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An interesting class of ideals in subalgebras of $C(X)$ containing $C^*(X)$

Sudip Kumar Acharyya, Dibyendu De

Abstract. In the present paper we give a duality between a special type of ideals of subalgebras of $C(X)$ containing $C^*(X)$ and $z$-filters of $\beta X$ by generalization of the notion $z$-ideal of $C(X)$. We also use it to establish some intersecting properties of prime ideals lying between $C^*(X)$ and $C(X)$. For instance we may mention that such an ideal becomes prime if and only if it contains a prime ideal. Another interesting one is that for such an ideal the residue class ring is totally ordered if and only if it is prime.

Keywords: Stone-Čech compactification, rings of continuous functions, maximal ideals, $z^\beta_A$-ideals

Classification: 54D35

1. Introduction

Throughout the paper all topological spaces are assumed to be Tychonoff. For a space $X$, $C(X)$ stands for the ring of all real valued continuous functions on $X$, $C^*(X)$ is the subring of $C(X)$ consisting of all bounded functions and $\Sigma(X)$ will denote the collection of all subalgebras of $C(X)$ containing $C^*(X)$.

It is a fascinating fact in the theory of rings of continuous functions that for a space $X$ the structure spaces of both $C(X)$ and $C^*(X)$ produce the Stone-Čech compactification $\beta X$ of that space. Plank [7] has proved that the structure space of any subalgebra of $C(X)$ containing $C^*(X)$ also produces the Stone-Čech compactification $\beta X$ of $X$ in an analogous manner. In this course an analogous study of arbitrary subalgebra of $C(X)$ containing $C^*(X)$ becomes important. The study of ideals in $C(X)$ depends strongly on the fact that if $I$ is a proper ideal in $C(X)$ then $Z(I) = \{Z(f) : f \in I\}$ becomes a $z$-filter on $X$. But in case of an arbitrary $A(X) \in \Sigma(X)$ the analogous statement is not necessarily true. H.L. Byun and S. Watson [2] introduced a method for studying ideals in arbitrary $A(X) \in \Sigma(X)$. For each ideal $I$ in $A(X)$, they associated a family of subsets of $X$ given by $Z_A[I] = \bigcup \{Z_A(f) : f \in I\}$, where for each $f \in A(X)$, $Z_A(f) = \{E \in Z(X) : \exists g \in A(X) \text{ with } f \cdot g|_{X-E} = 1\}$, which latter turned out to be a $z$-filter on $X$. Further they called an ideal $I$ in $A(X)$ a $B$-ideal if $Z_A^{-1}[Z_A[I]] = I$. But the map $Z_A$, which relates ideals in $A(X)$ to $z$-filters on $X$, lacks the sensitivity for distinguishing prime ideals. In fact even in case of
$A(X) = C(X)$ also, it follows that $Z_C[O_C^p] = Z_C[M_C^p]$ for all $p \in \beta X$, where $O_C^p = \{ f \in C(X) : p \in \text{int}_{\beta X}\{\text{cl}_{\beta X} Z(f)\}\}$. More generally, if $P$ is a prime ideal contained in a maximal ideal $M_A^p$ in $A(X)$ then $Z_A[P] = Z_A[M_A^p]$. So by this definition of $\mathcal{B}$-ideal there does not exist any non-maximal prime $\mathcal{B}$-ideal. In this article we introduce a new type of ideals in $A(X)$ called $z_A^\beta$-ideals, and a correspondence $z_A^\beta$ from the set of all ideals in $A(X)$ to the set of a special type of filters in $\beta X$ in such a way that the correspondence $z_A^\beta$ retains the sensitivity of distinguishing prime ideals to some extent. In fact we shall show that there exists a non-maximal prime $z_A^\beta$-ideal in $A(X)$. Following Plank [7], for any $f \in A(X)$ we denote $\{ p \in \beta X : (f \cdot g)^*(p) = 0 \text{ for all } g \in A(X) \}$ as $S_A(f)$ and $Z_A^\beta[I] = \{ S_A(f) : f \in I \}$. Throughout this article we shall call $S_A(f)$ an $A$-zeroset in $\beta X$, and the set $\{ S_A(f) : f \in A(X) \}$ will be denoted by $Z_A^\beta[X]$.

