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ABSTRACTS OF THESES* IN MATHEMATICS

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PIERCING PROBLEMS AND TOPOLOGICAL METHODS

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(April 17, 1998; supervisor J. Matoušek)

The celebrated Helly's theorem, discovered in 1913, is one of the basic results in combinatorial geometry. Various related questions have received much attention, among them being the piercing problems. In the present thesis, we study the piercing problem for several types of geometric objects, as well as another problem that is even more closely tied to Helly's theorem. Since our basic results rely on the use of facts from algebraic topology, it would not be without reason to regard the text as a contribution to the study of topological aspects of Helly's theorem and its relatives. This is supplemented by purely combinatorial proofs of similar results in those cases where such proofs have been found.

MODULES COMMUTING IN FUNCTOR HOM WITH LIMITS

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(April 24, 1998; supervisor T. Kepka)

Let $\triangle$ be a diagram (i.e., a small category) of modules. Given a module M, we have natural (connecting) homomorphism

$$\Phi : \mathrm{colim}(\mathrm{Hom}_R(\triangle, M)) \to \mathrm{Hom}_R(\lim(\triangle), M).$$

It may happen that Φ is an isomorphism, whenever $\triangle$ is a diagram of certain type. In such a case, we say that M commutes (in Hom) with limits of the considered diagrams. The subject of the present work is the study of modules commuting with direct products, pull-backs and limits of downwards-directed spectra. The corresponding modules are fully described in the cases of pull-backs and limits of downwards-directed spectra. In the more complicated case of direct products the cardinality of the index set of the product is to be distinguished. We call the modules commuting with products *slim modules*. When only the products with cardinality of index set less then $\mathfrak{a} \in \mathrm{Cn}$ are considered, we call the corresponding modules $\mathfrak{a}$-*slim*. The $\aleph_1$-slim modules are known under the name *slender modules*. In the present work slender commutative noetherian rings are characterized and some results are established for slender prime rings and slender small (i.e. with less then $2^{\aleph_0}$ elements) semiprime rings. Further, for a regular cardinal $\mathfrak{a}$, examples

*An equivalent to PhD.

of $\mathfrak{a}$-slim modules are given. The existence of slim module depends on the set theory.

BIFURCATIONS FOR REACTION-DIFFUSION SYSTEMS WITH CONDITIONS GIVEN BY INCLUSIONS

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(June 22, 1998; supervisor M. Kučera)

In this thesis, there are studied bifurcations of nontrivial solutions to reaction-diffusion systems

$$(1)\qquad \begin{aligned} u_t &= d_1\Delta u + b_{11}u + b_{12}u + n_1(u,v),\\ v_t &= d_2\Delta v + b_{21}u + b_{22}v + n_2(u,v) \end{aligned} \qquad \text{on } [0,+\infty)\times\Omega$$

with multivalued boundary conditions

$$(2)\quad u = v = 0 \text{ on } \Gamma_D,\ \frac{\partial u(x)}{\partial n} \in -\frac{m_1(x,u(x))}{d_1},\ \frac{\partial v(x)}{\partial n} \in -\frac{m_2(x,v(x))}{d_2} \text{ on } \Gamma_N,$$

where $\Omega \subset \mathbb{R}^N$, $\Gamma_D \cup \Gamma_N = \partial\Omega$, $d_1, d_2 > 0$ are diffusion parameters and m_1, m_2 are multivalued functions of certain types.

The existence of a bifurcation point $d^I = [d_1^I, d_2^I]$ is proved. Moreover, under certain assumptions, the destabilizing effect of multivalued conditions is showed (the bifurcation for the system (1) with (2) occurs in some sense sooner then for (1) with classical Dirichlet and Neumann boundary conditions).
For the proof, the method of M. Kučera — based on the penalty technique — was used.

The first part of the thesis was published in the joint paper with M. Kučera *Spatial patterns for reaction-diffusion systems with conditions described by inclusions*, Applications of Mathematics **42** (1997), no. 6, 421–449.

The second part was submitted to Nonlinear Analysis, Theory, Methods, Applications.

SOLUTION OF OPTIMIZATION PROBLEM ON ATTAINABLE SETS OF EXTREMAL SEPARABLE OPERATORS

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(July 7, 1998; supervisor K. Zimmermann)

The thesis is concerned with the solution of different optimization problems on the solubility sets which are defined on extremal separable operators. We have concentrated on the case when those solubility sets are empty. Some direct algorithms are described to solve the given optimization problems approximately. Also some computer codes are given to illustrate the described algorithms.

