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On the properties of the fundamental dispersions of the equation $y' = \lambda q(t)y$

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ON THE PROPERTIES OF THE FUNDAMENTAL DISPERSIONS OF THE EQUATION $y'' = \lambda q(t) y$

SVATOSLAV STANĚK
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Dedicated to Academician O. Borůvka on his 80th birthday

1. Introduction

In this paper we consider a differential equation

$$y'' = \lambda q(t) y$$

with $q \in C^0(j), j = (a, b)$ $(a < b \leq \infty)$ where $\lambda$ is a real parameter. The object of our study is to investigate the zero distribution of solutions and the zero distribution of the derivative of solutions of $(\lambda q)$, described as functions $\varphi(t, \lambda)$, $\psi(t, \lambda)$, $\chi(t, \lambda)$ and $\omega(t, \lambda)$. On applying the “generalized Wronskian” $w := y_0y_1' - y_0'y_1$, where $y_0$ and $y_1$ are respectively the solutions of the equations $(\lambda_0 q)$ and $(\lambda_1 q)$, we prove in analogy with [3] some results on the monotony of the functions $\varphi, \psi, \chi$ and $\omega$ with respect to the variable $\lambda$, which are well known in case of $q(t) \equiv 0$ on $j$.

2. Basic definitions, relations and notation

Let $q \in C^0(j)$ and let $\lambda$ be a (real) number. Throughout our discussion we exclude the trivial solution of $(\lambda q)$. Suppose that $x \in j$ and $u, v$ are solutions of $(\lambda q)$ satisfying the condition $u(x) = 0, v'(x) = 0$. Denote by $\varphi(x, \lambda)$ $(\chi(x, \lambda); \omega(x, \lambda))$ the first zero (if any) of the function $u$ ($u'; v$) lying to the right of the point $x$. The function $\varphi$ is called the 1st kind fundamental dispersion of $(\lambda q)$ and in case of $q(t) \not\equiv 0$ the functions $\chi$ and $\omega$ are called respectively the 3rd and 4th kind fundamental dispersion of $(\lambda q)$. (Cf. [1, 2]). The functions $\chi$ and $\omega$ are introduced in analogy with [5].

Say, a function $p \in C^0(j)$ possesses the property H if there is not a cluster point of zeros of $p$ lying on $j$. If the function $p$ possesses the property H and $\lambda \neq 0$, then $\lambda p$ possesses this property, too.
Lemma 1. Let a function $p$ possess the property $H$ and let $u$ be a solution of $y'' = p(t)y$. Then the zeros of $u'$ have no cluster point on $j$.

Proof. Suppose that the function $p$ possesses the property $H$ and there exists a nontrivial solution $u$ of $y'' = p(t)y$ together with a sequence $\{t_n\}$, $t_n \in j$, $t_n \to c$ as $n \to \infty$. Then $u'(c) = 0$, $u''(c) = 0$ and because of $u(c) \neq 0$ we have $p(c) = 0$. According to the assumption, $p$ possesses the property $H$ and therefore there exists a number $\varepsilon > 0$ such that $u(t) \neq 0$ for $t \in (c - \varepsilon, c + \varepsilon)$ and $p(t) \neq 0$ for $t \in (c - \varepsilon, c + \varepsilon) - \{c\}$. Then $u'(t_n) = \frac{1}{c} \int_{t_n}^{c} p(t)u(t)dt \neq 0$ holds for all $n$ for which $t_n \in (c - \varepsilon, c + \varepsilon)$, which is a contradiction.

Lemma 2. Let $(\lambda_0q)$ be an oscillatory equation. Then $\chi(t_1, \lambda_0) < \chi(t_0, \lambda_0)$ for $t_1 < t_0$, $t_1 \in j$, $t_0 \in j$.