2. $z_A^\beta$-filter on $\beta X$

Like $z$-filters in $X$, we define $z_A^\beta$-filters in $\beta X$ in the following way.

**Definition 2.1.** A non empty subset $F$ of $Z_A^\beta[X]$ is called a $z_A^\beta$-filter on $\beta X$ provided that

1. $\varphi \notin F$,
2. if $Z_1, Z_2$ are in $F$ then $Z_1 \cap Z_2 \in F$,
3. if $Z$ is in $F$ and $Z' \in Z_A^\beta[X]$ with $Z' \supset Z$ then $Z' \in F$.

Now we can easily see that if $f$ is a unit of $A(X)$ then $\frac{1}{f} \in A(X)$ so that $(f \cdot \frac{1}{f})^*(p) = 1$ for all $p \in \beta X$ and therefore $S_A(f) = \varphi$. Again for each $p \in \beta X$ there exists $g_p \in A(X)$ such that $(f \cdot g_p)^*(p) \neq 0$. This means that $f$ is missed by every maximal ideal in $A(X)$, so that $f$ is not a unit of $A(X)$. Therefore we have the following lemma.

**Lemma 2.2.** Suppose $A(X) \in \Sigma(X)$. Then for any $f \in A(X)$, $S_A(f) = \varphi$ if and only if $f$ is a unit of $A(X)$.

The above lemma discovers the duality existing between the ideals of $A(X)$ and $z_A^\beta$-filters on $\beta X$.

**Theorem 2.3.** For any $A(X) \in \Sigma(X)$ the following holds.

1. If $I$ is an ideal in $A(X)$ then the family $Z_A^\beta[I] = \{ S_A(f) : f \in I \}$ is a $z_A^\beta$-filter on $\beta X$.
2. If $F$ is a $z_A^\beta$-filter on $\beta X$ then the family $Z_A^\beta^{-1}[F]$ given as $\{ f \in A(X) : S_A(f) \in F \}$ is an ideal in $A(X)$.
Before talking about the duality between maximal ideals in $A(X)$ and maximal $z_A^\beta$-filter in $\beta X$ we simply write down the following results, whose proofs can also be given by using the well-known routine arguments. First we introduce the following notion.

**Definition 2.4.** A $z_A^\beta$-ultrafilter on $\beta X$ is a $z_A^\beta$-filter on $\beta X$ which is not contained in any other $z_A^\beta$-filter on $\beta X$.

**Theorem 2.5.** For any $A(X) \in \Sigma(X)$ the followings are equivalent.

1. Every $z_A^\beta$-filter on $\beta X$ can be extended to a $z_A^\beta$-ultrafilter on $\beta X$.
2. Every subfamily of $Z_A^\beta[X]$ with finite intersection property can be extended to a $z_A^\beta$-ultrafilter on $\beta X$ and therefore a $z_A^\beta$-ultrafilter on $\beta X$ is a subfamily of $Z_A^\beta[X]$ which is maximal with respect to having finite intersection property. Conversely a subfamily $F$ of $Z_A^\beta[X]$ enjoying finite intersection property and maximal with respect to this property is necessary a $z_A^\beta$-ultrafilter on $\beta X$.
3. A $z_A^\beta$-filter $F$ on $\beta X$ is a $z_A^\beta$-ultrafilter on $\beta X$ if and only if for any $Z \in Z_A^\beta[X]$, $Z \cap Z' \neq \varphi$ for any $Z' \in F$, implies that $Z \in F$.

As a straightforward consequence of the above theorem, taking into account the maximality of $M$ and $F$, we have the following theorem.

**Theorem 2.6.** Suppose $A(X) \in \Sigma(X)$. Then

1. if $M$ is a maximal ideal in $A(X)$ then $Z_A^\beta[M]$ is a $z_A^\beta$-ultrafilter on $\beta X$,
2. if $\mathfrak{Z}$ is a $z_A^\beta$-ultrafilter on $\beta X$ then $Z_A^{\beta-1}[\mathfrak{Z}]$ is a maximal ideal in $A(X)$.

Using the duality between maximal ideals in $A(X)$ and ultrafilters in $\beta X$ we have the following theorem.

