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DIRECTED CONVEX SUBGROUPS OF ORDERED GROUPS

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In this paper are studied the set Γ of all directed convex subgroups of a (partially) ordered group G and the set Δ of all convex subsemigroups of G^+ that contain 0. There is given (Theorem 2.1) the isomorphism φ between the sets Γ and Δ ordered by inclusion (φ: $A \in \Gamma \to A^+ \in \Delta$, φ^{-1}: $S \in \Delta \to \langle S \rangle \in \Gamma$). Then Γ, Δ are isomorphic complete lattices whose properties depend on properties of an order of G (there are considered Riesz groups and l-groups).

The other section concerns the set $\Gamma_1 \subseteqq \Gamma$ of all o-ideals of an ordered group G and the set $\Delta_1 \subseteqq \Delta$ of all convex invariant subsemigroups of G^+ that contain 0 (if need be the set Δ'_1 in which invariancy in G is made up for invariancy in G^+). In Theorem 3.1 is proved: a restriction of the mapping φ (from Theorem 2.1) on Γ_1 is an isomorphism between Γ_1 and Δ_1. There holds again that Γ_1, Δ_1 are isomorphic complete lattices. If G is directed, we can obtain similar results for Γ_1 and Δ'_1. In particular, we can obtain results for Riesz groups and l-groups.

In accordance with these results now follows the known correspondence in an l-group G between l-ideals and invariant convex subsemigroups of G^+ that contain 0.

1. In this section we shall remind some basic concepts and relations. G will always denote *a (partially) ordered group* $[G, +, \leqq]$ and G^+ will denote *the positive cone* of G that is the set of all elements $a \in G$, $a \geqq 0$. If A is a subset of G, then $G^+ \cap A$ will be denoted by A^+. For each subgroup A of G, A is an ordered group $[A, +, \leqq]$ and A^+ is the positive cone of A. A subset $A \subseteqq G$ is *convex* in G if $a, b \in A$, $x \in G$, $a \leqq x \leqq b$ imply that $x \in A$. As is known, a subgroup A is a convex subgroup of G if and only if A^+ is a convex subset of G^+. G is *directed* if $U(a, b) \neq \emptyset$ for each $a, b \in G$, where $U(a, b) = \{x \in G: a \leqq x, b \leqq x\}$. G is directed if and only if $G = G^+ - G^+$. It is known too that G is directed if and only if for each $a \in G$ there exists $y \in G^+$ such that $a \leqq y$.

A directed convex normal subgroup of G will be called *an o-ideal* of G. If G is a lattice-ordered group (notation: l-group), then a subgroup A of G which is also a sublattice of the lattice G, will be called *an l-subgroup*. A convex normal l-subgroup will be called *an l-ideal.*

Remind also the concept of a Riesz group ([4], I. V. 13). We shall call *G a Riesz group* if G is directed and if the following is satisfied: For any elements a_1, a_2, b_1, b_2 in G such that $a_i \leqq b_j$ ($i = 1, 2; j = 1, 2$) there exists c in G such that $a_i \leqq c \leqq b_j$ ($i = 1, 2$; $j = 1, 2$). Each l-group is evidently a Riesz group; but there exist Riesz groups which are not l-groups.

2. In this section we shall investigate a relation among directed convex subgroups of an ordered group G and convex subsemigroups of G^+ that contain 0.

If A is an arbitrary subset $\emptyset \neq A \subseteqq G$, we shall denote $A - A$ by $\bar{A}$ and $\langle A \rangle$ will always denote the subgroup of G, generated by A.

Lemma 2.1. (ŠIK [5]) *If G is an ordered group, S a convex subsemigroup of G^+ containing 0, then $(\bar{S})^+ = S$.*

Proof: Let $x \in (\bar{S})^+$. By our assumption $x = y_1 - y_2$, where $y_1, y_2 \in S$. Therefore $y_1 \geqq y_1 - y_2 = x \geqq 0$ and since S is convex, $x \in S$. Hence $(\bar{S})^+ \subseteqq S$. The converse is evident.

