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ON A STRUCTURE OF THE INTERSECTION OF THE SET OF DISPERSIONS OF TWO SECOND-ORDER LINEAR DIFFERENTIAL EQUATIONS

SVATOSLAV STANĚK

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Dedicated to Prof. Miroslav Laitoch on the occasion of his 60th birthday

1. Introduction

O. Borůvka [3] investigated in his lectures the structures of the intersection of groups of dispersions of two oscillatory equations $(q_1) : y'' = q_1(t)\,y$, $(q_2) : y'' = q_2(t)\,y$ and found necessary and sufficient conditions for this intersection to be one-parametric continuous group. This problem is considered also in the present paper. Here the structure of the intersection of sets of dispersions of equations (q_1) and (q_2) is entirely described and namely under the assumption that (q_1) is a oscillatory equation, $q_1 \in C^0(\mathbf{R})$, $q_1 - q_2 \in C^2(\mathbf{R})$ and $q_1(t) \neq q_2(t)$ for $t \in \mathbf{R}$.

2. Basic notions and notation

In the interest of brevity we shall write hereafter $qX(t)$, $\alpha^{-1}\varepsilon\alpha(t)$ etc. instead of $q[X(t)]$, $\alpha^{-1}[\varepsilon(\alpha(t))]$ etc. If there exists a function inverse to the function f, we will denote it by f^{-1}.

We investigate differential equations of the type

$$y'' = q(t)\,y, \qquad q \in C^0(\mathbf{R}). \tag{q}$$

Say that a function $\alpha \in C^0(\mathbf{R})$ is the (first) phase of (q) if there exist independent solutions u, v of (q) such that

$$\operatorname{tg} \alpha(t) = u(t)/v(t) \qquad \text{for } t \in \mathbf{R} - \{t \in \mathbf{R}, v(t) = 0\}.$$

Every phase α of (q) has the following properties:

$$\alpha \in C^3(\mathbf{R}), \qquad \alpha'(t) \neq 0 \qquad \text{and} \qquad q(t) = -\{\alpha, t\} - \alpha'^2(t),$$

where $\{\alpha, t\} := \alpha'''(t)/2\alpha''(t) - \frac{3}{4}(\alpha''(t)/\alpha'(t))^2$ is the Schwarz derivative of the function α at the point t.

Equation (q) is oscillatory exactly if any (and then every) phase of (q) maps $\mathbf{R}$ onto $\mathbf{R}$.

The set of phases of the equation $y'' = -y$ will be written as $\mathfrak{E}$. If α is a phase of (q) then $\mathfrak{E}\alpha := \{\varepsilon\alpha, \varepsilon \in \mathfrak{E}\}$ is the set of phases of (q).

Say that a function $X \in C^3(\mathrm{j})$, $X'(t) \neq 0$ for $t \in \mathrm{j} \subset \mathbf{R}$, is a dispersion (of the 1st kind) of (q) exactly if X is a maximal solution of a nonlinear differential equation of the 3[nd] order

$$-\{X, t\} + X'^2(t) \,.\, qX(t) = q(t).$$

The dispersion $X : \mathrm{j} \to \mathbf{R}$ of (q) has the following characteristic property: for every solution u of (q) $uX(t)/\sqrt{|X'(t)|}$ is again a solution of this equation (in the interval j).

Let α be a phase of (q). Then $\alpha^{-1}\mathfrak{E}\alpha$ is the set of dispersions of (q), that is: if X is a dispersion of (q) defined in j, then there exists $\varepsilon \in \mathfrak{E}$ such that $X(t) = \alpha^{-1}\varepsilon\alpha(t)$ for $t \in \mathrm{j}$ and also conversely, for every $\varepsilon \in \mathfrak{E}$ the function $\alpha^{-1}\varepsilon\alpha$ is a dispersion of (q) and namely in the interval where the composite function $\alpha^{-1}\varepsilon\alpha$ is defined.