**Theorem 2.7.** Let $A(X) \in \Sigma(X)$ and $f \in A(X)$. If $M$ is a maximal ideal in $A(X)$ and $S_A(f)$ meets every member of $Z_A^\beta[M]$ then $f \in M$.

3. $z_A^\beta$-ideals in $A(X)$ and its properties

For any $A(X) \in \Sigma(X)$ and for any $z_A^\beta$-filter $\mathfrak{Z}$ on $\beta X$, it is obvious that $\mathfrak{Z} = Z_A^\beta[Z_A^{\beta-1}[\mathfrak{Z}]]$; therefore $Z_A^\beta$ can be considered to be a mapping from the set of all ideals in $A(X)$ onto the set of all $z_A^\beta$-filters on $\beta X$. Furthermore, for any ideal $I$ in $A(X)$, we have $I \subset Z_A^{\beta-1}[Z_A^\beta[I]]$. The inclusion in the above relation may be proper. In fact in the ring $C(\mathbb{R})$ if we consider the ideal $I = \langle i \rangle$, the smallest ideal in $C(\mathbb{R})$ generated by the identity mapping $i$, we can easily observe that the mapping $i^{1/3}$ is in $Z_C^{\beta-1}[Z_C^\beta[I]]$ but it does not belong to $I$. This motivates to introduce the following definition.
Definition 3.1. An ideal $I$ in $A(X) \in \Sigma(X)$ is said to be a $z_{A}^{\beta}$-ideal if for any $f \in A(X)$, $S_{A}(f) \subset Z_{A}^{\beta}[I]$ implies that $f \in I$, that is, $I = Z_{A}^{\beta-I}[Z_{A}^{\beta}[I]]$.

Clearly if $f$ is a $z_{A}^{\beta}$-filter on $\beta X$ then $I = Z_{A}^{\beta-I}[\exists]$ is a $z_{A}^{\beta}$-ideal in $A(X)$, in fact $\exists = Z_{A}^{\beta-I}[Z_{A}^{\beta-I}[\exists]]$. Further for any $p \in \beta X$, $O_{A}^{p} = \{f \in A(X) : p \in \text{int}_{\beta X} S_{A}(f)\}$ is a $z_{A}^{\beta}$-ideal. It is also evident that the intersection of any nonempty collection of $z_{A}^{\beta}$-ideals in $A(X)$ is again a $z_{A}^{\beta}$-ideal. Again from Theorem 2.7 we can prove that for any maximal ideal $M$ in $A(X)$, $M = Z_{A}^{\beta-I}[Z_{A}^{\beta}[M]]$. Thus we have the following theorem.

Theorem 3.2. Suppose $A(X) \in \Sigma(X)$. Then every maximal ideal in $A(X)$ is a $z_{A}^{\beta}$-ideal in $A(X)$.

The following theorem shows that like maximal prime ideals, i.e. maximal ideals, minimal prime ideals in $A(X)$ are also $z_{A}^{\beta}$-ideals.

Theorem 3.3. If $I$ is a $z_{A}^{\beta}$-ideal in $A(X)$ and $P$ is minimal in the class of prime ideals containing $I$, then $P$ is a $z_{A}^{\beta}$-ideal.

Proof: Let $J$ be a prime ideal containing $I$ which is not a $z_{A}^{\beta}$-ideal. Then to prove the theorem it is sufficient to show that $J$ is not minimal in the class of prime ideals containing $I$. Since $J$ is not a $z_{A}^{\beta}$-ideal there exists an $f \in J$ and a $g \in A(X)$ with $g \notin J$ such that $S_{A}(f) = S_{A}(g)$. Now consider the set $S = (A(X) - J) \cup \{hf^{n} : h \notin J, n \in \mathbb{N}\}$. Since $J$ is a prime ideal, $S$ is closed under multiplication. Furthermore $S$ does not meet $I$. In fact $hf^{n} \in I$ for some $h \in J$, $n \in \mathbb{N}$ implies that $h \cdot g \in J$, which contradicts that $J$ is a prime ideal. Hence there exists a prime ideal containing $I$ and disjoint from $S$ and, hence, contained in $J$ properly. Therefore $J$ is not minimal.

