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ALGEBRAICAL LOOPS AND TRANSFORMATION OF VARIABLES

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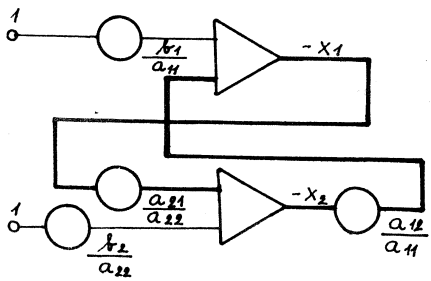
Algebraical loop is ment a closed loop, which contains inverters (adders) and potentionmeters only. E.g. the program diagram in the Fig.1 for solving of the system of the linear



algebraical equations

$$a_{11}x_1 + a_{12}x_2 = b_1 \qquad (1)$$

$$a_{21}x_1 + a_{22}x_2 = b_2$$

programmed in the form

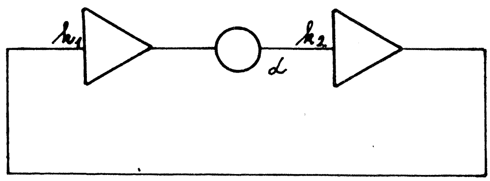
$$x_1 = \frac{b_1}{a_{11}} - \frac{a_{12}}{a_{11}} x_2$$

$$x_2 = \frac{b_2}{a_{22}} - \frac{a_{21}}{a_{22}} x_1$$

contains the algebraical loop. These algebraical loops used to be very frequently unstable. The general gain S of disjoin loop is the decisive criterion for stability of loop, in the case of the Fig.1 the general gain is

$$S = \frac{a_{21}}{a_{22}} \cdot \frac{a_{12}}{a_{11}} ,$$

the gain of the loop in the Fig.2 is $S = k_1 k_2 \alpha$. Two cases are



needed to distinquish for the first criticism of stability of an algebraical loop according to the number of inverters or adders in the loop. If even number of these units is included in the loop then the border of stability is $S_{theor} = 1$ with maximum theoretical gain. Algebraical loops containing odd number of inverters or adders would be stable in every case with using ideal computational units. The actually border of stability e.g. for the computer MEDA 41TA is following as to properties of real units.

The number of inverters in the loop	S_{real}
3	7
5	3
7	2
even	0,98

Algebraical loops are often occured during solving of differential equations. E.g. the system of the differential equations is given

$$a_2y_1'' + a_1y_1' + a_0y_1 + b_2y_2'' + b_1y_2' + b_0y_2 = 0 \qquad (2)$$

$$c_2y_1'' + c_1y_1' + c_0y_1 + d_2y_2'' + d_1y_2' + d_0y_2 = 0$$

with initial conditions $y'_{1(0)}$, $y_{1(0)}$, $y'_{2(0)}$, $y_{2(0)}$.

The decesion which variable from which equation will be counted is not unambiguous. We will count e.g. the quantity y_1 from the 1st equation of the system (2) and the quantity y_2 from the 2nd one, i.e.

