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TWO APPLICATIONS OF AN INTEGRAL FORMULA

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We are going to present two consequences of a general integral formula presented in [1].

1. Harmonic mappings of Riemannian manifolds

Be given a Riemannian manifold $(M, \mathrm{d}s^2)$, $\dim M = m$. In a suitable domain $U \subset M$, let us write $(i, j, \ldots = 1, \ldots, m)$

$$\mathrm{d}s^2 = \sum_i (\omega^i)^2, \tag{1.1}$$

ω^i being linearly independent 1-forms on U. Then there are on U 1-forms ω_i^j such that

$$\mathrm{d}\omega^i = \sum_j \omega^j \wedge \omega_j^i, \qquad \omega_i^j + \omega_j^i = 0; \tag{1.2}$$

the forms ω_i^j are uniquely determined by (1.2). The components of the curvature tensor be introduced by

$$\mathrm{d}\omega_i^j = \sum_k \omega_i^k \wedge \omega_k^j - \tfrac{1}{2} R_{ikl}^j \omega^k \wedge \omega^l, \qquad R_{ikl}^j + R_{ilk}^j = 0; \tag{1.3}$$

they satisfy the symmetry relations

$$R_{ikl}^j + R_{jkl}^i = 0, \qquad R_{ikl}^j = R_{kij}^l, \qquad R_{ikl}^j + R_{ilj}^k + R_{ijk}^l = 0. \tag{1.4}$$

Let $v_1, \ldots, v_m$ be the field of orthonormal frames on U dual to the field of coframes $\omega^1, \ldots, \omega^m$. Denote by $K(v_i, v_j)$, $i \neq j$, the sectional curvature of the 2-plane $\{v_i, v_j\}$; of course, $K(v_i, v_j) = R_{iij}^j$.

Further, be given another Riemannian manifold $(N, \mathrm{d}\sigma^2)$, $\dim N = n$, and a mapping $f: M \to N$. Consider a neighbourhood $V \subset N$ such that $f(U) \subset V$ and there are 1-forms φ^α $(\alpha, \beta, \ldots = 1, \ldots, n)$ satisfying

$$\mathrm{d}\sigma^2 = \sum_\alpha (\varphi^\alpha)^2. \tag{1.5}$$

Let

$$\tau^\alpha := f^*\varphi^\alpha = \sum_i A_i^\alpha \omega^i, \qquad \tau_\alpha^\beta := f^*\varphi_\alpha^\beta. \tag{1.6}$$

The exterior differentiation of (1.6_1) yields

$$\sum_i (\mathrm{d}A_i^\alpha - \sum_j A_j^\alpha \omega_i^j + \sum_\beta A_i^\beta \tau_\beta^\alpha) \wedge \omega^i = 0, \tag{1.7}$$

and, according to E. Cartan's lemma, we get the existence of functions A_{ij}^α on U satisfying

$$\mathrm{d}A_i^\alpha - \sum_j A_j^\alpha \omega_i^j + \sum_\beta A_i^\beta \tau_\beta^\alpha = \sum_j A_{ij}^\alpha \omega^j, \qquad A_{ij}^\alpha = A_{ji}^\alpha. \tag{1.8}$$

A furhter exterior differentiation implies

$$\begin{aligned}\sum_j (\mathrm{d}A_{ij}^\alpha - \sum_k A_{ik}^\alpha \omega_j^k - \sum_k A_{kj}^\alpha \omega_i^k + \sum_\beta A_{ij}^\beta \tau_\beta^\alpha) \wedge \omega^j = \\ = \tfrac{1}{2} \sum_{j,k} (\sum_l A_l^\alpha R_{ijk}^l - \sum_{\beta,\gamma,\delta} A_i^\beta A_j^\gamma A_k^\delta S_{\beta\gamma\delta}^\alpha)\, \omega^j \wedge \omega^k,\end{aligned} \tag{1.9}$$