Remark 3.4. Since the ideal $(0)$ in any $A(X)$ is a $z_{A}^{\beta}$-ideal, every minimal prime ideal in an arbitrary $A(X)$ is a $z_{A}^{\beta}$-ideal.

It is well known that every $z$-ideal in $C(X)$ is the intersection of all prime ideals containing it. The basic fact behind the result is that $Z(f^{n}) = Z(f)$ for all $n \in \mathbb{N}$. In our setting of $A(X)$ we also see that $S_{A}(f^{n}) = S_{A}(f)$ for all $n \in \mathbb{N}$ and therefore we get the following theorem.

Theorem 3.5. Every $z_{A}^{\beta}$-ideal in $A(X)$ is the intersection of all prime ideals in $A(X)$ containing it.

Remark 3.6. Using Theorem 3.3 and Theorem 3.5 it is easy to observe that every $z_{A}^{\beta}$-ideal in $A(X)$ is the intersection of all minimal prime ideals containing it.

The following theorem shows that $z_{A}^{\beta}$-ideals in $A(X)$ are actually $A$-analogues of $z$-ideals in $C(X)$.
**Theorem 3.7.** In $C(X)$, an ideal $I$ is a $z$-ideal if and only if it is a $z_C^β$-ideal.

**Proof:** Let $I$ be a $z$-ideal in $C(X)$ and $f \in C(X)$ be such that $S_C(f) \in Z_C^β[I]$. Then there exists $g \in I$ such that $S_C(f) = S_C(g)$. Since it is well known that for any $f \in C(X)$, $S_C(f) = cl_{βX} Z(f)$ and $cl_{βX} Z(f) \cap X = Z(f)$, the above relation implies that $Z(f) = Z(g) \in Z[I]$. Hence $f \in I$, as $I$ is a $z$-ideal. Therefore every $z$-ideal in $C(X)$ is also a $z_C^β$-ideal.

Conversely, let $I$ be a $z_C^β$-ideal in $C(X)$ and $f \in C(X)$ with $Z(f) \in Z[I]$. Then there exists an element $g$ of $I$ such that $Z(f) = Z(g)$, so that $cl_{βX} Z(f) = cl_{βX} Z(g) \in Z_C^β[I]$. Since $I$ is a $z_C^β$-ideal, it follows that $f \in I$, proving that $I$ is a $z$-ideal in $C(X)$. $\Box$

It is known that in case of $C(X)$, an intersection of prime ideals need not be a $z$-ideal, see Example 2G.1 of [5]. So Theorem 3.7 shows that the converse of Theorem 3.5 is not valid. But like $z$-ideals in $C(X)$, a $z_C^β$-ideal in an arbitrary $A(X) \in Σ(X)$ can also be described as a purely algebraic object.

**Theorem 3.8.** An ideal $I$ in $A(X) \in Σ(X)$ is a $z_A^β$-ideal if and only if given $f \in A(X)$ there exists $g \in I$ such that whenever $f$ belongs to every maximal ideal in $A(X)$ containing $g$, then $f \in I$.

**Proof:** Let $I$ be a $z_A^β$-ideal in $A(X)$ and $f \in A(X)$. Again let $g \in I$ be such that $f$ belongs to every maximal ideal in $A(X)$ containing $g$. Then $S_A(g) \subset S_A(f)$ so that $S_A(f) \in Z_A^β[I]$. Since $I$ is a $z_A^β$-ideal in $A(X)$, we have $f \in I$.

For the converse, let us assume that the given condition holds and $S_A(f) \in Z_A^β[I]$ for some $f \in A(X)$. Taking $f = g$ we see that $f$ belongs to every maximal ideal in $A(X)$ that contains $g$. Hence $f \in I$ so that $I$ is a $z_A^β$-ideal. $\Box$

Now we present an example which shows that the notion of $B$-ideal in $A(X)$ [2], already described in Introduction, does not coincide with the notion of $z_A^β$-ideal even with the choice $A(X) = C(X)$.