Lemma 2.2. *If G is an ordered group, S a convex subsemigroup of G^+ containing 0, then $\bar{S} = \langle S \rangle$.*

Proof: Let $x_1, x_2 \in \bar{S}$. Then there exist $a_1, b_1, a_2, b_2 \in S$ such that $x_1 = a_1 - b_1$, $x_2 = a_2 - b_2$. Therefore

$$x_1 - x_2 = a_1 - b_1 + b_2 - a_2 = a_1 + b_2 - b_2 - b_1 + b_2 - a_2 =$$
$$= (a_1 + b_2) - [a_2 + (-b_2 + b_1 + b_2)].$$

Furthermore, $b_1 + b_2 \geqq -b_2 + b_1 + b_2 \geqq 0$ and since S is convex, $-b_2 + b_1 + b_2 \in S$. That is to say $x_1 - x_2 \in \bar{S}$, and hence $\bar{S}$ is a subgroup of G. Thus $\langle S \rangle \subseteqq \bar{S}$. The converse is evident.

Lemma 2.3. *Let G be an ordered group, S a convex subsemigroup of G^+ containing 0. Then $\langle S \rangle^+ = S$.*

Proof: Lemma is an immediate consequence of Lemmata 2.1 and 2.2.

Now, let G be an ordered group. We shall denote the set of all directed convex subgroups of G by Γ. Similarly we shall denote the set of all convex subsemigroups of G^+ containing *0* by Δ.

We can prove the following theorem:

Theorem 2.1. *Let G be an ordered group. Then the mapping φ of the set Γ into the set of all subsemigroups of G^+ defined by $A\varphi = A^+$ for each $A \in \Gamma$ is a isomorphism of the ordered set Γ onto the ordered set Δ. (Γ, Δ are ordered by inclusion.) The inverse mapping φ^{-1} is the mapping $\psi : \Delta \to \Gamma$ defined by $S\psi = \langle S \rangle$ for each $S \in \Delta$.*

Proof: Let $A \in \Gamma$. We shall show $A^+ \in \Delta$. Clearly, $0 \in A^+$. Since A is a convex subgroup of G, A^+ is a convex subsemigroup of G^+. Now suppose that A, $B \in \Gamma$ and $A^+ = B^+$. A^+, B^+ is the positive cone of A, B respectively implies (by the direction) $A = A^+ - A^+$, $B = B^+ - B^+$ and hence $A = B$. That is to say φ is the injection of Γ into Δ. Now consider arbitrary $S \in \Delta$. Thus S is convex in G^+ and by Lemma 2.3 $\langle S \rangle^+ = S$. Therefore $\langle S \rangle$ is convex in G. And since the ordered group $\langle S \rangle$ is generated by its positive cone, $\langle S \rangle$ is directed ([4], I. II. 1) that is $\langle S \rangle \in \Gamma$. And since $\langle S \rangle \varphi = S$, φ is a bijection of Γ onto Δ.

Show that $\psi = \varphi^{-1}$. If $A \in \Gamma$, then $A^+ \in \Delta$, $\langle A^+ \rangle \in \Gamma$. Since A is directed, $A = \langle A^+ \rangle$. Thus $A\varphi\psi = A^+\psi = \langle A^+ \rangle = A$. Similarly $S\psi\varphi = \langle S \rangle \varphi = \langle S \rangle^+ = S$ for $S \in \Delta$. Finally it is evident that φ is an isomorphism between the ordered sets Γ and Δ.

Theorem 2.2. *Let G be an ordered group. Then Δ ordered by inclusion is a complete lattice (in which the intersection is an infimum).*

Proof: Let $\{S_i : i \in I\}$ be an arbitrary system of convex subsemigroups of G^+ that contain 0. Then

(1) $0 \in \bigcap_{i \in I} S_i$;

(2) $\bigcap_{i \in I} S_i$ is (as a non-void intersection of convex subsemigroups) a convex subsemigroup of G^+ . G^+ is the unit in Δ.

The following theorem is an immediate consequence of Theorems 2.1 and 2.2.

Theorem 2.3. *If G is an ordered group, then Γ ordered by inclusion is a complete lattice isomorphic to the complete lattice Δ.*

Consider now the case where an ordered group G is a Riesz group.

Lemma 2.4. *A directed group G is a Riesz group if and only if it holds: If $a \in G$ satisfies $0 \leqq a \leqq b_1 + \dots + b_m$, where $0 \leqq b_i$ $(i = 1, 2, \dots, m)$, then there exist such elements $a_i \in G$ that $0 \leqq a_i \leqq b_i$ $(i = 1, \dots, m)$ and $a = a_1 + \dots + a_m$.* (See [4], I. V. 13.)