LONG-TIME BEHAVIOUR OF SOLUTIONS OF EVOLUTION PROBLEMS

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(August 27, 1998; supervisor E. Feireisl)

In the first part of the thesis we study the parabolic partial differential equation

$$\begin{aligned} u_t - u_{xx} &= f(u), \quad t > 0, \quad x \in \mathbb{R}, \\ u(t, \pm\infty) &= 0, \quad t > 0, \\ u(0, x) &= \lambda u_0(x), \quad x \in \mathbb{R}. \end{aligned}$$

Because of the lack of compactness which comes from the unboundedness of the domain, it is not straightforward that bounded trajectories are convergent. Let the assumptions on f be those which ensure the ground state solution. We show that every trajectory emanating from a compactly supported nonnegative initial value either converges as $t \to \infty$ to a stationary solution or is unbounded in certain integral norm, and that convergence to the ground state is threshold when λ varies. We also study the problem on $\mathbb{R}_+$ with Dirichlet and Neumann boundary conditions.

In the second part of the thesis we study the second order abstract integrodifferential equation

$$\begin{aligned} \ddot{u} + Au + \int_0^t dB(s)\dot{u}(t-s)\,ds &= f(u), \quad t > 0 \\ u(0) = u_0, \quad \dot{u}(0) &= u_1, \quad t = 0. \end{aligned}$$

Here A, $B(t)$ are self-adjoint positive definite operators, the kernel dB is dissipative, so that $dB \star \dot{u}$ is a damping term. Some models for viscoelastic materials with memory can be written in this form. Under some hypotheses on f, A and dB we show that any solution converges to a time-independent solution as $t \to \infty$, provided the set of stationary solutions is discrete. We study the corresponding linear problem, too. Our results can be viewed as generalization of dissipative dynamical systems to Volterra equations with, e.g., fractional derivative damping term.

Selected references.

[1] Fašangová E., *Asymptotic analysis for a nonlinear parabolic equation on* $\mathbb{R}$, Comment. Math. Univ. Carolinae **39.3** (1998), 525–544.

[2] Fašangová E., Prüss J., *Evolution equations with dissipation of memory type*, in: Topics in Nonlinear Analysis, J. Escher, G. Simonett Eds., Birkhäuser (1999), 213–250.

MULTIPLE STATE MODELS OF PERMANENT HEALTH INSURANCE ESTIMATING PER CAPITA EXPENSES

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(September 10, 1998; supervisor P. Mandl)

In the thesis a method was presented to compute the expected amount of annual claims in the health insurance. A regenerative is introduced and described by means of which a general methodology to compute the premium is obtained. This methodology is applicable not only to the permanent health insurance but to other types of insurance as well. The methods used today can be divided into three groups: the method of the decrement tables, the Manchester Unity method and the multiple state model. It is shown how to compute from the data of the multiple state model the basis for the Manchester Unity and for the decrement tables methods. The numerical results obtained are compared with those computed and published for the permanent health insurance in the Continuous Mortality Investigation Reports No. 12 (1992).

LAPLACIANS IN HILBERT SPACES AND SEQUENCES IN BANACH SPACES

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The thesis consists of two loosely connected parts. Part I, composed of the first four chapters, is devoted to discussing various ways how to introduce Laplace's operator on infinite dimensional spaces, especially on Hilbert spaces. Part II, made up only of the last fifth chapter, is concerned with summability of sequences in Banach spaces.

Chapter 1 is preliminary. Chapter 2 is concerned with the theory of the laplacian studied by P. Lévy in his book *Problèmes concrets d'analyse fonctionnelle* (Gauthier-Villars, Paris, 1951). Given an orthonormal basis $E = \{e_n\}_{n=1}^{\infty}$ in a real separable Hilbert space $\mathbf{H}$, the *Lévy laplacian* $\boldsymbol{L}^E$ is defined as follows: For a real functional f having the second Fréchet derivative $f''(x)$ at a point $x \in \mathbf{H}$,

$$\boldsymbol{L}^E f(x) = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} D_{jj} f(x) \tag{1}$$

if the limit exists (where $D_{jj}f(x) = f''(x)(e_j, e_j)$). P. Lévy introduced spherical and volume means of functionals on $\mathbf{H}$ and showed that for uniformly continuous functionals volume and spherical means coincide and that any continuous functional depending only on a finite number of coordinates in $\mathbf{H}$ possesses the mean

value property. In the thesis, an example of an infinitely Fréchet differentiable harmonic functional on $\mathbf{H}$ which has only the local mean value property on $\mathbf{H}$ is given. Moreover, the Dirichlet and the Poisson problem are discussed for the class of uniformly continuous functionals.