Proof. We may assume without any loss of generality that $a \leq t_1 < t_0 < \chi(t_1, \lambda_0)$. Suppose that $u, v$ are solutions of $(\lambda_0q)$, $u(t_1) = v(t_0) = 0$, $u'(t_1) = v'(t_0) = 1$. Let $\chi(t_0, \lambda_0) \leq \chi(t_1, \lambda_0)$. Then $u'(t) > 0$ for $t \in (t_1, \chi(t_0, \lambda_0))$. We put $w(t) := u(t)v'(t) - u'(t)v(t)$, $t \in j$. Then $w(t) = k$ ($= a$ constant $\neq 0$) and next $k = u(t_0)$, $k = -u'(\chi(t_0, \lambda_0))v(\chi(t_0, \lambda_0))$. Because of $u(t_0) > 0$ we have $k > 0$ and since $v(\chi(t_0, \lambda_0)) > 0$, we have $u'(\chi(t_0, \lambda_0)) < 0$, i.e. a contradiction.

Convention. In so far as a function at $x_0$ passing to an infinite expression of the type "$0/0$" occurs in our consideration, the value of such a function at $x_0$ will be defined as its limit (if any).

In closing this section let us remark the following observation: If there exists an interval $(c, d) \subset j$ with $q(t) < 0$, then every solution of $(\lambda q)$ possesses at least two zeros on $(c, d)$ for a sufficiently large $\lambda$.

3. Main results

Theorem 1. Assume $(\lambda_0q)$ to be oscillatory. If:

a) $\lambda_0 > 0$, then $(\lambda q)$ is oscillatory also for every $\lambda \geq \lambda_0$ and $\varphi(t, \lambda_1) > \varphi(t, \lambda_2)$ for $\lambda_0 \leq \lambda_1 < \lambda_2$, $t \in j$;
b) \( \lambda_0 < 0 \), then \((\lambda q)\) is oscillatory also for every \( \lambda \leq \lambda_0 \) and \( \varphi(t, \lambda_1) > \varphi(t, \lambda_2) \) for \( \lambda_2 < \lambda_1 \leq \lambda_0 \), \( t \in j \).

Proof. Suppose \((\lambda_0 q)\) to be oscillatory and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \), which means that either \( 0 < \lambda_0 < \lambda \) or \( 0 > \lambda_0 > \lambda \). Let \( x \in j \) and let \( y_0 \) and \( y_1 \) be solutions of \((\lambda_0 q)\) and \((\lambda q)\), respectively, with \( y_0(x) = y_1(x) = 0 \), \( y_0'(x) = y_1'(x) = 1 \). Then \( y_0(\varphi(x, \lambda_0)) = 0 \) and \( y_0(t) > 0 \) for \( t \in (x, \varphi(x, \lambda_0)) \). Assume \( \varphi(x, \lambda_0) \leq \varphi(x, \lambda) \), consequently \( y_1(t) > 0 \) for \( t \in (x, \varphi(x, \lambda_0)) \). We set \( w(t) := y_0(t) y_1'(t) - y_0'(t) y_1(t), \ t \in j \). Then \( w = (\lambda - \lambda_0) q y_0 y_1 \) and \( w(x) = 0 \). This gives

\[
0 < \int x y_0^2(t) dt = y_0(t) y_0'(t) \int x \varphi(x, \lambda_0) - \lambda_0 \int x q(t) y_0^2(t) dt = - \frac{\lambda_0}{\lambda - \lambda_0} \int x y_0(t) w'(t) dt = - \lambda_0 \left[ \frac{y_0(t) w(t)}{y_1(t)} \right]_{x}^{\varphi(x, \lambda_0)} + \int x \left( \frac{w(t)}{y_1(t)} \right)^2 dt
\]

which, however, contradicts the assumption \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). Consequently \( \varphi(t, \lambda) < \varphi(t, \lambda_0) \) for \( t \in j \) and \((\lambda q)\) is oscillatory for every \( \lambda \) where \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). The rest of this proof is carried out writing \( \lambda_1 \) and \( \lambda_2 \) for \( \lambda_0 \) and \( \lambda \) into the above part of the proof.