The set of the dispersions of (q) will be written as $\mathscr{L}_q$, the set of increasing (decreasing) dispersions of (q) as $\mathscr{L}_q^+$ ($\mathscr{L}_q^-$). Equation (q) is oscillatory exactly if all their dispersions are mapping $\mathbf{R}$ onto $\mathbf{R}$. In case of an oscillatory equation (q), the sets $\mathscr{L}_q$ and $\mathscr{L}_q^+$ generate groups under the composition of functions.

If id_j denotes the identical mapping j ($\subset \mathbf{R}$) on j, then there is $\mathrm{id}_\mathbf{R} \in \mathscr{L}_q^+$ for every equation (q).

The reader is referred to [1, 2] for all definitions and results.

Let (q) be an oscillatory equation (on $\mathbf{R}$) and $\mathscr{P}^+ \subset \mathscr{L}_q^+$. Say that $\mathscr{P}^+$ is a continuous one-parametric group if $\mathscr{P}^+$ is a subgroup $\mathscr{L}_q^+$ and through the every point $(t_0, x_0) \in \mathbf{R} \times \mathbf{R}$ passes exactly one function from $\mathscr{P}^+$.

By $\mathscr{P}_{q_1q_2}$ ($\mathscr{P}^+_{q_1q_2}$; $\mathscr{P}^-_{q_1q_2}$) we denote the set $\mathscr{L}_{q_1} \cap \mathscr{L}_{q_2}$ ($\mathscr{L}^+_{q_1} \cap \mathscr{L}^+_{q_2}$; $\mathscr{L}^-_{q_1} \cap \mathscr{L}^-_{q_2}$). For any two equations (q_1) and (q_2) is $\mathrm{id}_\mathbf{R} \in \mathscr{P}^+_{q_1q_2}$.

Say that a function f belongs to the set $\mathscr{M}$ iff $f : \mathrm{j} \to \mathbf{R}$ for an interval $\mathrm{j} \in \mathbf{R}$ and there exists a number ϱ with $f(-t + \varrho) = f(t)$ for $t \in \mathrm{j}$. Obviously $f \in \mathscr{M}$ exactly if the graph of the function f is symmetric with respect to a line parallel with the axis of ordinates.

3. Principle results

Say that functions q_1 and q_2 satisfy the assumption (L) if

$$q_1 \in C^0(\mathbf{R}), \quad q_1 - q_2 \in C^2(\mathbf{R}), \quad q_1(t) \neq q_2(t) \quad \text{for } t \in \mathbf{R} \tag{L}$$

and the equation (q_1) is oscillatory.

Lemma 1. *Let functions q_1, q_2 satisfy the assumption* (L). *Let X be a dispersion of* (q_1). *Let $t_0 \in \mathbf{R}$ and put $\gamma(t) := \int_{t_0}^{t} \sqrt{|q_1(s) - q_2(s)|}\, \mathrm{d}s$ for $t \in \mathbf{R}$ and $\mathrm{j} := \gamma(\mathbf{R})$. Then*

(i) *$X \in \mathscr{P}_{q_1 q_2}$ exactly if for a number k is*

$$\gamma X(t) = \operatorname{sign} X' . \gamma(t) + k, \qquad t \in \mathbf{R}. \tag{1}$$

(ii) *If $X \in \mathscr{P}^+_{q_1 q_2}$ and $X \neq \mathrm{id}_{\mathbf{R}}$, then $\mathrm{j} = \mathbf{R}$.*

(iii) *If $\mathrm{j} \neq \mathbf{R}$, then $\mathscr{P}^+_{q_1 q_2} = \{\mathrm{id}_{\mathbf{R}}\}$ and $\mathscr{P}^-_{q_1 q_2}$ containes one element at most.*

(iv) *If $X \in \mathscr{P}^-_{q_1 q_2}$, then $\mathscr{P}^-_{q_1 q_2} = X\mathscr{P}^+_{q_1 q_2}$ $(= \mathscr{P}^+_{q_1 q_2}X)$.*

Proof. Let the assumptions of Lemma 1 be fulfilled and let $\sigma = \operatorname{sign} X'$.

