) \geq m$ if there exist m disjoint dense subsets with respect to $\mathcal{S}$ (i.e. intersecting each $S \in \mathcal{S}$). We say that $rd_{\mathcal{B}}(\mathcal{S}) \geq m$ if there exist m disjoint subsets belonging to $\mathcal{B}$, all dense in $\mathcal{S}$.

4.3 Proposition. *For every infinite T_1-space X and for the space $exp(X)$ of all nonempty closed subsets of X with the Vietoris' topology we have $rd_{Borel}(exp(X)) \geq rd(X)$.*

PROOF: is very easy: let $\mathcal{Y}$ be a disjoint system of dense subsets of X. For every $Y \in \mathcal{Y}$ let $Fin(Y)$ be the family of all finite subsets of Y. It is evident that $Fin(Y)$ is an F_σ-set which is dense in $exp(X)$. □

Let us formulate the following proposition for the completeness of our exposition.

4.4 Proposition. *For every infinite T_1-space X and for the space $exp(X)$ of closed subsets of X with the Vietoris' topology we have $rd_{Borel}(exp(X)) \geq \aleph_0$.*

PROOF: Let a_k be the k-th in order prime natural number, $k \in \omega$. Put $A_k = \{a_k^i : i = 1, 2, \dots\}$. It is clear that $A_k \cap A_m = \emptyset$ if $k \neq m$. For every $k \in \omega$ let

$E_k = \{Y : Y \subset X, |Y| \in A_k\} \subset expX$. Then $E_k \cap E_m = \emptyset$ if $k \neq m$ and each E_k is dense in $expX$. Moreover each E_k is an F_σ-subset of $expX$. □

4.5 Proposition. *Let each subset of the first category in a space X be a nowhere dense subset. Then X is not Baire resolvable, hence, not Borel resolvable.*

PROOF: Let A be a dense subset with the Baire property, i.e. $A = V \Delta N$, where V is an open subset and N is a subset of the first category. Then $A = (V \setminus N_1) \cup N_2$, where N_1, N_2 are subsets of the first category. If $V \neq \emptyset$ and N is a subset of the first category and, by hypothesis, it is nowhere dense, then $IntA \neq \emptyset$, so $X \setminus A$ is nowhere dense. If $V = \emptyset$ and N_2 is a nowhere dense set, then A is a nowhere dense set, so we have a contradiction to the assumption of density of A. □

4.6 Corollary. *The following spaces are not Baire resolvable,* (*hence, not Borel resolvable*)*, because in each of these spaces subsets of the first category are nowhere dense.*

(1) *The Stone-Čech remainder $\omega^\star = \beta\omega \backslash \omega$ of the countable discrete space.*
(2) *The Stone space of the Boolean algebra of Lebesgue measurable subsets of the segment* $[0, 1]$.
(3) *A Souslin continuum.*

Let us note that a Souslin continuum has always some nonempty interval that satisfies the condition of Proposition 4.5.

We point out that all these spaces are Hausdorff compacta and that a Souslin continuum is even a perfectly normal compact space. Thus we see a big difference between Borel resolvability and usual resolvability, in which each compact space is maximally resolvable.

4.7 Proposition. *Let X be a Tychonoff space and $C(X)$ be the set of all continuous real-valued functions. Then*

(a) *the space $C(X)$ with the uniform convergence topology u is maximally Borel resolvable,*
(b) *the space $C(X)$ with the pointwise convergence topology p is $2^{\aleph_0}$-Borel resolvable.*

PROOF: (a) is true because $C_u(X)$ is a metric space and by Corollary 4.2 it is maximally Borel resolvable. Let us prove (b). Let Y be a countably infinite subset of X. Then the restriction map $f \in C_p(X) \longrightarrow f|Y \in C_p(Y)$ is an open map. But the image of this map (it is a subset of $C_p(Y)$) is a metric space and hence, by Corollary 4.2 it is maximally, (in this case, $2^{\aleph_0}$) -Borel resolvable. Let $\{K_\alpha : \alpha \in 2^{\aleph_0}\}$ be a disjoint system of dense F_σ-subsets in it. Thus $\{f^{-1}(K_\alpha) : \alpha \in 2^{\aleph_0}\}$ are disjoint dense F_σ-subsets in $C_p(X)$. □

4.8 Problem. Is $C_p(X)$ maximally Borel resolvable?

Let us note that V.V. Tkachuk proved maximal resolvability of $C_p(X)$ for each Tychonoff space X. His proof is not very difficult.