Remark 1. Suppose \((\lambda_0 q)\) to be oscillatory. Then the statement of Theorem 1 on the oscillation of \((\lambda q)\), where \( \lambda_0 \leq \lambda \) and \( \lambda \leq \lambda_0 \) are respectively \( \lambda_0 > 0 \) and \( \lambda_0 < 0 \), follows also from Theorem 2. 60 [7, p. 105] or from Lemma 3 [4].

Corollary 1. Let \( \lambda_0 > 0 \) and let \((\lambda q)\) be an oscillatory equation. Then

\[
\lim_{\lambda \to \infty} \varphi(t, \lambda) = \Phi_q(t), \quad t \in j,
\]

where

\[
\Phi_q(t) = \begin{cases} t & \text{when } q(t) < 0, \\ \inf \{ x; x \in j, t < x, q(x) < 0 \} & \text{when } q(t) \geq 0. \\ \end{cases}
\]

Proof. Let \( x \in j \). By Theorem 1 \( \varphi(x, \lambda) \) is a decreasing function on the interval \((\lambda_0, \infty)\). There exists therefore \( \lim_{\lambda \to \infty} \varphi(x, \lambda) \) whose value we denote as \( c \); \( \lim_{\lambda \to \infty} \varphi(x, \lambda) = c \). Let \( q(x) < 0 \). Then there exists \( \varepsilon > 0 \) with \( q(t) < 0 \) for \( t \in (x, x + \varepsilon) \) and hence necessarily \( c = x = \Phi_q(x) \). Let \( q(x) \geq 0 \). Then \( q(t) \geq 0 \) for \( t \in (x, \Phi_q(x)) \).
Corollary 2. Suppose \( \lambda_0 > 0 \) and let \((\lambda_0 q)\) be oscillatory with \( \Phi_q(t) \) being the function defined in terms of Corollary 1. Then \( \lim_{\lambda \to \infty} \Phi_q(t) \) uniformly on every compact subinterval of \( j \) exactly if \( \Phi_q(t) = t \) for \( t \in \langle \Phi_q(a), b \rangle := j_1 \), i.e. \( q(t) \leq 0 \) for \( t \in j_1 \) and \( q(t) \) nonvanishing on every interval \((c, \infty)\).

Proof. Suppose \( \lim_{\lambda \to \infty} \Phi_q(t) \) to be uniformly converging on every compact subinterval of \( j \). Then \( \Phi_q(t) = \lim_{\lambda \to \infty} \Phi_q(t) \) is a continuous function on \( j \).

According to Theorem 1, the function \( \Phi_q(t) \) is a decreasing one in the variable \( \lambda \) on the interval \( \langle \lambda_0, \infty \rangle \) and since \( \Phi_q(t) \) is a continuous function for every \( \lambda \in \in \langle \lambda_0, \infty \rangle \) on \( j \), then by the generalized wellknown Dini’s theorem \( \lim_{\lambda \to \infty} \Phi_q(t) \) uniformly on every compact subinterval of \( j \). It is evident from the definition of \( \Phi_q(t) \) that this function is continuous on \( j \) exactly if \( \Phi_q(t) = t \) for \( t \in j_1 \) (= \( \langle \Phi_q(a), b \rangle \)) which occurs precisely in case of \( q(t) \leq 0 \) for \( t \in j_1 \) and \( q(t) \) nonvanishing on every interval \((c, \infty)\).

Remark 2. If \( \lambda_0 = 0 \), then \((\lambda_0 q)\) is a nonoscillatory equation and it is easy to verify that the domain of the function \( \Phi_q(t, \lambda_0) \) is an empty set. There is, however, such a function \( q \) to be found where \( \Phi_q(t, \lambda) \) is defined on the set \( j \times R_0 \) with \( j = \langle a, \infty \rangle \), \( R_0 = (-\infty, \infty) \) \(- \{0\} \). From [6] that say \( q \) may be replaced by any function \( q \in C^0(j) \), \( q(t) \equiv 0, q(t + \pi) = q(t) \) for \( t \in j \) and \( \int_{x_0}^{x_0 + \pi} q(t) dt = 0 \) (\( x_0 \in j \)).