$S_{\beta\gamma\delta}^\alpha$ being the components of the curvature tensor of $(N, \mathrm{d}\sigma^2)$. Thus there are functions A_{ijk}^α such that

$$\mathrm{d}A_{ij}^\alpha - \sum_k A_{ik}^\alpha \omega_j^k - \sum_k A_{kj}^\alpha \omega_i^k + \sum_\beta A_{ij}^\beta \tau_\beta^\alpha = \sum_k A_{ijk}^\alpha \omega^k, \qquad A_{ijk}^\alpha = A_{jik}^\alpha, \tag{1.10}$$

$$A_{ijk}^\alpha - A_{ikj}^\alpha = \sum_l A_l^\alpha R_{ikj}^l - \sum_{\beta,\gamma,\delta} A_i^\beta A_k^\gamma A_j^\delta S_{\beta\gamma\delta}^\alpha. \tag{1.11}$$

Let us consider on U the 1-forms

$$\varphi_1 = \sum_{\alpha,i,j} A_i^\alpha A_{ij}^\alpha \omega^j, \qquad \varphi_2 = \sum_{\alpha,i,j} A_j^\alpha A_{ii}^\alpha \omega^j. \tag{1.12}$$

It is easy to see that the forms (1.12) are globally defined over all of M. The usual $*$-operator be defined by

$$*\omega^i = (-1)^{i+1} \omega^1 \wedge \ldots \wedge \omega^{i-1} \wedge \omega^{i+1} \wedge \ldots \wedge \omega^n, \tag{1.13}$$

$$\text{i.e., } \mathrm{d}o := \omega^1 \wedge \ldots \wedge \omega^n = \omega^i \wedge * \omega^i.$$

Now,

$$\begin{aligned}\mathrm{d} * \varphi_1 &= \sum_{\alpha,i,j} \{(A_{ij}^\alpha)^2 + A_i^\alpha A_{jij}^\alpha\}\, \mathrm{d}o, \\ \mathrm{d} * \varphi_2 &= \sum_{\alpha,i,j} (A_{ii}^\alpha A_{jj}^\alpha + A_i^\alpha A_{jji}^\alpha)\, \mathrm{d}o,\end{aligned} \tag{1.14}$$

and, according to (1.11),

$$\mathrm{d} * (\varphi_1 - \varphi_2) = \sum_{\alpha,i,j} \{(A_{ij}^\alpha)^2 - A_{ii}^\alpha A_{jj}^\alpha + \sum_k A_i^\alpha A_k^\alpha R_{jji}^k - \sum_{\beta,\gamma,\delta} A_j^\alpha A_i^\beta A_i^\gamma A_j^\delta S_{\beta\gamma\delta}^\alpha\}\, \mathrm{d}o. \tag{1.15}$$

Let us turn our attention to the geometrical interpretation of the above introduced invariants. Let $p \in U \subset M$ be a given point. The Euclidean connection on M or N resp. is given by

$$\begin{aligned}\nabla m &= \sum_i \omega^i v_i, \qquad \nabla v_i = \sum_j \omega_i^j v_j \qquad \text{or} \\ \nabla^* n &= \sum_\alpha \varphi^\alpha w_\alpha, \qquad \nabla^* w_\alpha = \sum_\beta \varphi_\alpha^\beta w_\beta \qquad \text{resp.;}\end{aligned} \tag{1.16}$$

here, $w_1, \dots, w_n$ is the dual basis to $\varphi^1, \dots, \varphi^n$. Evidently,

$$\mathrm{d}f_p(v_i) = A_i^\alpha w_\alpha. \tag{1.17}$$

Let $v \in T_p(M)$ be a non-zero vector. Choose a curve $\gamma : (-\varepsilon, \varepsilon) \to M$ such that $\gamma(0) = p$; let s be its arc and v its tangent vector at p. Denote by $\gamma^* = f \circ \gamma : (-\varepsilon, \varepsilon) \to$ $\to N$ the corresponding curve. Then it is easy to see that

$$\frac{\nabla^* n}{\mathrm{d}s^2} - \mathrm{d}f_p\left(\frac{\nabla^2 m}{\mathrm{d}s^2}\right) = \frac{L(v)}{|v|^2}, \tag{1.18}$$

where $|v|^2 = \sum_i (\omega^i(v))^2$ and

$$L(v) = A_{ij}^\alpha(p)\, \omega^i(v)\, \omega^j(v)\, w_\alpha(f(p)). \tag{1.19}$$