**Example.** Let us consider the $z$-ideal $O_0 = \{f \in C(X) : 0 \in \text{int}_X Z(f)\}$. Then the $z$-filter $Z_C(i) = \{Z \in Z(\mathbb{R}) : \exists g \in C(\mathbb{R}) \text{ with } i \cdot g|_{\mathbb{R} - Z} = 1\} \subset Z_C[O_0]$. In fact if $Z \in Z_C(i)$ then there exists $g \in C(\mathbb{R})$ such that $i \cdot g|_{\mathbb{R} - Z} = 1$, which implies that $i \cdot g(cl_{β}[\mathbb{R} - Z]) = \{1\}$. It then clearly follows that $0 \notin cl_{β}[\mathbb{R} - Z]$. Therefore there exists a $δ > 0$ such that $(\mathbb{R} - Z) \cap (-δ, δ) = \emptyset$. We define $h \in C(\mathbb{R})$ as follows: if $|x| ≤ \frac{δ}{2}$ then $h(x) = 0$, if $\frac{δ}{2} ≤ x ≤ δ$ then $h(x) = \frac{g(δ)}{δ}(2x - δ)$, if $|x| ≥ δ$ then $h(x) = g(x)$, and if $-δ ≤ x ≤ -\frac{δ}{2}$ then $h(x) = \frac{g(-δ)}{-δ}(2x + δ)$. Then clearly $h \in O_0$ and $i \cdot h|_{\mathbb{R} - Z} = 1$, so that $Z \in Z_C(h)$. Hence $Z \in Z_C[O_0]$. But as $i \notin O_0$, $O_0$ cannot be a $B$-ideal in $C(\mathbb{R})$.

Next we recall the definition of $e$-ideal [5]. An ideal $I$ in $C^*(X)$ is called an $e$-ideal if $E_\epsilon(f) \in E(I) = \bigcup_\epsilon E_\epsilon(f)$ for all $\epsilon > 0$ implies that $f \in I$, where
Theorem 3.9. Suppose the following statements are equivalent.

Example. In the ring $C^*(\mathbb{R})$ let us consider the ideal $O_0 = \{ f \in C^*(\mathbb{R}) : 0 \in \text{int}_{\beta \mathbb{R}} Z(f^\beta) \}$. Since $Z(f^\beta) = S_{C^*}(f)$ for any $f \in C^*(\mathbb{R})$, it is easy to see that $O_0$ is a $z_{C^*}^\beta$-ideal in $C^*(\mathbb{R})$. Now taking $f = (i \lor -1) \land 1$ we see that $E_\epsilon(f) \in E(O_0)$ for all $\epsilon > 0$, but $f \notin O_0$. Hence $O_0$ is not an $\epsilon$-ideal.

In case of $C(X)$ it is well known that a $z$-ideal need not be prime. In fact if $X$ is not an $F$-space then there exists some $p \in \beta X$ such that $O^p_C$ is not a prime ideal. But $O^p_C$ is a $z$-ideal for every $p \in \beta X$, i.e. a $z_{C^*}^\beta$-ideal. The following theorem tells us that if a $z_{A}^\beta$-ideal contains a prime ideal then it becomes prime.

Theorem 3.9. Suppose $A(X) \in \Sigma(X)$ and let $I$ be a $z_{A}^\beta$-ideal in $A(X)$. Then the following statements are equivalent.

(1) $I$ is a prime ideal in $A(X)$.
(2) $I$ contains a prime ideal in $A(X)$.
(3) For all $g$, $h$ in $A(X)$, $g \cdot h = 0$ implies that $g \in I$ or $h \in I$.
(4) For every $f \in A(X)$ there exists an $A$-zero set $Z$ in $Z_{A}^{\beta}[I]$ such that either

$$M^p_A(f) \geq 0 \forall p \in Z \text{ or } M^p_A(f) \leq 0 \forall p \in Z.$$  

Proof: (1) $\Rightarrow$ (2) is trivial.

(2) $\Rightarrow$ (3) Let us assume that $P$ is a prime ideal in $A(X)$ contained in $I$. Now for any two $g$, $h$ in $A(X)$ if $g \cdot h = 0$ then $g \cdot h \in P$. So either $g \in P$ or $h \in P$, that is, either $g \in I$ or $h \in I$.