(i) According to the definition of the dispersion is $X \in \mathscr{P}_{q_1 q_2}$ iff

$$-\{X, t\} + X'^2(t) . q_i X(t) = q_i(t), \qquad t \in \mathbf{R}, i = 1, 2,$$

which is equivalent to

$$X'(t)\sqrt{|q_1 X(t) - q_2 X(t)|} = \sigma\sqrt{|q_1(t) - q_2(t)|}, \qquad t \in \mathbf{R}. \tag{2}$$

Integrating (2) from t_0 to t, using the substitution method and respecting the definition γ, gives

$$\gamma X(t) = \sigma . \gamma(t) + k, \qquad t \in \mathbf{R}, \tag{3}$$

where $k := \gamma X(t_0) = \int_{t_0}^{X(t_0)} \sqrt{|q_1(s) - q_2(s)|}\, \mathrm{d}s$. Let the dispersion X of (q_1) satisfy (3), where k is a number. Then by differentiating (3) we obtain (2) whence $X \in \mathscr{P}_{q_1 q_2}$.

(ii) Let $X \in \mathscr{P}^+_{q_1 q_2}$, $X \neq \mathrm{id}_{\mathbf{R}}$. Then, by (i), there exists a number k, $k \neq 0$, such that $\gamma X(t) = \gamma(t) + k$ for $t \in \mathbf{R}$ and therefrom $\gamma(\mathbf{R}) = \mathbf{R}$, hence $\mathrm{j} = \mathbf{R}$.

(iii) Let $\mathrm{j} \neq \mathbf{R}$. We have from (ii) that $\mathrm{id}_{\mathbf{R}}$ is the single element of $\mathscr{P}^+_{q_1 q_2}$. Let $Y_1, Y_2 \in \mathscr{P}^-_{q_1 q_2}$, $Y_1 \neq Y_2$. Then, by (i), there exist numbers k_1, k_2, $k_1 \neq k_2$, such that $\gamma Y_1(t) = -\gamma(t) + k_1$, $Y_2(t) = -\gamma(t) + k_2$. From this $\gamma Y_1 Y_2(t) = -\gamma Y_2(t) + k_1 = \gamma(t) + k_1 - k_2$. Putting $Y(t) := Y_1 Y_2(t)$, $t \in \mathbf{R}$, $k := k_1 - k_2$ then $\operatorname{sign} Y' = 1, k \neq 0, \gamma Y(t) = \gamma(t) + k$. Hence $Y \neq \mathrm{id}_{\mathbf{R}}$, $Y \in \mathscr{P}^+_{q_1 q_2}$. By (ii) then $\mathrm{j} = \mathbf{R}$ which contradicts our assumption.

(iv) Let $X \in \mathscr{P}^-_{q_1 q_2}$. If $\mathscr{P}^+_{q_1 q_2} = \{\mathrm{id}_{\mathbf{R}}\}$, then with respect to (iii) the assertion (iv) is true. Let $Y \in \mathscr{P}^+_{q_1 q_2}$, $Y \neq \mathrm{id}_{\mathbf{R}}$. Then there exist numbers $k_1, k_2 \neq 0$: $\gamma X = -\gamma + k_1$, $\gamma Y = \gamma + k_2$. From here $\gamma XY = -\gamma Y + k_1 = -\gamma + k_1 - k_2$, $\gamma YX = \gamma X + k_2 = -\gamma + k_1 + k_2$ and by (i)we have XY, $YX \in \mathscr{P}^-_{q_1 q_2}$. This proves that $X\mathscr{P}^+_{q_1 q_2} \subset \mathscr{P}^-_{q_1 q_2}$, $\mathscr{P}^+_{q_1 q_2}X \subset \mathscr{P}^-_{q_1 q_2}$. Let $X_1 \in \mathscr{P}^-_{q_1 q_2}$, $X \neq X_1$ and $\gamma X_1 = -\gamma + k_3$ for a number k_3. If we put $Y_1 := X^{-1}X_1$, $Y_2 := X_1 X^{-1}$, then $\operatorname{sign} Y_1' = \operatorname{sign} Y_2' = 1$ and from $\gamma Y_1 = \gamma X^{-1} X_1 = -\gamma X_1 + k_1 = \gamma + k_1 - k_3$, $\gamma Y_2 = \gamma X_1 X^{-1} = -\gamma X^{-1} + k_3 = \gamma + k_3 - k_1$ and from (i) we get $Y_1, Y_2 \in \mathscr{P}^+_{q_1 q_2}$. Therefore