This gives the geometrical interpretation of the quadratic mapping

$$L : T_p(M) \to T_{f(p)}(N). \tag{1.20}$$

Let $L(.,.)$ be the corresponding bilinear mapping.

At p, let us choose an orthonormal frame v_i, let w_α be an orthonormal frame at $f(p)$. Then

$$L(v_i, v_j) = A_{ij}^\alpha w_\alpha \tag{1.21}$$

and the expressions

$$\sum_{i,j} |L(v_i, v_j)|^2 = \sum_{i,j,\alpha} (A_{ij}^\alpha)^2, \qquad |\sum_i L(v_i)|^2 = \sum_{\alpha,i,ji} A_{ii}^\alpha A_{jj}^\alpha \tag{1.22}$$

do not depend on the choice of the frames v_i and w_α. In the same way, the vector

$$t = \sum_i L(v_i) \tag{1.23}$$

is invariant; the mapping

$$t : M \to T(N), \qquad t(p) \in T_{f(p)}(N) \tag{1.24}$$

is the so-called tension field. The mapping $f : M \to N$ is said to be harmonic if $t = 0$ for each $p \in M$.

The frames $(v_1, \dots, v_m)$ and $(w_1, \dots, w_n)$ at p and $f(p)$ resp. are called adapted to f if

$$\begin{aligned} \mathrm{d}f_p(v_i) &= A_i w_i && \text{for } i = 1, \dots, m \text{ in the case } m \leqq n \text{ and} \\ \mathrm{d}f_p(v_\alpha) &= A_\alpha w_\alpha && \text{for } \alpha = 1, \dots, n, \\ \mathrm{d}f_p(v_\varrho) &= 0 && \text{for } \varrho = n + 1, \dots, m \text{ in the case } m > n. \end{aligned} \tag{1.25}$$

Thus, we may always write (25_1) setting $w_i = 0$ for $i > n$. The adapted bases exist for each couple $(p, f(p))$. In the adapted bases, we have

$$\sum_{\alpha,i,j} \sum_k A_i^\alpha A_k^\alpha R_{jji}^k = \sum_i (A_i)^2 \sum_{j \neq i} K(v_j, v_i), \tag{1.26}$$

$$\sum_{\alpha,i,j} \sum_{\beta,\gamma,\delta} A_j^\alpha A_i^\beta A_i^\gamma A_j^\delta S_{\beta\gamma\delta}^\alpha = 2 \sum_{i \neq j} (A_i A_j)^2 K^*(w_i, w_j). \tag{1.27}$$

Further,

$$\varphi_1(v_i) = \sum_j \langle \mathrm{d}f(v_j), L(v_i, v_j)\rangle, \qquad \varphi_2(v_i) = \langle \mathrm{d}f(v_i), t\rangle, \tag{1.28}$$

$\langle , \rangle$ being the scalar product in $T_{f(p)}(N)$.