Theorem 2.4. *Let G be a Riesz group. Then Γ ordered by inclusion is a distributive sublattice of the lattice of all subgroups of G.*

Proof: (1) Let A, $B \in \Gamma$. Since A, B are convex, $A \cap B$ is also convex in G. Let x, $y \in A \cap B$. Since A, B are directed, there exist $a \in A$, $b \in B$ such that $x \leqq a$, $y \leqq a$, $x \leqq b$, $y \leqq b$. Since G is a Riesz group, there exists an element $c \in G$ such that $x \leqq c$, $y \leqq c$, $c \leqq a$, $c \leqq b$. And since A, B are convex, $c \in A \cap B$. Thus $A \cap B$ is directed. Therefore $A \cap B \in \Gamma$.

(2) Let A, $B \in \Gamma$. Let x, $y \in \langle A, B \rangle$. We can express x, y in the form $x = \alpha_1 + \dots + \alpha_n$, $y = \beta_1 + \dots + \beta_m$, where $\alpha_i \in A_i$ $(i = 1, \dots, n)$, $A_i = A$ or B, $A_i \neq A_{i+1}$ $(i = 1, \dots, n - 1)$. Similarly for β_j $(j = 1, \dots, m)$. Consider the set $\{\alpha_i\}$ of all sum-

mands of x and the set $\{\beta_j\}$ of all summands of y. Since A, B are directed, there exist elements $\gamma_1 \in A$, $\gamma_2 \in B$ such that $\gamma_1 \in U(\{\alpha_i\} \cap A, \{\beta_j\} \cap A, 0)$, $\gamma_2 \in U(\{\alpha_i\} \cap B, \{\beta_j\} \cap B, 0)$. We can suppose that it holds $\alpha_1 \in A$, $\beta_1 \in A$, $n = m$. (In the other case we can add zeros.) Then

$$x = \alpha_1 + \alpha_2 + \dots + \alpha_i + \dots + \alpha_n \leqq \gamma_1 + \gamma_2 + \dots + \gamma^{(i)} + \dots + \gamma^{(n)} = x',$$
$$y = \beta_1 + \beta_2 + \dots + \beta_i + \dots + \beta_n \leqq \gamma_1 + \gamma_2 + \dots + \gamma^{(i)} + \dots + \gamma^{(n)} = x',$$

where $\gamma^{(i)}$ is equal γ_1, γ_2 alternately ($i = 1, \dots, n$). Therefore $\langle A, B\rangle$ is directed.

We shall prove the convexity of $\langle A, B\rangle$. Let $u \in G$, $0 \leqq u \leqq x$, where $x \in \langle A, B\rangle$. We shall express the element x in the form $x = \alpha_1 + \dots + \alpha_n$ as in the precedent. Since A, B are directed, we can suppose that α_i ($i = 1, \dots, n$) are positive elements and x precedes their sum. By Lemma 2.4 there exist elements $a_i \in G$ such that $0 \leqq a_i \leqq \alpha_i$ ($i = 1, \dots, n$) and $u = a_1 + \dots + a_n$. Since A, B are convex, $a_i \in A$ or $a_i \in B$ ($i = 1, \dots, n$). Thus $u \in \langle A, B\rangle^+$ and hence $\langle A, B\rangle$ is convex. Thus Γ is a lattice with a supremum $\langle A, B\rangle$ and an infimum $A \cap B$.

(3) We shall prove that the lattice Γ is distributive. According to (1), (2), we have to prove $C \cap \langle A, B\rangle \subseteqq \langle C \cap A, C \cap B\rangle$ for each $A, B, C \in \Gamma$. Let $x \in C \cap \langle A, B\rangle$. Then x can be expressed in the form $x = \alpha_1 + \dots + \alpha_n$ as in (2). Without loss of generality we may suppose that $0 < x$. (Each element of the directed subgroup $C \cap \langle A, B\rangle$ can be expressed by a difference of positive elements.) Since A, B are directed, there exist $\delta_1 \in A$, $\delta_2 \in B$ such that $\delta_1 \in U(\{\alpha_i\} \cap A, 0)$, $\delta_2 \in U(\{\alpha_i\} \cap B, 0)$. Thus $x \leqq \delta^{(1)} + \delta^{(2)} + \dots + \delta^{(n)}$, where $\delta^{(i)} = \delta_1$ or δ_2 ($i = 1, \dots, n$). According to Lemma 2.4 there exist elements $\varepsilon^{(1)}, \dots, \varepsilon^{(n)} \in G$ such that $0 \leqq \varepsilon^{(i)} \leqq \delta^{(i)}$ ($i = 1, \dots, n$), $x = \varepsilon^{(1)} + \varepsilon^{(2)} + \dots + \varepsilon^{(n)}$. Since A, B are convex, $\varepsilon^{(i)} \in A$ or B ($i = 1, \dots, n$). And since $0 \leqq \varepsilon^{(i)} \leqq x$, $\varepsilon^{(i)} \in C$ ($i = 1, \dots, n$). Therefore $x \in \langle C \cap A, C \cap B\rangle$. And thus $C \cap \langle A, B\rangle \subseteqq \langle C \cap A, C \cap B\rangle$.