Theorem 2. Suppose that \((\lambda_0 q)\) is oscillatory. If:

a) \( \lambda_0 > 0 \), then the function \( \chi(t, \lambda) \) is defined at every point \((t, \lambda) \in j \times \langle \lambda_0, \infty \rangle \) and \( \chi(t, \lambda) \) is a continuous function for every \( t \in \langle \lambda_0, \infty \rangle \) on \( j \).

b) \( \lambda_0 < 0 \), then the function \( \chi(t, \lambda) \) is defined at every point \((t, \lambda) \in j \times (-\infty, \lambda_0) \) and \( \chi(t, \lambda) \) is a continuous function for every \( t \in \langle \lambda_0, \infty \rangle \) on \( j \).

Proof. Let \( x \in j \) and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). Let next \( y_0 \) and \( y_1 \) be solutions of \((\lambda_0 q)\) and \((\lambda_0 q)\), respectively, with \( y_0(x) = y_1(x) = 0, y_0(x) = y_1(x) = 1 \). Then \( y_0(y(x, \lambda_0)) = 0 \) and \( y_0(t) > 0 \) for \( t \in (x, y(x, \lambda_0)) \). Assume that \( y_1(t) > 0 \) for \( t \in (x, y(x, \lambda_0)) \) and therefore \( \chi(x, \lambda_0) \leq \chi(x, \lambda) \). We set \( w(t) := y_0(t)y_1(t) - y_0(t)y_1(t), t \in j \) and get \( w' = (\lambda - \lambda_0)qy_0y_1, w(x) = 0 \). This gives
\begin{align*}
0 < \int x y_0^2(t) \, dt = y_0(t) y'_0(t) \left|_{x}^{x(x, \lambda_0)} \right. - \lambda_0 \int x q(t) y_0^2(t) \, dt = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \int x \frac{y_0(t) w(t)}{y_1(t)} \, dt = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \left[ \frac{y_0(t) w(t)}{y_1(t)} \left|_{x}^{x(x, \lambda_0)} \right. + \int x \left( \frac{w(t)}{y_1(t)} \right)^2 \, dt \right] = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \int x \left( \frac{w(t)}{y_1(t)} \right)^2 \, dt,
\end{align*}

which yields a contradiction since \( \int x \left( \frac{w(t)}{y_1(t)} \right)^2 \, dt > 0 \) and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). Consequently \( \chi(t, \lambda) < \chi(t, \lambda_0) \) and thus the function \( \chi(t, \lambda) \) is defined at the points \((t, \lambda)\), where \( t \in j \) and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). Writing \( \lambda_1 \) and \( \lambda_2 \) in the above part of the proof for \( \lambda_0 \) and \( \lambda \) satisfying the assumptions of the Theorem, we prove the remaining part of its statement.

**Theorem 3.** Suppose that \((\lambda_0 q)\) is oscillatory. If:

a) \( \lambda_0 > 0 \), then the function \( \omega(t, \lambda) \) is defined at every point \((t, \lambda) \in j \times (\lambda_0, \infty) \) and \( \omega(t, \lambda_1) > \omega(t, \lambda_2) \) for \( \lambda_0 \leq \lambda_1 < \lambda_2 \), \( t \in j \).

b) \( \lambda_0 < 0 \), then the function \( \omega(t, \lambda) \) is defined at every point \((t, \lambda) \in j \times (-\infty, \lambda_0) \) and \( \omega(t, \lambda_1) > \omega(t, \lambda_2) \) for \( \lambda_2 < \lambda_1 \leq \lambda_0 \), \( t \in j \).