Choosing for each couple $(p, f(p))$ the adapted bases, we have the integral formula

$$\int_{\partial M} *(\varphi_1 - \varphi_2) = \int_M \{ \sum_{i,j} | L(v_i, v_j)|^2 - |t|^2 +$$
$$+ \sum_i (A_i)^2 \sum_{j \neq i} K(v_j, v_i) - 2 \sum_{i \neq j} (A_i A_j)^2 K^*(w_i, w_j)\} \,\mathrm{d}v. \tag{1.29}$$

Thus we get the following

Theorem. Let M, N be Riemannian manifolds and $f : M \to N$ a harmonic mapping. Let N have non-positive sectional curvatures and let M have, at each point $p \in M$ and for each unit vector $v \in T_p(M)$ the following property: $v_1, \ldots, v_{m-1}, v$ being an orthonormal basis of $T_p(M)$, we have $\sum_{r=1,\ldots,m-1} K(v, v_r) > 0$. Let $\varphi_1 = \varphi_2$ on the boundary ∂M of M. Then f is a constant mapping.

2. *Holomorphic curves in the Hermitian plane*

Be given a Hermitian plane H^2 and let $m : D \to H^2$ be a holomorphic curve, $D \subset \mathscr{C}$ being a bounded domain. To each its point $m(d)$, $d \in D$, let us associate an orthonormal frame $\{m, w_1, w_2\}$. Then we have the equations

$$\begin{gathered} \mathrm{d}m = \tau^1 w_1 + \tau^2 w_2, \\ \mathrm{d}w_1 = \tau_1^1 w_1 + \tau_1^2 w_2, \qquad \mathrm{d}w_2 = \tau_2^1 w_1 + \tau_2^2 w_2; \end{gathered} \tag{2.1}$$

clearly $(i, j, \ldots = 1, 2)$

$$\tau_i^j + \bar{\tau}_j^i = 0, \tag{2.2}$$

$$\mathrm{d}\tau^i = \tau^j \wedge \tau_j^i, \qquad \mathrm{d}\tau_i^j = \tau_i^k \wedge \tau_k^j. \tag{2.3}$$

Let us restrict ourselves to the tangent frames satisfying

$$\tau^2 = 0. \tag{2.4}$$

By successive exterior differentiations we get the existence of functions $R, S, T, U : D \to \mathscr{C}$ such that

$$\tau_1^2 = R\tau^1, \tag{2.5}$$

$$\mathrm{d}R + R(\tau_2^2 - 2\tau_1^1) = S\tau^1, \tag{2.6}$$

$$\mathrm{d}S + S(\tau_2^2 - 3\tau_1^1) + 3R^2\bar{R}\bar{\tau}^1 = T\tau^1, \tag{2.7}$$

$$\mathrm{d}T + T(\tau_2^2 - 4\tau_1^1) + 10R\bar{R}S\bar{\tau}^1 = U\tau^1. \tag{2.8}$$

Let us consider another field of orthonormal frames

$$u_1 = e^{i\alpha} w_1, \qquad u_2 = e^{i\beta} w_2; \qquad \alpha, \beta : D \to \mathscr{R}; \tag{2.9}$$

let us write

$$dm = \varphi^1 u_1, \qquad du_1 = \varphi_1^1 u_1 + \varphi_1^2 u_2, \qquad du_2 = \varphi_2^1 u_1 + \varphi_2^2 u_2. \tag{2.10}$$

Then it is easy to see that

$$\varphi^1 = e^{-i\alpha}\tau^1, \tag{2.11}$$

$$\varphi_1^1 = \tau_1^1 + i\,d\alpha, \qquad \varphi_2^2 = \tau_2^2 + i\,d\beta, \qquad \varphi_1^2 = e^{i(\alpha-\beta)}\tau_1^2. \tag{2.12}$$

Write

$$\varphi_1^2 = R'\varphi_1, \tag{2.13}$$

$$dR' + R'(\varphi_2^2 - 2\varphi_1^1) = S'\varphi^1, \tag{2.14}$$

$$dS' + S'(\varphi_2^2 - 3\varphi_1^1) + 3R'^2\bar{R}'\bar{\varphi}^1 = T'\varphi^1, \tag{2.15}$$

$$dT' + T'(\varphi_2^2 - 4\varphi_1^1) + 10R'\bar{R}'S'\bar{\varphi}^1 = U'\varphi^1. \tag{2.16}$$