$X_1 = XY_1 \in X\mathscr{P}^+_{q_1q_2}$, $X_1 = Y_2X \in \mathscr{P}^+_{q_1q_2}X$ and consequently $\mathscr{P}^-_{q_1q_2} \subset X\mathscr{P}^+_{q_1q_2}, \mathscr{P}^-_{q_1q_2} \subset \subset \mathscr{P}^+_{q_1q_2}X$. From this $\mathscr{P}^-_{q_1q_2} = X\mathscr{P}^+_{q_1q_2}$ and $X\mathscr{P}^+_{q_1q_2} = \mathscr{P}^+_{q_1q_2}X$.

Remark. According to (iii) of Lemma 1 we have from the assumption $\mathrm{j} \neq \mathbf{R}$ that $\mathscr{P}^-_{q_1q_2}$ has one element at most. The following example shows that there exist functions q_1, q_2 fulfilling the assumptions of Lemma 1 and such that $\mathscr{P}^-_{q_1q_2}$ is a one-element set.

Example 1. Let $q_1(t) := -1$, $q_2(t) := -1 + (1 + t^4)^{-1}$, $t \in \mathbf{R}$. The functions q_1, q_2 satisfy the assumptions of Lemma 1 and since the improper integrals $\int_{t_0}^{\infty} (1 + t^4)^{-1/2}\,\mathrm{d}t$, $\int_{-\infty}^{t_0} (1 + t^4)^{-1/2}\,\mathrm{d}t$ converge, we get $\mathrm{j} \neq \mathbf{R}$. Let us put $X(t) := -t$, $t \in \mathbf{R}$. Then $-\{X, t\} + X'^2(t) \,.\, q_iX(t) = q_i(t)$, $t \in \mathbf{R}$, $i = 1, 2$. Thus X is a dispersion of both equations (q_1), (q_2) and according to (iii) $\mathscr{P}^-_{q_1q_2}$ is a one-element set.

Lemma 2. *Let functions q_1, q_2 fulfil the assumption* (L). *Let α_1 be a phase of* (q_1) *and X be its dispersion. Let $t_0 \in \mathbf{R}$ and let us put* $\gamma(t) := \int_{t_0}^{t} \sqrt{|q_1(s) - q_2(s)|}\,\mathrm{d}s$ *for $t \in \mathbf{R}$ and $\beta(t) := \alpha_1\gamma^{-1}(t)$ for $t \in \mathrm{j} := \gamma(\mathbf{R})$. Then $X \in \mathscr{P}_{q_1q_2}$ and $X = \alpha_1^{-1}\varepsilon\alpha_1$, $\varepsilon \in \mathfrak{E}$, iff for a number k*

$$\beta(t \,.\, \operatorname{sign} X' + k) = \varepsilon\beta(t), \qquad t \in \mathrm{j}. \tag{4}$$