**Proof.** Let \( x \in j \) and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). Let next \( y_0 \) and \( y_1 \) be solutions of \((\lambda_0 q)\) and \((\lambda q)\), respectively, with \( y_0(x) = y_1(x) = 1 \), \( y'_0(x) = y'_1(x) = 0 \). Then \( y_0(\omega(x, \lambda_0)) = 0 \) and \( y_0(t) > 0 \) for \( t \in (x, \omega(x, \lambda_0)) \). Assume that \( y_1(t) > 0 \) for \( t \in (x, \omega(x, \lambda_0)) \) and therefore \( \omega(x, \lambda_0) \leq \omega(x, \lambda) \). We set \( w(t) := y_0(t) y'_1(t) - y'_0(t) y_1(t) \), \( t \in j \) and get \( w' = (\lambda - \lambda_1) q y_0 y_1 \), \( w(x) = 0 \). Then

\begin{align*}
0 < \int x y_0^2(t) \, dt = y_0(t) y'_0(t) \left|_{x}^{x(x, \lambda_0)} \right. - \lambda_0 \int x q(t) y_0^2(t) \, dt = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \int x \frac{y_0(t) w(t)}{y_1(t)} \, dt = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \left[ \frac{y_0(t) w(t)}{y_1(t)} \left|_{x}^{x(x, \lambda_0)} \right. + \int x \left( \frac{w(t)}{y_1(t)} \right)^2 \, dt \right] = \\
= -\frac{\lambda_0}{\lambda - \lambda_0} \int x \left( \frac{w(t)}{y_1(t)} \right)^2 \, dt,
\end{align*}

which is a contradiction. Therefore \( \omega(t, \lambda) < \omega(t, \lambda_0) \), \( t \in j \) and thus the function
\( \omega(t, \lambda) \) is defined at every point \((t, \lambda)\), where \( t \in j \) and \( \frac{\lambda_0}{\lambda - \lambda_0} > 0 \). If we replace \( \gamma_0 \) and \( \lambda \) in the above part of the proof by \( \lambda_1 \) and \( \lambda_2 \) satisfying the assumptions of the Theorem, we prove so the remaining part of its statement.

**Lemma 3.** Let \( x \in j \) and \( q \) possess the property \( H \). Further let \((\lambda_0 q)\) be oscillatory and \( \psi(x, \lambda_0) > \omega(x, \lambda_0) \). If:

a) \( \lambda_0 > 0 \), then \( \psi(x, \lambda) > \omega(x, \lambda) \) for \( \lambda > \lambda_0 \),

b) \( \lambda_0 < 0 \), then \( \psi(x, \lambda) > \omega(x, \lambda) \) for \( \lambda < \lambda_0 \).

**Proof.** Let \( x \in j, \psi(x, \lambda_0) > \omega(x, \lambda_0) \). Let \( \psi(x, \lambda_1) < \omega(x, \lambda_1) \) for a number \( \lambda_1 \) satisfying the inequality \( \frac{\lambda_0}{\lambda_1 - \lambda_0} > 0 \) and thus also the inequality \( \frac{\lambda_1}{\lambda_1 - \lambda_0} > 0 \).

Let \( y_0 \) and \( y_1 \) be solutions of \((\lambda_0 q)\) and \((\lambda_1 q)\), respectively, \( y_0(x) = y_1(x) = 1 \), \( y_0'(x) = y_1'(x) = 0 \). Then \( y_1'(\psi(x, \lambda_1)) = 0 \), \( y_1(t) > 0 \), \( y_1'(t) < 0 \) for \( t \in (x, \psi(x, \lambda_1)) \) and \( y_0'(t) < 0 \) for \( t \in (x, \psi(x, \lambda_1)) \), since by Theorem 3 we have \( \omega(x, \lambda_0) > \omega(x, \lambda_1) \).

Setting \( w(t) := y_0(t) y_1'(t) - y_0'(t) y_1(t), t \in j \) gives \( w' = (\lambda_1 - \lambda_0) q y_0 y_1 \) and \( w(x) = 0 \).