Then

$$\begin{aligned} R' &= e^{i(2\alpha-\beta)}R, & S' &= e^{i(3\alpha-\beta)}S, \\ T' &= e^{i(4\alpha-\beta)}T, & U' &= e^{i(5\alpha-\beta)}U. \end{aligned} \tag{2.17}$$

The mappings $B^{(k)} : T_m \to N_m$ be introduced by

$$\begin{gathered} B(zw_1) = z^2Rw_2, \qquad B^{(1)}(zw_1) = z^3Sw_2, \\ B^{(2)}(zw_1) = z^4Tw_2; \qquad z \in \mathscr{C}. \end{gathered} \tag{2.18}$$

These mappings are invariant. Indeed: Let $w = zw_1 = z'u_1$, then $z' = e^{-i\alpha}z$ and $z'^2R'u_2 = z^2Rw_2$; similarly for $B^{(k)}$. Let $S^1 = \{w \in T_m;\ \langle w, w\rangle = 1\}$, i.e., $S^1 = \{zw_1;\ |z|^2 = 1\}$. Then $B^{(k)}(S^1)$ is a circle; the radius of $B(S^1)$ is equal to $|R|^{1/2}$, the radius of $B^{(1)}(S^1)$ is equal to $|S|^{1/2}$, etc. The geometrical interpretation of the mappings $B^{(k)}$ will be presented later on.

The area element of m is given by

$$do = \tfrac{1}{2}i\tau^1 \wedge \bar{\tau}^1. \tag{2.19}$$

The Hodge operator be introduced by

$$*\tau^1 = -i\tau^1, \qquad *\bar{\tau}^1 = i\bar{\tau}^1. \tag{2.20}$$

Let $f : D \to \mathscr{R}$ be a function. Then its Laplacian Δf is given, as usually, by

$$\Delta f\, do = d * df. \tag{2.21}$$

The straightforward calculations lead to ($n \geqq 1$)

$$d\,|R|^{2n} = 2n\,|R|^{2n-2}\,\mathrm{Re}\,(\bar{R}S\tau^1), \tag{2.22}$$

$$\Delta\,|R|^{2n} = 4n\,|R|^{2n-2}(n\,|S|^2 - 3\,|R|^4), \tag{2.23}$$

$$d\,|S|^{2n} = 2n\,|S|^{2n-2}\,\mathrm{Re}\,\{(\bar{S}T - 3SR\bar{R}^2)\,\tau^1\}, \tag{2.24}$$

$$\Delta \mid S\mid^{2n} = 4n\{\mid S\mid^{2n-2}(n\mid T\mid^2 - 16\mid S\mid^2\mid R\mid^2 + 9n\mid R\mid^6) - - 6(n-1)\mid S\mid^{2n-4}\mid R\mid^2 \operatorname{Re}(\bar{S}^2RT)\}. \tag{2.25}$$

Especially,

$$\begin{aligned} \Delta \mid R\mid^2 &= 4(\mid S\mid^2 - 3\mid R\mid^4), \\ \Delta \mid R\mid^4 &= 8\mid R\mid^2(2\mid S\mid^2 - 3\mid R\mid^4), \\ \Delta \mid S\mid^2 &= 4(\mid T\mid^2 - 16\mid S\mid^2\mid R\mid^2 + 9\mid R\mid^6) \end{aligned} \tag{2.26}$$

and

$$\Delta(\mid S\mid^2 + 4\mid R\mid^4) = 4(\mid T\mid^2 - 15\mid R\mid^6). \tag{2.27}$$

Lemma. Let $S = 0$ on D. Then $m(D)$ is a part of a straight line of H^2.

Proof. The equation (2.7) implies $R = 0$. QED.