Proof. Let the assumptions of Lemma 2 be fulfilled and let $\sigma = \operatorname{sign} X'$. Let next $X \in \mathscr{P}_{q_1q_2}$ and $X = \alpha_1^{-1}\varepsilon\alpha_1$, $\varepsilon \in \mathfrak{E}$. Then according to the assertion (i) of Lemma 1 there exists a number k: $\gamma X(t) = \sigma \,.\, \gamma(t) + k$, $t \in \mathbf{R}$. From this we have $\gamma\alpha_1^{-1}\varepsilon\alpha_1(t) = \sigma \,.\, \gamma(t) + k$ and consequently $\varepsilon\beta(t) = \beta(\sigma t + k)$ for $t \in \mathrm{j}$. Let for a number k and $\varepsilon \in \mathfrak{E}$ the relation (4) hold. Then for the dispersion $X := \alpha_1^{-1}\varepsilon\alpha_1$ of (q_1) we get $\gamma X(t) = \sigma \,.\, \gamma(t) + k$, $t \in \mathbf{R}$, and from the assertion (i) of Lemma 1 we have $X \in \mathscr{P}_{q_1q_2}$.

Corollary 1. *Let the assumptions of Lemma 2 be fulfilled and let the function $\beta : \mathrm{j} \to \mathbf{R}$ be a phase of* (p). *Then*

(i) *$X \in \mathscr{P}^+_{q_1q_2}$ and $X \neq \mathrm{id}_{\mathbf{R}}$ iff p is a periodic function on* $\mathbf{R}$,

(ii) *$X \in \mathscr{P}^-_{q_1q_2}$ iff $p \in \mathscr{M}$ and $p : \mathrm{j} \to \mathbf{R}$.*

Proof. Let the assumptions of Lemma 2 be fulfilled and let β be a phase of (p) and $\sigma = \operatorname{sign} X'$. By Lemma 2 is $X \in \mathscr{P}_{q_1q_2}$ iff for a number k and $\varepsilon \in \mathfrak{E}$ the relation (4) holds. From the theory of phases now follows that (4) is equivalent to the assertion saying that (p) has also a phase $\beta(\sigma t + k)$, which is again equivalent to the equality $p(\sigma t + k) = p(t)$ for $t \in \mathrm{j}$.

(i) If $X \in \mathscr{P}^+_{q_1q_2}$ and $X \neq \mathrm{id}_{\mathbf{R}}$, then $\mathrm{j} = \mathbf{R}$ (see (ii) of Lemma 1) and p is a periodic function with the period $k \neq 0$ on $\mathbf{R}$. Reversely, if p is a periodic function with a period $k \neq 0$ on $\mathbf{R}$, then for any $\varepsilon \in \mathfrak{E}$ we have $\beta(t + k) = \varepsilon\beta(t)$. Hence $X := \alpha^{-1}\varepsilon\alpha_1 \in \mathscr{P}^+_{q_1q_2}$, $X \neq \mathrm{id}_{\mathbf{R}}$.

(ii) If $X \in \mathscr{P}^-_{q_1q_2}$ then $p(-t + k) = p(t)$ for $t \in \mathrm{j}$ and $p \in \mathscr{M}$. Let $p \in \mathscr{M}$ and $p : \mathrm{j} \to \mathbf{R}$. Then there exists a number k such that $p(-t + k) = p(t)$ for $t \in \mathrm{j}$ and consequently for an $\varepsilon \in \mathfrak{E}$ we have $\beta(-t + k) = \varepsilon\beta(t)$ for $t \in \mathrm{j}$ and by Lemma 2 we have $X := \alpha_1^{-1}\varepsilon\alpha_1 \in \mathscr{P}^-_{q_1q_2}$.

Lemma 3. *Let functions* q_1, q_2 *fulfil the assumption* (L). *Let* α_1 *be a phase of* (q_1), $t_0 \in \mathbf{R}$ *and put* $\gamma(t) := \int_{t_0}^{t} \sqrt{|q_1(s) - q_2(s)|}\,\mathrm{d}s$ *for* $t \in \mathbf{R}$ *and* $\beta(t) := \alpha_1\gamma^{-1}(t)$ *for* $t \in \mathrm{j} := \gamma(\mathbf{R})$. *Let* β *be a phase of* (p). *Then*

(i) $\mathscr{P}^+_{q_1q_2}$ *is a one-parametric continuous group iff* $\mathrm{j} = \mathbf{R}$ *and* $p(t) =$ *constant*,