It is easily seen that the Lévy laplacian of any functional depending only on a finite number of coordinates in $\mathbf{H}$ is identically zero. If this is true for a differential operator $\boldsymbol{D}$ of the second order in $\mathbf{H}$, then $\boldsymbol{D}$ is called *irregular*. In the thesis, examples of such differential operators different from the Lévy laplacian are given.

The dependence of the spherical mean and the Lévy laplacian on the choice of an orthonormal basis in $\mathbf{H}$ is thoroughly studied. It is shown there that if the Lévy laplacian $\boldsymbol{L}^E f(x)$ of a functional f at a point $x \in \mathbf{H}$ exists for any orthonormal basis E in $\mathbf{H}$, then the value $\boldsymbol{L}^E f(x)$ is independent of the choice of an orthonormal basis E in $\mathbf{H}$. On the other hand, if this is not the case, then there is an orthonormal basis E in $\mathbf{H}$ such that the values of the Lévy laplacian of f at x fill a non-degenerate closed interval while the orthonormal basis E is being rearranged.

Chapter 3 is concerned with the potential theory on abstract Wiener spaces developed by L. Gross. On Hilbert spaces the laplacian introduced by L. Gross is the same as that studied first by Yu.L. Daletzky. A differential operator $\boldsymbol{D}$ of the second order in $\mathbf{H}$ is said to be *regular* provided that there are an orthonormal basis $E = \{e_n\}_{n=1}^{\infty}$ in $\mathbf{H}$ and a sequence $\{a_n\}_{n=1}^{\infty}$ of real numbers such that $\sum |a_n| < \infty$ and

$$\boldsymbol{D}f(x) = \sum_{n=1}^{\infty} a_n D_{nn} f(x) \tag{2}$$

whenever a functional f has the second Fréchet derivative $f''(x)$ at a point $x \in \mathbf{H}$. If any a_n is strictly positive, then such a differential operator $\boldsymbol{D}$ is called a *Daletzky laplacian*.

In Chapter 4, the author's article (*The Lévy laplacian and differential operators of 2-nd order in Hilbert spaces*, Comment. Math. Univ. Carolinae 39 (1998), 115–135) is presented. The article deals with differential operators of the second order in a real separable Hilbert space $\mathbf{H}$. It is shown there that every differential operator of the second order in $\mathbf{H}$ can be decomposed into a regular and an irregular differential operator. In spite of being differential operators of the second order irregular differential operators behave like those of the first order. A characterization of irregularity of differential operators by means of independence on the choice of an orthonormal basis in $\mathbf{H}$ is also given. Moreover, differential operators satisfying maximum principle are characterized and, for an irregular differential operator satisfying maximum principle, the corresponding Dirichlet and the Poisson problem are discussed; in particular, weakly continuous functionals are 'harmonic'.

In Chapter 5, the author's article (*Limit points of arithmetic means of sequences in Banach spaces*, preprint) is presented. In the article, the following statements are proved: Given a sequence $\{a_n\}_{n=1}^{\infty}$ in a Banach space $\mathbf{X}$ enjoying the weak Banach-Saks property, there is a subsequence (or a permutation) $\{b_n\}_{n=1}^{\infty}$ of the sequence $\{a_n\}_{n=1}^{\infty}$ such that

$$\lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} b_j = a$$

whenever a belongs to the closed convex hull of the set of weak limit points of $\{a_n\}_{n=1}^{\infty}$. In case $\mathbf{X}$ has the Banach-Saks property and $\{a_n\}_{n=1}^{\infty}$ is bounded the converse assertion holds too. Of course, in case $\mathbf{X} = \mathbf{R}$ this assertion plays an important role in discussing dependence of the Lévy laplacian on the choice of an orthonormal basis in $\mathbf{H}$. Furthermore, a new characterization of reflexive spaces in terms of limit points and cores of bounded sequences is also given.