Theorem. Let $S = 0$ on ∂D and

$$3\mid R\mid^4 \geqq \mid S\mid^2 \qquad \text{on } D. \tag{2.28}$$

Then $m(D)$ is a part of a straight line of H^2.

Proof. Obviously, $*\mathrm{d}\mid R\mid^{2n} = 0$ on ∂D. From the integral formula

$$0 = \int_M \Delta\mid R\mid^2 \mathrm{d}v \tag{2.29}$$

we get, because of (2.26),

$$3\mid R\mid^4 = \mid S\mid^2 \qquad \text{on } D. \tag{2.30}$$

The integral formula

$$\int_{\partial M} *\,\mathrm{d}\mid R\mid^4 = 8\int_M \mid R\mid^2(2\mid S\mid^2 - 3\mid R\mid^4)\,\mathrm{d}v \tag{2.31}$$

reduces to

$$0 = \int_M \mid R\mid^6 \mathrm{d}v, \tag{2.32}$$

and we get $R = 0$. QED.

The formulas (2.23), (2.25), (2.27) imply new characterizations of straight lines of H^2. It is sufficient to suppose $S = 0$ on ∂D and, for ex.,

$$14\mid R\mid^6 \geqq \mid T\mid^2 \tag{2.33}$$

or

$$\mid T\mid^2 \geqq 8\mid R\mid^2(2\mid S\mid^2 - \mid R\mid^4) \tag{2.34}$$

on D; see (2.27) and (2.26_3).

Now, the geometrical description of the mappings $B^{(k)}$ is given in [1]. To do this, let us consider H^2 as a space over m, i.e., H^2 becomes E^4. Write

$$\begin{gathered} v_1 = w_1, \qquad v_2 = iw_1, \qquad v_3 = w_2, \qquad v_4 = iw_2, \\ \tau^1 = \omega^1 + i\omega^2, \qquad \tau^2 = \omega^3 + i\omega^4, \qquad \tau_1^2 = \omega_1^3 + i\omega_1^4, \\ \tau_1^1 = i\omega_1^2, \qquad \tau_2^2 = i\omega_3^4, \end{gathered} \tag{2.35}$$

i.e.,

$$\begin{aligned}
\mathrm{d}m &= \omega^1 v_1 + \omega^2 {}_2, \\
\mathrm{d}v_1 &= \qquad\quad \omega_1^2 v_2 + \omega_1^3 v_3 + \omega_{1\text{-}4}^4, \\
\mathrm{d}v_2 &= -\omega_1^2 v_1 \qquad\quad - \omega_1^4 v_3 + \omega_1^3 \iota_4, \\
\mathrm{d}v_3 &= -\omega_1^3 v_1 + \omega_1^4 v_2 \qquad\quad + \omega_3^4 v_4, \\
\mathrm{d}v_4 &= -\omega_1^4 v_1 - \omega_1^3 v_2 - \omega_3^4 v_3
\end{aligned} \tag{2.36}$$

and

$$\omega_1^3 = R_1 \omega^1 - R_2 \omega^2, \qquad \omega_1^4 = R^2 \omega^1 + R_1 \omega^2 \tag{2.37}$$

with $R_1 = \operatorname{Re} R$, $R_2 = \operatorname{Im} R$. In E^4, consider a general surface

$$\begin{aligned}
\mathrm{d}n &= \varrho^1 v_1 + \varrho^2 v_2, \\
\mathrm{d}v_1 &= \qquad\quad \varrho_1^2 v_2 + \varrho_1^3 v_3 + \varrho_1^4 v_4, \\
\mathrm{d}v_2 &= -\varrho_1^2 v_1 \qquad\quad + \varrho_2^3 v_3 + \varrho_2^4 v_4, \\
\mathrm{d}v_3 &= -\varrho_1^3 v_1 - \varrho_2^3 v_2 \qquad\quad + \varrho_3^4 v_4, \\
\mathrm{d}v_4 &= -\varrho_1^4 v_1 - \varrho_2^4 v_2 - \varrho_3^4 v_3
\end{aligned} \tag{2.38}$$