The following theorem is a consequence of Theorems 2.3 and 2.4.

Theorem 2.5. *Let G be a Riesz group. Then Δ ordered by inclusion is a complete distributive lattice (in which the intersection is an infimum) isomorphic to the lattice Γ.*

Now, let an ordered group G be an l-group.

Lemma 2.5. *If G is an l-group, then each directed convex subgroup A of G is a convex l-subgroup of G and conversely.*

Proof: Since A is directed, for $a, b \in A$ there exists $c \in A$ such that $a \leqq c$, $b \leqq c$. Therefore $a \vee b \leqq c$. Since A is convex, $a \vee b \in A$. The converse is evident.

Consider now the set Γ' of all convex l-subgroups of an l-group G. By [3], [6] Γ' ordered by inclusion is a complete distributive lattice in which $\bigcap_{i \in I} A_i$ is an infimum of an arbitrary system $\{A_i : i \in I\}$ of l-subgroups and $\langle A_i : i \in I\rangle$ is a supremum of this system. By Lemma 2.5 is now $\Gamma' = \Gamma$, thus in the case of an l-group, Γ is a closed distributive sublattice of the lattice of all subgroups of G.

3. Now, G be again an arbitrary ordered group. In this section we shall study the same types of subgroups and subsemigroups as in Section 2 but they will be invariant besides. First prove some lemmata.

Lemma 3.1. *Let G be an ordered group. Then $\langle G^+ \rangle = G^+ - G^+$.*

Proof: Evidently, G^+ is a convex subsemigroup of G^+ containing 0 and hence by Lemma 2.2 the proof is completed.

Lemma 3.2. *If G is an ordered group, then $\langle G^+ \rangle$ is a directed convex normal subgroup of G.*

Proof: Let $x \in G$, $c \in \langle G^+ \rangle$. By Lemma 3.1 holds $c = a - b$, where $a, b \in G^+$. We have

$$-x + (a - b) + x = (-x + a + x) - (-x + b + x) = p - q,$$

where $p, q \in G^+$, thus $\langle G^+ \rangle$ is normal. By Lemma 3.1 $\langle G^+ \rangle = G^+ - G^+$ and hence $\langle G^+ \rangle$ is directed. $\langle G^+ \rangle = G^+ \varphi^{-1}$ is evidently convex in G.

Lemma 3.3. *Let G be an (abstract) group. Then a non-void intersection of an arbitrary system of invariant subsemigroups of G is an invariant subsemigroup of G.*

Lemma 3.4. *Let A be a normal subgroup of an ordered group G.*
Then (1) *A^+ is an invariant subsemigroup of G;*
(2) *A^+ is the positive cone of an order of G.*

Proof: $0 \in A^+$. $A^+ = G^+ \cap A$ is by Lemma 3.3 an invariant subsemigroup of G. Since $A^+ \subseteqq G^+$, $A^+ \cap -(A^+) = 0$.

Lemma 3.5. *Let S be an invariant subsemigroup of an ordered group G, $S \subseteqq G^+$, $0 \in S$. Then*

(1) *S is the positive cone of an order $\preccurlyeq$ of G;*

(2) *$\langle S \rangle$ is normal subgroup of G and $\langle S \rangle$ is directed in the order $\leqq$.*

Proof: The proposition (1) is evident. According to (1) and Lemma 3.2, $\langle S \rangle$ is normal. Simultaneously, $\langle S \rangle$ is directed with respect to the order $\preccurlyeq$ of G. The order $\leqq$ is an extension of the order $\preccurlyeq$, therefore $\langle S \rangle$ is also directed in the order $\leqq$.