From this

\[
0 < \int_x y_0'(t) y_1'(t) \ dt = y_0'(t) y_0(t) \left| \frac{\psi(x, \lambda_1)}{\psi(x, \lambda_2)} - \frac{\lambda_1}{\lambda_1 - \lambda_0} \int_x q(t) y_0(t) y_1(t) \ dt \right.
\]

\[
= - \frac{\lambda_1}{\lambda_1 - \lambda_0} \int_x w'(t) \ dt = - \frac{\lambda_1}{\lambda_1 - \lambda_0} w(\psi(x, \lambda_1)) = \frac{\lambda_1}{\lambda_1 - \lambda_0} y_0'(\psi(x, \lambda_1)) y_1(\psi(x, \lambda_1))
\]

i.e. a contradiction to the fact that \( y_0'(\psi(x, \lambda_1)) y_1(\psi(x, \lambda_1)) < 0 \).

**Theorem 4.** Let \( x \in j \) and \( q \) be possessing the property \( H \). Let \((\lambda_0 q)\) be oscillatory with \( \psi(x, \lambda_0) > \omega(x, \lambda_0) \). If:

a) \( \lambda_0 > 0 \), then \( \psi(x, \lambda_1) > \psi(x, \lambda_2) \) for \( \lambda_0 \leq \lambda_1 < \lambda_2 \),

b) \( \lambda_0 < 0 \), then \( \psi(x, \lambda_1) > \psi(x, \lambda_2) \) for \( \lambda_2 < \lambda_1 \leq \lambda_0 \).

**Proof.** Suppose that \( x \in j \) and \( \psi(x, \lambda_0) > \omega(x, \lambda_0) \). Let \( 0 < \lambda_0 \leq \lambda_1 < \lambda_2 \). Then, from Lemma 3, we obtain \( \psi(x, \lambda_1) > \omega(x, \lambda_1), \psi(x, \lambda_2) > \omega(x, \lambda_2) \) and consequently \( \psi(x, \lambda_1) = \chi(\omega(x, \lambda_1), \lambda_1), \psi(x, \lambda_2) = \chi(\omega(x, \lambda_2), \lambda_2) \). Theorem 2 and Lemma 3 imply \( \psi(x, \lambda_1) = \chi(\omega(x, \lambda_1), \lambda_1) > \chi(\omega(x, \lambda_2), \lambda_1) > \chi(\omega(x, \lambda_2), \lambda_2) = \psi(x, \lambda_2) \), hence \( \psi(x, \lambda_1) > \psi(x, \lambda_2) \). We proceed similarly even in case of \( 0 > \lambda_0 \geq \lambda_1 > \lambda_2 \).

**Theorem 5.** Let \( x \in j \) and \( q \) be possessing the property \( H \). Let \((\lambda_0 q)\) be oscillatory with \( \lambda_0 q(x) > 0 \). If:

a) \( \lambda_0 > 0 \), then \( \psi(x, \lambda_1) > \psi(x, \lambda_2) \) for \( \lambda_0 \leq \lambda_1 < \lambda_2 \),

b) \( \lambda_1 < 0 \), then \( \psi(x, \lambda_1) > \psi(x, \lambda_2) \) for \( \lambda_2 < \lambda_1 \leq \lambda_0 \).
Proof. Let \( x \in j, \lambda_0 q(x) > 0 \) and \( 0 < \lambda_0 \leq \lambda_1 < \lambda_2 \). Let \( y_1 \) and \( y_2 \) be solutions of \((\lambda_1 q)\) and \((\lambda_2 q)\), respectively, \( y_1(x) = y_2(x) = 1, y'_1(x) = y'_2(x) = 0 \) and \( \psi(x, \lambda_1) \leq \psi(x, \lambda_2) \). According to the assumption \( \lambda_1 q(x) > 0, \lambda_2 q(x) > 0 \) and therefore \( y'_1(t) > 0, y'_2(t) > 0 \) for \( t \in (x, \psi(x, \lambda_1)) \); \( y'_1(t) > 0, y'_2(t) > 0 \) for \( t \in (x, \psi(x, \lambda_2)) \). Setting \( w(t) := y_1(t) y'_2(t) - y'_1(t) y_2(t), t \in j \), gives \( w' = (\lambda_2 - \lambda_1) q y_1 y_2, w(x) = 0 \). From this it follows that