FINANCIAL MANAGEMENT IN INSURANCE COMPANIES: APPLICATION IN LIFE INSURANCE

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Insurance companies are very fruitful institutions for mathematical applications. Besides actuarial mathematics also financial mathematics has wide usage and use of such disciplines like econometrics, statistics and optimization depends on individual approach. The aim of this work is to study two main topics. Both concern financial management in insurance companies. Thesis focuses on life insurance. The scope of the thesis is practical usage of achieved theoretical results. The first problem concerns sensitivity of life insurance liabilities from a traditional life insurance product. The concept of Macauley duration as a measure for an interest rate risk with respect to life insurance liabilities is derived. Convexity, variability and portfolio approach are not omitted. Three models are used to assess the problem. The concept is discussed with respect to its usefulness for asset liability management of life insurance companies. The derivation of formulae is completed with many numerical examples and analysis of parameter influence. The second topic is about mathematical modelling of cash flows within life insurance. An appropriate model is an important instrument for a management of the insurance company. Various aspects of building model suitable for forecasting and testing of scenarios are discussed. Equations are estimated by econometric means on the basis of the real data.

AN OBJECT-ORIENTED APPROACH TO MULTISTAGE STOCHASTIC PROGRAMMING: MODELS AND ALGORITHMS

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(November 30, 1998; supervisor J. Dupačová)

In this thesis, the following results have been achieved: First, a detailed overview of different aspects of stochastic programming (static programs, two-stage programs, multistage programs, successful applications, related software, and parallel approaches) has confirmed the implicit need for an advanced modelling approach that will allow for a combination of existing tools and remain flexible and open for future enhancements.

Second, as an OOA theoretical background, certain basic concepts of multistage stochastic programming are generalized, and then a syntactical concept of DOP is presented. It is shown that this concept may serve for a description of existing multiperiod deterministic and multistage stochastic programs. The concept of DOP semantics for certain classes of programs presents the semantical equivalence between DOP descriptions and stochastic programs. Hence, the discussion continues with the possibilities to solve programs in their distributed form.

Third, based on the original interpretation of general object-oriented principles, implementable objects for stochastic programming are designed. Internal objects are derived from the basic inheritance tree. The original ownership scheme is defined and further extended for external objects.

Fourth, melt control problems are carefully studied. Verbal and pictorial metallurgy-related descriptions are transformed into general external and internal schemes. They allow a description of any metal production in any furnace (or combination of furnaces) with use of any technology. As a result, a general multistage underlying program is derived. Its elements are further analysed, especially, the case of decision-dependent randomness. A general algorithm in the form of several nested outer loops is designed. The inner procedure may compute either the individual EV equivalent or the HN scenario-based equivalent using the PHA algorithm and is completed with special-purpose subroutines. Different algorithm versions are implemented using OOA, specifically Delphi and GAMS.

INVARIANT DIFFERENTIAL OPERATORS ON SPINOR-VALUED DIFFERENTIAL FORMS

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The dissertation studies properties of natural differential operators, acting on spinor-valued differential form bundles on a *Spin* manifold and invariant with respect to action of *Spin* group. These operators are rising from (polynomials of) the exterior Levi-Civita covariant derivative, its dual operator, the twisted Dirac operator and proper invariant projections onto the *Spin* irreducible submodules

of the spinor-valued forms space.

The dissertation is divided into two parts; the first of them is devoted to the introduction of notions from Riemann and *Spin* geometry with some necessary basics of the representation theory.

The first section of the second part develops algebraic properties of spinor-valued forms. An $sl(2, C)$ realization, induced by the fundamental form — a natural homomorphism of *Spin* modules — and its dual is properly used to decompose the spinor-valued forms into *Spin* irreducible submodules. As a byproduct the invariant projections onto the irreducible pieces are obtained.

The second section extends the $sl(2, C)$ action to the first order differential operators (so called 'elementary'), given by projections of the covariant derivative and the twisted Dirac. Results like limitation of the target space of elementary operators are obtained.

The third section further extends the $sl(2, C)$ action to higher degree polynomials of the elementary operators. Parts of the Riemann curvature appears among operators obtained in such way; the action of the Riemann curvature on an irreducible submodule is described by decomposition into isometry-invariant elements. Especially simple picture is obtained for action on those modules, isomorphic to the spinor spaces.

The last two sections contain an application of the above derived: the section four, dealing with elementary operators allowing eigenspaces, investigates an embedding among parts of spectra of those operators; for example an embedding of spectra of the Dirac and the Rarita-Schwinger operator on an Einstein manifold is obtained. The fifth section then uses the derived tools to obtain an explicit form of operators occurring in the Bernstein-Gelfand-Gelfand resolution of the spinor space.