Let us remind that we have denoted by Γ the set of all directed convex subgroups of an ordered group G, by Δ the set of all convex subsemigroups of G^+ that contain 0. Now, let $\Gamma_1 = \{A \in \Gamma\colon A \text{ is a normal subgroup of } G\}$; Γ_1 is thus the set of all o-ideals of G. Similarly, let $\Delta_1 = \{S \in \Delta\colon S \text{ is an invariant subsemigroup of } G\}$.

Theorem 3.1. *Let G be an ordered group. Then the mapping $\varphi_1\colon A \in \Gamma_1 \to A^+ \in \Delta$ is an isomorphism of the set Γ_1 ordered by inclusion onto the set Δ_1 ordered by inclusion. The inverse mapping φ_1^{-1} is the mapping $\psi_1\colon S \in \Delta_1 \to \langle S \rangle \in \Gamma_1$.*

Proof: By Theorem 2.1 each $A \in \Gamma_1 \subseteqq \Gamma$ is in a one-one correspondence with $A^+ \in \Delta$. By Lemma 3.4 $A^+ \in \Delta_1$, thus $\Gamma_1 \varphi \subseteqq \Delta_1$. Conversely, by Theorem 2.1 arbitrary

$S \in \Delta_1 \subseteqq \Delta$ is in a one-one correspondence with the subgroup $\langle S \rangle \in \Gamma$. $\langle S \rangle$ is by Lemma 3.5 a normal subgroup of G, therefore $\Delta_1 \varphi^{-1} \subseteqq \Gamma_1$. An isomorphism of the ordered sets Γ_1 and Δ_1 is now evident.

Theorem 3.2. *Let G be an ordered group. Then the set Δ_1 ordered by inclusion is a complete lattice in which the intersection is an infimum.*

Proof: Let $\{S_i : i \in I\}$ be an arbitrary system of elements in Δ_1. By Theorem 2.2 it holds $\bigcap_{i \in I} S_i \in \Delta$. Since S_i $(i \in I)$ are invariant in G, $\bigcap_{i \in I} S_i$ is also invariant in G. Thus $\bigcap_{i \in I} S_i \in \Delta_1$. G^+ is the unit in Δ_1.

Corollary 3.1. *If G is an ordered group, then Γ_1 ordered by inclusion is a complete lattice isomorphic to Δ_1.*

Now, let us denote by Δ'_1 the set $\{S \in \Delta : S$ is an invariant subsemigroup in $G^+\}$. The invariancy of S in G^+ means that $x + s - x \in S$ and $-x + s + x \in S$ are valid for arbitrary elements $x \in G^+$, $s \in S$. Evidently $\Delta_1 \subseteqq \Delta'_1$.

Lemma 3.6. *Let G be a directed group. Then $\Delta_1 = \Delta'_1$.*

Proof: Let S be invariant in G^+ and let $y \in G$, $s \in S$. Since G is directed, y may be expressed in the form $y = x_1 - x_2$, where $x_1, x_2 \in G^+$. Therefore

$$y + s - y = (x_1 - x_2) + s - (x_1 - x_2) = x_1 + (-x_2 + s + x_2) - x_1,$$

and by the assumption it holds $-x_2 + s + x_2 = s_1 \in S$. It holds further $x_1 + s_1 - x_1 = s_2 \in S$ and hence $y + s - y = s_2 \in S$.

Therefore it holds:

Theorem 3.3. *If G is a directed group and if the sets Γ_1, Δ'_1 are ordered by inclusion, then the mapping φ_1 (from Theorem 3.1) of the set Γ_1 is an isomorphism of Γ_1 onto Δ'_1.*

Corollary 3.2. *If G is a directed group, then the set Δ'_1 ordered by inclusion is a complete lattice in which the intersection is an infimum.*

Now, let G be a Riesz group. Then Γ_1 forms with respect to inclusion a distributive sublattice in the lattice of all subgroups of G ([4], I. V. 13). Clearly, Γ_1 is also a sublattice of the lattice Γ. By Corollary 3.1 Γ_1 is a complete lattice.