(ii) $\mathscr{P}^+_{q_1q_2}$ *is an infinite cyclic group iff* $\mathrm{j} = \mathbf{R}$ *and* p *is an in constant periodic function.*

Proof. Let the assumptions of Lemma 3 be satisfied. Note first that from (i) and (ii) follows that $\mathscr{P}^+_{q_1q_2}$ contains at least two elements and $X \in \mathscr{P}^+_{q_1q_2}$, $X \neq \mathrm{id}_\mathbf{R}$ if $\mathrm{j} = \mathbf{R}$ and for a number $k \neq 0$ we have $X = \gamma^{-1}(\gamma + k)$.

Let $Y \in \mathscr{P}^+_{q_1q_2}$, $Y \neq \mathrm{id}_\mathbf{R}$. Then by Corollary 1, p is a periodic function on $\mathbf{R}$. With respect to the continuity of the function p there may occur two possibilities:

a) $p(t) =$ constant. Then $\beta(t + k)$ is a phase of (p) for every $k \in \mathbf{R}$ and $\mathscr{P}^+_{q_1q_2} = \{\gamma^{-1}(\gamma(t) + k), k \in \mathbf{R}\}$ follows from Lemma 2. It is easily verified that exactly one function passes through each point $(t_0, x_0) \in \mathbf{R} \times \mathbf{R}$ – hence $\mathscr{P}^+_{q_1q_2}$ is a one-periodic continuous group.

b) $p(t) \neq$ constant and $r > 0$ is its main period. Then $\beta(t + k)$ is a phase of (p) iff $k = nr$ for an integer n. Hence $\mathscr{P}^+_{q_1q_2} = \{\gamma^{-1}(\gamma(t) + nr), n = 0, \pm 1, \pm 2, \ldots\}$. Again, it is easy to verify that $\mathscr{P}^+_{q_1q_2}$ is an infinite cyclic group.

Let $\mathscr{P}^+_{q_1q_2}$ be a one-parametric continuous group. Then $\mathrm{j} = \mathbf{R}$ and because of the elements $\mathscr{P}_{q_1q_2}$ being of the form $\gamma^{-1}(\gamma + k)$, where k is a number, it is necessarily $\mathscr{P}^+_{q_1q_2} = \{\gamma^{-1}(\gamma(t) + k), k \in \mathbf{R}\}$, hence every number is a period of the function p and with respect to its continuity, necessarily $p(t) =$ constant. Let $\mathscr{P}^+_{q_1q_2}$ be an infinite cyclic group and $\gamma^{-1}(\gamma + r)$ be one of the generators of the group $\mathscr{P}^+_{q_1q_2}$. Then $\mathrm{j} = \mathbf{R}$ and p is necessarily an in constant function where $|r|$ is its main period.

From Lemmas 1–3 and from Corollary 1 now follows

Theorem 1. *Let functions* q_1, q_2 *fulfil the assumption* (L) *and let* α_1 *be a phase of* (q_1). *Let* $t_0 \in \mathbf{R}$ *and put* $\gamma(t) := \int_{t_0}^{t} \sqrt{|q_1(s) - q_2(s)|}\,\mathrm{d}s$ *for* $t \in \mathbf{R}$ *and* $\beta(t) := \alpha_1\gamma^{-1}(t)$ *for* $t \in \mathrm{j} := \gamma(\mathbf{R})$. *Let* β *be a phase of* (p).

Then $\mathscr{P}^+_{q_1q_2}$ *is either a one-parametric continuous group or it is an infinite cyclic group or* $\mathscr{P}^+_{q_1q_2} = \{\mathrm{id}_\mathbf{R}\}$ *and it holds*:

(i) $\mathscr{P}^+_{q_1q_2}$ *is an one-parametric continuous group*, $\mathscr{P}^-_{q_1q_2}$ *is an non-empty set and* $\mathscr{P}^-_{q_1q_2} = X\mathscr{P}^+_{q_1q_2}$ *for a* $X \in \mathscr{P}^-_{q_1q_2}$ *iff* $p(t) =$ *constant for* $t \in \mathbf{R}$,