$$a_2y_1'' = -a_1y_1' - a_0y_1 - b_2y_2'' - b_1y_2' - b_0y_2 \qquad (3)$$

$$d_2y_2'' = -d_1y_2' - d_0y_2 - c_2y_1'' - c_1y_1' - c_0y_1 .$$

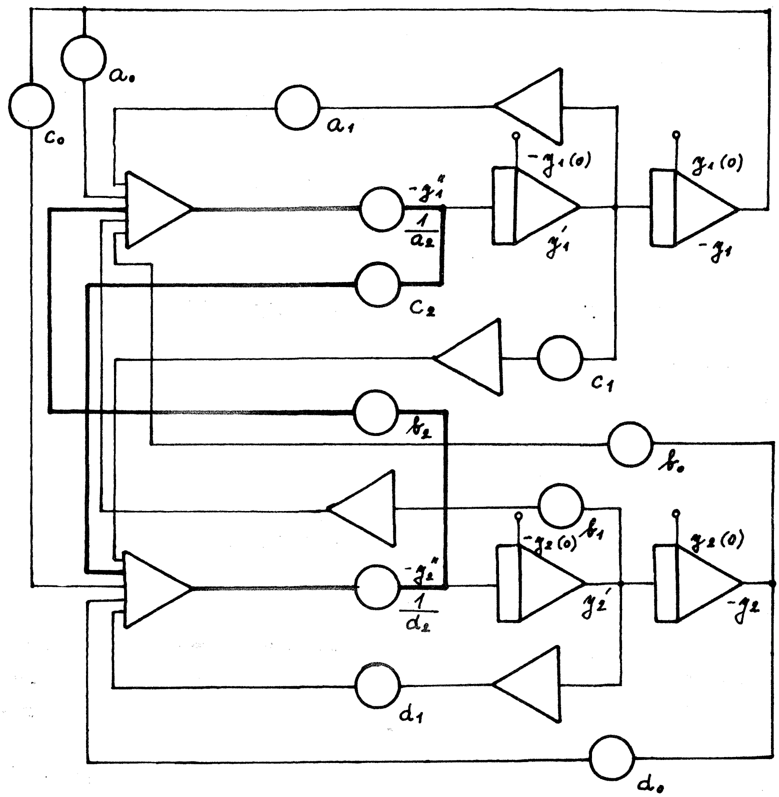
The program diagram for the solving of the system (3) is in the Fig.3. It contains the algebraical loop with even number of adders or inverters. Theoretical border of stability of this loop is given by maximum theoretical value of the gain of the loop, i.e.

$$S_{theor} = \frac{b_2c_2}{a_2d_2} = 1 . \qquad (4)$$

In the case that we will count the variable y_2 from the 1st equation of the system (2) and the variable y_1 from the second equation, i.e.

$$b_2y_2'' = -b_1y_2' - b_0y_2 - a_2y_1'' - a_1y_1' - a_0y_1 \qquad (5)$$

$$c_2y_1'' = -c_1y_1' - c_0y_1 - d_2y_2'' - d_1y_2' - d_0y_2$$



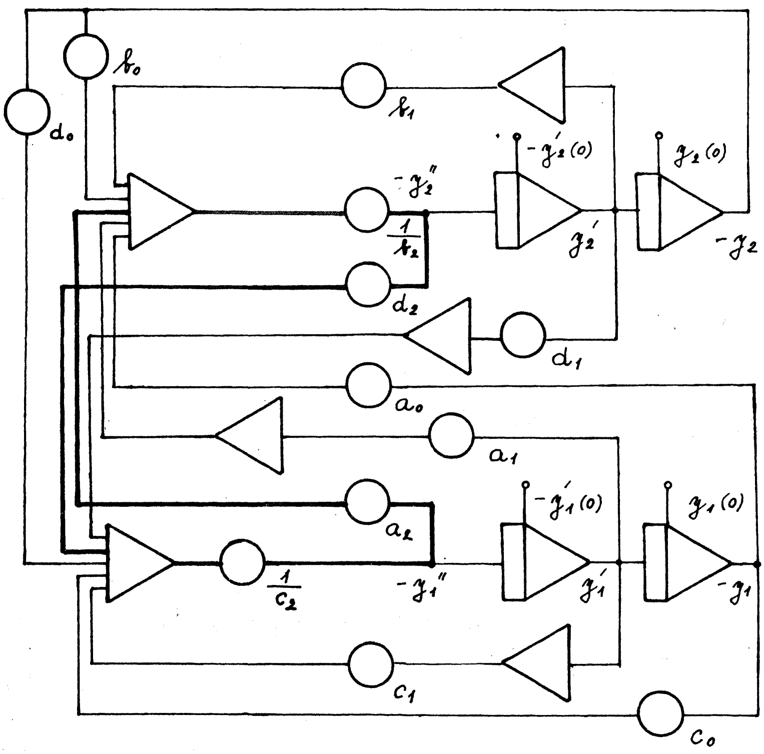
we will receive corresponding program diagram according to the Fig.4. Algebraical loop doesn't disappear, but it has the gain $S_2 = \frac{a_2 d_2}{b_2 c_2}$. If in the first case the gain $S_1 = \frac{b_2 c_2}{a_2 d_2} > 1$, then $S_2 = \frac{1}{S_1} < 1$ and the algebraical loop is stable. If the system (3) is programmed with the scale ratios M_1 respectively M_2, i.e. $Y_1^{(j)} = M_1 y^{(j)}$, $Y_2^{(j)} = M_2 y_2^{(j)}$ the system (3) has form