Therefore it holds:

Theorem 3.4. *Let G be a Riesz group. Then the set Γ_1 ordered by inclusion is a complete distributive lattice that is a sublattice of Γ.*

Corollary 3.3. *If G is a Riesz group, then the set Δ'_1 ordered by inclusion is a complete distributive lattice in which the intersection is an infimum.*

Now let us suppose that G is an l-group. By Lemma 2.5 o-ideals and l-ideals in an l-group coincide. Therefore the following theorem is an immediate consequence of Theorem 3.1.

Theorem 3.5. *Let G be an l-group. Let us order the set Γ'_1 of all l-ideals of G and the set Δ'_1 by inclusion. Then the mapping ν: $\Gamma'_1 \to \Delta$ defined by $A\nu = A^+$ for each $A \in \Gamma'_1$ is an isomorphism of Γ'_1 onto Δ'_1.*

Remark. The proposition "ν is a bijection of Γ'_1 onto Δ'_1" is proved in [4], I. V. 5, partially also in [1].

As is known, (see e.g. [4], I. V. 5), the set Γ'_1 ordered by inclusion is a complete infinitely-distributive sublattice of the lattice of all normal subgroups of an l-group G and hence by[3], [6] the same is true of the lattice Γ'.

Therefore it holds:

Theorem 3.6. *If G is an l-group, then the set Δ'_1 ordered by inclusion is a complete infinitely-distributive sublattice of the lattice Δ.*

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SHRNUTÍ

USMĚRNĚNÉ KONVEXNÍ PODGRUPY USPOŘÁDANÝCH GRUP

JIŘÍ RACHŮNEK

V práci je studována množina Γ všech konvexních usměrněných podgrup (částečně) uspořádané grupy G a množina Δ všech konvexních podpologrup z G^+ obsahujících 0. Je ukázán (věta 2.1) izomorfismus φ mezi inkluzí uspořádanými množinami Γ a Δ (φ: $A \in \Gamma \to A^+ \in \Delta$, φ^{-1}: $S \in \Delta \to \langle S \rangle \in \Gamma$). Γ, Δ jsou pak izomorfními úplnými svazy, jejichž vlastnosti závisí na vlastnostech uspořádání G. (Uvažují se Rieszovy grupy a l-grupy.)

Další část se týká množiny $\Gamma_1 \subseteqq \Gamma$ všech o-ideálů z uspořádané grupy G a množiny $\Delta_1 \subseteqq \Delta$ všech konvexních invariantních podpologrup s 0 z G^+. Ve větě 3.1 se dokazuje, že restrikce zobrazení φ z věty 2.1 na Γ_1 je izomorfismem mezi Γ_1 a Δ_1. Opět platí, že Γ_1, Δ_1 tvoří izomorfní úplné svazy. Speciální výsledky se opět dostanou pro Rieszovy grupy a l-grupy. Důsledkem je známá korespondence v l-grupě G mezi l-ideály a invariantními konvexními podpologrupami s 0 z G^+.

РЕЗЮМЕ

НАПРАВЛЕННЫЕ ВЫПУКЛЫЕ ПОДГРУППЫ УПОРЯДОЧЕННЫХ ГРУПП

ИРЖИ РАХУНЕК

В работе рассматривается множество Γ всех выпуклых направленных подгрупп из (частично) упорядоченной группы G и множество Δ всех выпуклых подполугрупп из G^+, содержащих 0. Показывается (теорема 2.1) изоморфизм φ множеств Γ и Δ упорядоченных отношением включения ($\varphi : A \in \Gamma \to A^+ \in \Delta$, $\varphi^{-1} : S \in \Delta \to \langle S \rangle \in \Gamma$). Γ, Δ образуют изоморфные полные структуры, свойства которых зависят от свойств порядка на G. (Рассматриваются группы Рисса и l-группы.)

В дальнейшей части изучается множество $\Gamma_1 \subseteqq \Gamma$ всех o-идеалов из упорядоченной группы G и множество $\Delta_1 \subseteqq \Delta$ всех инвариантных выпуклых подполугрупп с 0 из G^+. В теореме 3.1 показывается, что сужение отображения φ из теоремы 2.1 на Γ_1 является изоморфизмом Γ_1 на Δ_1 . Γ_1, Δ_1 образуют изоморфные полные структуры. В частности получаем результаты для групп Рисса и l-групп. Следствием является известное соответствие в l-группе G между l-идеалами и инвариантными выпуклыми подполугруппами с 0 из G^+.