\[
0 < \int_x y_1'^2(t) \, dt = y_1(t) y'_1(t) \int_x \psi(x, \lambda_2) - \lambda_1 \int_x q(t) y_1'^2(t) \, dt =
\]

\[
- \frac{\lambda_1}{\lambda_2 - \lambda_1} \int_x y_1(t) w(t) y_2(t) \psi(x, \lambda_1) + \int_x \left( \frac{w(t)}{y_2(t)} \right)^2 \, dt =
\]

\[
- \frac{\lambda_1}{\lambda_2 - \lambda_1} \int_x y_1^2(\psi(x, \lambda_1)) y_2^2(\psi(x, \lambda_1)) + \psi(x, \lambda_1) \left( \frac{w(t)}{y_2(t)} \right)^2 \, dt
\]

contrary to \( \frac{\lambda_1}{\lambda_2 - \lambda_1} > 0 \) and \( \frac{y_1^2(\psi(x, \lambda_1)) y_2^2(\psi(x, \lambda_1))}{y_2(\psi(x, \lambda_1))} \geq 0 \). In an analogous fashion we proceed in case of \( 0 > \lambda_0 \geq \lambda_1 > \lambda_2 \).

Remark 3. It becomes apparent from the proof of Theorem 5 that the assumption \( \lambda_0 q(x) > 0 \) may be replaced by a weaker one: \( \lambda_0 q(x) \geq 0 \) and \( \lambda_0 q(t) > 0 \) in a right neighbourhood of the point \( x \).

BIBLIOGRAPHY

VLASTNOSTI ZÁKLADNÍCH DISPERSÍ ROVNICE

\[ y'' = \lambda q(t) y \]

SVATOSLAV STANĚK

V práci je vyšetřováno rozložení nulových bodů řešení a nulových bodů derivace řešení rovnice

\[ y'' = \lambda q(t) y, \quad q \in C^0(j), \quad (\lambda q) \]

kde \( j = (a, b) \) \((a < b \leq \infty)\), které je popsáno pomocí základní centrální disperse 1. druhu \( \phi(t, \lambda) \) rovnice \((\lambda q)\) a pomocí jistých funkcí \( \psi(t, \lambda), \chi(t, \lambda) \) a \( \omega(t, \lambda) \), které v případě \( q(t) \neq 0 \) \((t \in f)\) odpovídají postupně základním centrálním disperzim 2., 3. a 4. druhu rovnice \((\lambda q)\). Užitím „zobecněného wronskiánu“ \( w := y_0 y_1' - y_0' y_1 \), kde \( y_0 \) a \( y_1 \) jsou řešení rovnice \((\lambda_0 q)\) a \((\lambda_1 q)\), je dokázána monotonnost funkcí \( \phi, \psi, \chi \) a \( \omega \) vzhledem k proměnné \( \lambda \).

Резюме

СВОЙСТВА ОСНОВНЫХ ДИСПЕРСИЙ УРАВНЕНИЯ

\[ y'' = \lambda q(t) y \]

СВАТОСЛАВ СТАНЕК

В работе исследовано расположение корней решений и корней их производных для уравнения

\[ y'' = \lambda q(t) y, \quad q \in C^0(j), \quad (\lambda q) \]

где \( j = (a, b) \) \((a < b \leq \infty)\). Их расположение описано при помощи основной дисперсии 1-го рода \( \phi(t, \lambda) \) уравнения \((\lambda q)\) и некоторых функций \( \psi(t, \lambda), \chi(t, \lambda) \) и \( \omega(t, \lambda) \), которые в случае \( q(t) \neq 0 \) для \( t \in f \) соответствуют постепенно основным дисперсиям 2-го, 3-го и 4-го родов уравнения \((\lambda q)\). С помощью „обобщенного вронскиана“ \( w := y_0 y_1' - y_0' y_1 \), где \( y_0 \) и \( y_1 \) решения уравнения \((\lambda_0 q)\) и \((\lambda_1 q)\), доказана монотонность функций \( \phi, \psi, \chi \) и \( \omega \) относительно переменного \( \lambda \).