with

$$\begin{aligned}
\varrho_1^3 &= a_1 \varrho^1 + a_2 \varrho^2, \qquad \varrho_2^3 = a_2 \varrho^1 + a_3 \varrho^2, \\
\varrho_1^4 &= b_1 \varrho^1 + b_2 \varrho^2, \qquad \varrho_2^4 = b_2 \varrho^1 + b_3 \varrho^2,
\end{aligned} \tag{2.39}$$

$$\begin{aligned}
\mathrm{d}a_1 - 2a_2 \varrho_1^2 - b_1 \varrho_3^4 &= \alpha_1 \varrho^1 + \alpha_2 \varrho^2, \\
\mathrm{d}a_2 + (a_1 - a_3)\varrho_1^2 - b_2 \varrho_3^4 &= \alpha_2 \varrho^1 + \alpha_3 \varrho^2, \\
\mathrm{d}a_3 + 2a_2 \varrho_1^2 - b_3 \varrho_3^4 &= \alpha_3 \varrho^1 + \alpha_4 \varrho_2, \\
\mathrm{d}b_1 - 2b_2 \varrho_1^2 + a_1 \varrho_3^4 &= \beta_1 \varrho^1 + \beta_2 \varrho^2, \\
\mathrm{d}b_2 + (b_1 - b_3)\varrho_1^2 + a_2 \varrho_3^4 &= \beta_2 \varrho^1 + \beta_3 \varrho^2, \\
\mathrm{d}b_3 + 2b_2 \varrho_1^2 + a_3 \varrho_3^4 &= \beta_3 \varrho^1 + \beta_4 \varrho^2.
\end{aligned} \tag{2.40}$$

Then it is known [1] that, for

$$\begin{aligned}
\Phi = {} & (a_1\alpha_3 + a_2\alpha_4 - a_2\alpha_2 - a_3\alpha_3 + b_1\beta_3 + b_2\beta_4 - b_2\beta_2 - b_3\beta_3)\,\varrho^1 + \\
& + (\mathrm{a}_2\alpha_1 + a_3\alpha_2 - a_1\alpha_2 - a_2\alpha_3 + b_2\beta_1 + b_3\beta_2 - b_1\beta_2 - b_2\beta_3)\,\varrho^2,
\end{aligned}$$

we have

$$\begin{aligned}
\int_{\partial N} * \Phi = {} & \int_N [2(\alpha_1\alpha_3 + \alpha_2\alpha_4 - \alpha_2^2 - \alpha_3^2 + \beta_1\beta_3 + \beta_2\beta_4 - \beta_2^2 - \beta_3^2) - \\
& - \{(a_1 - a_3)^2 + 4a_2^2 + (b_1 - b_3)^2 + 4b_2^2\}(a_1 a_3 - a_2^2 + b_1 b_3 - b_2^2) + \\
& + 2\{b_2(a_1 - a_3) + a_2(b_3 - b_1)\}^2]\,\mathrm{d}v.
\end{aligned} \tag{2.42}$$

In our case, (2.42) is identical with

$$\int_{\partial N} * \mathrm{d}\,|R|^2 = \int_N \Delta\,|R|^2\,\mathrm{d}v. \tag{2.43}$$

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SOUHRN

DVĚ APLIKACE JEDNÉ INTEGRÁLNÍ FORMULE

ALOIS ŠVEC

V práci jsou vyloženy aplikace integrální formule [1] na teorii harmonických zobrazení a na teorii křivky v hermiteovské rovině.

РЕЗЮМЕ

ДВА ПРИМЕНЕНИЯ ОДНОЙ ИНТЕГРАЛЬНОЙ ФОРМУЛЫ

АЛОИС ШВЕЦ

В работе излагаются приложения интегральной формулы [1] на теорию гармонических отображений и на теорию кривых в пространствах Эрмита.