(3) $\Rightarrow$ (4) For any given $f \in A(X)$, $(f \lor 0) \cdot (f \land 0) = 0$. Hence from (3) it follows that $f \lor 0 \in I$ or $f \land 0 \in I$. If $f \lor 0 \in I$ then $S_A(f \lor 0) \in Z_{A}^{\beta}[I]$. In this case for any $p \in S_A(f \lor 0)$, we have $f \lor 0 \in M^p_A$, that is, $M^p_A(f) \lor 0 = 0$.

Clearly this implies that $M^p_A(f) \leq 0$ for all $p \in S_A(f \lor 0) \in Z_{A}^{\beta}[I]$. Similarly in case $f \land 0 \in I$ we have $M^p_A(f) \geq 0$ for all $p \in S_A(f \land 0) \in Z_{A}^{\beta}[I]$.

(4) $\Rightarrow$ (1) Let us assume $g \cdot h \in I$, $g, h \in A(X)$, and consider the function $|g| - |h|$ in $A(X)$. Then there exists an $A$-zeroset $Z$ such that $M^p_A(|g| - |h|) \geq 0$ for all $p \in Z$, say for definiteness. Then clearly

$$M^p_A(|g|) \geq [M^p_A(|h|)] \text{ for all } p \in Z.$$  

Now we claim that $Z \cap S_A(g \cdot h) = Z \cap S_A(h) \subset S_A(h)$. In fact, by the above relation, $p \in S_A(g) \cap Z$ implies that $p \in S_A(h) \cap Z$, here we use the absolute convexity of maximal ideals in $A(X)$. Now because $S_A(f \cdot g) \in Z_{A}^{\beta}[I]$, it follows that $S_A(h) \in Z_{A}^{\beta}[I]$. Therefore $I$ is a $z_{A}^\beta$-ideal and we have $h \in I$. Analogously, if
that every $zA$-ideal in $A(X)$ is totally ordered. The following theorem shows that
these are only when $zA$-ideals are prime. We recall that every prime ideal in arbitrary $A(X) \in \Sigma(X)$ is absolutely convex. From this it is easy to conclude that every $zA$-ideal is also absolutely convex.

**Theorem 3.11.** Suppose that $A(X) \in \Sigma(X)$ and that $I$ is a $zA$-ideal in $A(X)$. Then $A(X)/I$ is totally ordered if and only if $I$ is prime.

**Proof:** Let $A(X)/I$ be a totally ordered ring and $f \in A(X)$. We assume that $I(f) \geq 0$. Since $I$ is absolutely convex we have $f - |f| \in I$, and therefore $S_A(f) \subseteq Z_A[I]$. Hence for any $p \in S_A(f)$ it follows that $M_A^p(f - |f|) = 0$ that is $M_A^p(f) = M_A^p(|f|)$. This implies that $M_A^p(f) \geq 0$ for all $p \in Z = S_A(f - |f|) \subseteq Z_A[I]$. Therefore by Theorem 3.9 $I$ becomes a prime ideal.

Conversely let $I$ be a prime ideal in $A(X)$ and $f \in A(X)$. Then again by Theorem 3.9 there exists a $Z \in Z_A[I]$ such that either $M_A^p(f) \geq 0$ for all $p \in Z$ or $M_A^p(f) \leq 0$ for all $p \in Z$. Let us assume that $M_A^p(f) \geq 0$ for all $p \in Z$. This implies that $f - |f| \in M_A^p$ so that $M_A^p(f) = M_A^p(|f|)$ for all $p \in Z$. Hence $M_A(f - |f|) = 0$ for all $p \in Z$, that is $Z \subseteq S_A(f - |f|)$. Now as $Z_A[I]$ is a $zA$-filter on $\beta X$ and $I$ is a $zA$-ideal in $A(X)$ we have $f - |f| \in I$ and hence $I(f) \geq 0$. Similarly $M_A^p(f) \leq 0$ for all $p \in Z$ implies that $I(f) \leq 0$. Therefore $A(X)/I$ becomes totally ordered. \qed
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Department of Pure Mathematics, University of Calcutta, 35 Ballygunge Circular Road, Kolkata-700019, India

E-mail: sudipkumaracharyya@yahoo.co.in

Department of Mathematics, Krishnagar Women’s College, Krishnagar, Nadia-741101, India

E-mail: dibyendude@gmail.com

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