(ii) $\mathscr{P}^+_{q_1q_2}$ *is an infinite cyclic group,* $\mathscr{P}^-_{q_1q_2}$ *is an non-empty set and* $\mathscr{P}^-_{q_1q_2} = X\mathscr{P}^+_{q_1q_2}$ *for a* $X \in \mathscr{P}^-_{q_1q_2}$ *iff p is an ion-constant periodic function on* $\mathbf{R}$ *and* $p \in \mathscr{M}$,

(iii) $\mathscr{P}^+_{q_1q_2}$ *is an infinite cyclic group and* $\mathscr{P}^-_{q_1q_2}$ *is the empty set iff p is an inconstant periodic function on* $\mathbf{R}$ *and* $p \notin \mathscr{M}$,

(iv) $\mathscr{P}^+_{q_1q_2} = \{\mathrm{id}_{\mathbf{R}}\}$ *and* $\mathscr{P}^-_{q_1q_2}$ *is an non-empty set (then necessarily one-element) iff* $p \notin \mathscr{M}$ *and p is not a periodic function,*

(v) $\mathscr{P}^+_{q_1q_2} = \{\mathrm{id}_{\mathbf{R}}\}$ *and* $\mathscr{P}^-_{q_1q_2}$ *is an empty set iff* $p \notin \mathscr{M}$ *and p is not a periodic function.*

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Souhrn

STRUKTURA PRŮNIKU MNOŽINY DISPERSÍ DVOU LINEÁRNÍCH DIFERENCIÁLNÍCH ROVNIC 2. ŘÁDU

SVATOSLAV STANĚK

Řekneme, že funkce $X \in C^3(\mathrm{j})$, $X'(t) \neq 0$ pro $t \in \mathrm{j} := (a, b) \subset \mathbf{R}$, je disperse rovnice

$$y'' = q(t)\, y, \qquad q \in C^\circ(\mathbf{R}), \tag{q}$$

jestliže je řešením diferenciální rovnice

$$-\frac{1}{2}\frac{X'''}{X'} + \frac{3}{4}\left(\frac{X''}{X'}\right)^2 + X'^2 \,.\, q(X) = q(t).$$

Označme $\mathscr{L}_q$ množinu dispersí rovnice (q). Nechť $q_1 \in C^0(\mathbf{R})$, $q_1 - q_2 \in C^2(\mathbf{R})$, $q_1(t) \neq q_2(t)$ pro $t \in \mathbf{R}$ a nechť rovnice (q_1) je oscilatorická. V práci je vyšetřována algebraická struktura množiny $\mathscr{L}_{q_1} \cap \mathscr{L}_{q_2}$.

Резюме

СТРУКТУРА ПЕРЕСЕЧЕНИЯ МНОЖЕСТВ ДИСПЕРСИЙ ДВУХ ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ ВТОРОГО ПОРЯДКА

СВАТОСЛАВ СТАНЕК

Функция $X \in C^3(\mathrm{j})$, $X'(t) = 0$ для $t \in \mathrm{j} := (a, b) \subset \mathbf{R}$, называется дисперсией уравнения

$$y'' = q(t)y, \qquad q \in C^0(\mathbf{R}), \tag{q}$$

если X решением уравнения

$$-\frac{1}{2}\frac{X'''}{X'} + \frac{3}{4}\left(\frac{X''}{X'}\right)^2 + X'^2 \,.\, q(X) = q(t).$$

Множество всех дисперсий уравнения (q) обозначаем $\mathscr{L}_q$. Пусть $q_1 \in C^\circ(\mathbf{R})$, $q_1 - q_2 \in C^2(\mathbf{R})$, $q_1(t) \neq q_2(t)$ для $t \in \mathbf{R}$ и (q_1) колеблющиеся уравнение. В работе исследуется алгебраическая структура множества $\mathscr{L}_{q_1} \cap \mathscr{L}_{q_2}$.