A $C_p(X)$ is a pair $(C(X), p)$, where p is the topology of pointwise convergence. The topology of uniform convergence u is stronger than p. As this topology u is metric so it is maximally resolvable, i.e. $\Delta(u)$-resolvable. But it is easy to prove that $\Delta(p) = \Delta(u) = |C(X)|$. This implies maximal resolvability of $C_p(X)$.

4.9 Proposition. *A separable pseudocompact Tychonoff T_1-space is Borel resolvable.*

PROOF: Let S be a countable dense subspace in such a space X. Then S has empty interior and hence S and $X \setminus S$ are two disjoint dense Borel subsets of X. □

Let us note that it is still unknown if every pseudocompact Tychonoff T_1-space is resolvable in the usual sense (this question is due to Comfort [4]).

4.10 Proposition. *A dyadic compact space is Borel resolvable.*

PROOF: Let X be a dyadic compact space. Since the Souslin number of X is countable there exists a family $\mathcal{A}$ of closed G_δ-subsets with empty interior such that $\cup\mathcal{A}$ is dense in X. It is known ([12]) that in a dyadic compact space every family $\mathcal{A}$ of G_δ-subsets has a countable subfamily $\mathcal{B}$ such that $\overline{\cup\mathcal{B}} = \overline{\cup\mathcal{A}}$. So $K = \cup\mathcal{B}$ is dense in X. But $IntK = \emptyset$ because X is compact and we can apply Baire's theorem. Finally, we have a dense in X Borel subset, namely the F_σ-set K, with empty interior; hence, $X \setminus K$ is another dense Borel subset. □

Let us recall that the *Souslin number $c(X)$ of a space X is the supremum of cardinalities of disjoint families of non-empty open subsets of X.* A space X for which $c(X) = \aleph_0$ is said to satisfy the countable chain condition or to be a c.c.c. space.

Recall that when we consider (usual or set-theoretic) structure resolvability we may ask whether El'kin's criterion of irresolvability and Illanes' proposition hold. With the appropriate modification these questions also arise in the case of Borel and Baire resolvability of topological spaces.

Let us note that for countable T_1-spaces, usual resolvability and Borel resolvability are the same. So, all countable T_1-examples with respect to usual resolvability or irresolvability fit with respect to Borel resolvability or Borel irresolvability.

Let us now consider the main theorems of the theory of usual topological resolvability. Are they true for Borel resolvability?

Hewitt's criterion of irresolvability ([1]) states that *a space is irresolvable iff it contains a nonempty open subset in which every subspace dense in itself is irresolvable.*

4.11 Proposition. *Hewitt's criterion is not true for Borel irresolvability.*

PROOF: If this criterion were true, it would look like: if a space is Borel irresolvable then it has a nonempty open subset V in which it is impossible that $\overline{B}_1 = \overline{B}_2$ for two disjoint dense in itself Borel subsets $B_1, B_2 \subset V$. □

However, $\omega^\star$ is Borel irresolvable by Corollary 4.6, nevertheless each nonempty open subset in it contains a copy of the absolute pK of the simplest Cantor discontinuum $K = D^\omega$ in which there are two disjoint dense F_σ subsets. Indeed, the following lemma is valid.

4.12 Lemma. *The absolute pK is Borel resolvable.*

PROOF OF LEMMA: Let $p: pK \longrightarrow K$ be the irreducible (continuous) map from pK onto K. In K there are two disjoint dense F_σ-subsets M_1, M_2 which are a countable sums of nowhere dense subsets. Then $p^{-1}(M_1)$ and $p^{-1}(M_2)$ are two disjoint dense F_σ-subsets in pK. □

4.13 Question. Let X, Y be coabsolute spaces. Is it true that X is Borel resolvable if and only if Y is Borel resolvable?

However we have to note that one part of Hewitt's criterion remains true, of course: a space is Borel irresolvable if it contains a nonempty open subset V in which it is impossible that $\overline{B}_1 = \overline{B}_2$ for two disjoint Borel subsets $B_1, B_2 \subset V$.

Now we want to formulate an analog of Hewitt's criterion for Borel irresolvability, but we need a definition.

4.14 Definition. *A space is called hereditarily Borel irresolvable if each nonempty open subspace of it is Borel irresolvable.*

(This definition is analogous to Illanes' definition for usual irresolvability [3].)

4.15 Hewitt's criterion for Borel irresolvability for spaces with c.c.c. *A space with c.c.c. is Borel irresolvable iff this space contains a nonempty hereditarily Borel irresolvable open subset.*

PROOF: Let us suppose that a space X doesn't contain any nonempty hereditarily Borel irresolvable open subset. Then every nonempty open subset contains a nonempty Borel resolvable open subset. As $c(X) = \aleph_0$, we can find a countable family of such open Borel resolvable subsets, whose union is dense in X. It is easy to verify that we have obtained two disjoint dense Borel subsets. □

Theorem of Comfort and Feng ([10]) states that *if a space is covered by resolvable subspaces then it is also resolvable.*

4.16 Proposition. *Theorem of Comfort and Feng is not true for Borel resolvability.*

PROOF: Indeed, a Souslin continuum is a union of its closed separable subspaces that are Borel resolvable (because they are metric compact spaces). □

However we have to note that one part of Theorem of Comfort and Feng remains true: a Borel resolvable space is, of course, a union of two Borel subsets.

Now we formulate a weak analog of Theorem of Comfort and Feng for Borel resolvability.

4.17 Proposition. *If a space with c.c.c. is covered by a family of open Borel resolvable subspaces, then it is Borel resolvable.*

And what about **El'kin's criterion of irresolvability**?

This criterion has to look like (in the general case of $\mathcal{B}$-resolvability): a space (X, τ) is $\mathcal{B}$-irresolvable iff the topology τ contains a base for some ultrafilter in the family $\mathcal{B}$.

4.18 Proposition. *If a space (X, τ) contains a nonempty hereditarily Borel irresolvable open subspace, then El'kin's criterion of irresolvability is fulfilled for this space, i.e. the topology τ contains a base for some ultrafilter in the family of all Borel subsets.*

Proof: We may suppose that every non-empty open subspace is Borel irresolvable. Let ξ be a maximal centered family of open subsets. Let us prove that ξ contains a base for some ultrafilter in the family of all Borel subsets. Indeed, let Y, Z be two disjoint Borel subsets and $Y \cup Z = X$. Let $W = X \setminus (Int(Y) \cup Int(Z))$. If $Int(W) \neq \emptyset$, then $Int(Y \cap Int(W)) \subset Int(Y) \cap Int(W) = \emptyset$, hence $Int(Y \cap Int(W)) = \emptyset$. Analogously $Int(Z \cap Int(W)) = \emptyset$. So $Int(W)$ contains two mutually complementary Borel subsets with empty interior, hence $Int(W)$ is Borel resolvable — a contradiction. Thus, now we may assume that $Int(W) = \emptyset$, hence $Int(Y) \cup Int(Z)$ is dense in X, and one of these two subsets belongs to ξ. □

It is evident that ω^*, the Stone space of Boolean algebra of Lebesgue measurable subsets of the segment $[0, 1]$, and a Souslin continuum, all contain some nonempty open Borel irresolvable subspaces (see also Proposition 5). Hence, El'kin's criterion of irresolvability is fulfilled for these spaces.

Two more propositions on El'kin's criterion of irresolvability.

4.19 Proposition. *Let each subset of the first category in the space (X, τ) be a nowhere dense subset. Then this space is not Baire resolvable and τ contains a base for an ultrafilter in the family of all subsets with the Baire property.*

Proof: According to Proposition 4.5 each nonempty open subspace of (X, τ) is hereditarily Borel irresolvable. Therefore we may end our proof having referred to the previous Proposition 18. However we may give a direct proof as well.

Let ξ be a maximal centered family of open subsets in such a space. Let B be any subset with the Baire property. Then $B = V \Delta N$, where V is open and N is of the first category and hence nowhere dense. If $V \in \xi$ then $V \setminus N \in \xi$ hence $B \in \xi$ as $B \supset V \setminus N$. If $V \notin \xi$ then $V \cup N \notin \xi$, hence $B \notin \xi$ as $B \subset V \cup N$. So ξ is an ultrafilter in the family of all subsets with the Baire property, and of course, ξ is an ultrafilter in the family of all Borel subsets. □

We do not believe that El'kin's criterion is true for Borel resolvability in general topological spaces, but it is true for spaces with c.c.c.

4.20 Proposition. *A space with c.c.c. is Borel irresolvable iff its topology contains a base for some ultrafilter in the family of all Borel subsets.*

Proof: According to Hewitt's criterion (see Proposition 15) we may suppose that a space contains a non-empty open hereditarily Borel irresolvable subspace and we may refer now to Proposition 19. □

The situation with Illanes' assertion is analogous.

4.21 Proposition. *If a space with c.c.c. is m-Borel resolvable for each natural number m, then it is $\aleph_0$-Borel resolvable.*

Proof: It is in fact the repetition of Illanes' proof, with some changes. □

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