$$\frac{a_2}{M_1}\, Y_1'' \;=\; -\frac{a_1}{M_1}\, Y_1' - \frac{a_0}{M_1}\, Y_1 - \frac{b_2}{M_2}\, Y_2'' - \frac{b_1}{M_2}\, Y_2' - \frac{b_0}{M_2}\, Y_2 \qquad (6)$$

$$\frac{d_2}{M_2} \;=\; -\frac{d_1}{M_2}\, Y_2' - \frac{d_0}{M_2}\, Y_2 - \frac{c_2}{M_1}\, Y_1'' - \frac{c_1}{M_1}\, Y_1' - \frac{c_0}{M_1}\, Y_1 \,.$$

Program diagram for system (6) is identical (except extent of the coefficients and initial conditions) as for the system (3) in the Fig.3. Extent of the gain of the algebraical loops is

$$S \;=\; -\frac{\frac{c_2}{M_2}\,\frac{c_2}{M_1}}{\frac{a_2}{M_1}\,\frac{d_2}{M_2}} \;=\; \frac{b_2 c_2}{a_2 d_2} \,. \qquad (7)$$



The extent of the loop doesn't change by amplitude of the transformation.

If we program the system (3) with time scale change with coefficient M, i.e. $y_1^{(n)} = M^n Y_1^{(n)}$, $y_2^{(n)} = M^n Y_2^{(n)}$, system (3) has form

$$a_2M^2Y_1'' = -a_1MY_1' - a_0Y_1 - b_2M^2Y_2'' - b_1MY_2' - b_0Y_2 \qquad (8)$$

$$d_2M^2Y_2'' = -d_1MY_2' - d_0Y_2 - c_2M^2Y_1'' - c_1MY_1' - c_0Y_1 \; .$$

Program diagram for the system (8) correspond to (except coefficients and initial conditions) the program diagram in the Fig.3. The gain of the algebraical loop is in this case

$$S = \frac{b_2M^2c_2M^2}{a_2M^2d_2M^2} = \frac{b_2c_2}{a_2d_2} \; . \qquad (9)$$

The gain of the loop doesn't change by time scale change, too.

We can remove the algebraical loop by the following way: from the first equation of the system (2) we will compute quantity y_1'' and by substitution into the second equation we will get

$$a_2y_1'' + a_1y_1' + a_0y_1 + b_2y_2'' + b_1y_2' + b_0y_2 = 0 \qquad (10)$$

$$c_2 \frac{1}{a_2} (-a_1y_1' - a_0y_1 - b_2y_2'' - b_1y_2' - b_0y_2) +$$

$$+ c_1y_1' + c_0y_1 + d_2y_2'' + d_1y_2' + d_0y_2 = 0$$

after modification

$$a_2y_1'' + a_1y_1' + a_0y_1 + b_2y_2'' + b_1y_2' + b_0y_2 = 0 \qquad (11)$$

$$(c_1 - \frac{c_2}{a_2} a_1)y_1' + (c_0 - c_2 \frac{a_0}{a_2})y_1 + (d_2 - c_2 \frac{b_2}{a_2})y_2'' +$$

$$+ (d_1 - c_2 \frac{b_1}{a_2})y_2' + (d_0 - c_2 \frac{b_0}{a_2})y_2 = 0 \; .$$

The program diagram for solution of the system (11) is in the Fig.5. The system is programmed in the form

$$a_2 y_1'' = -a_1 y_1' - a_0 y_1 - b_2 y_2'' - b_1 y_2' - b_0 y_2$$

$$(d_2 - c_2 \frac{b_2}{a_2}) y_2'' = -(d_1 - c_2 \frac{b_1}{a_1}) y_2' - (d_0 - c_2 \frac{b_0}{a_2}) y_2 -$$

$$- (c_1 - \frac{c_2}{a_2} a_1) y_1' - (c_0 - c_2 \frac{a_0}{a_2}) y_1 \quad ,$$

algebraical loop doesn't occur in the program diagram. We would reach to the similar result, if we computed for example the quantity y_1'' from the second equation of the system (2) and substituted into the 1st equation of the system, we get

$$a_2 \frac{1}{c_2} (- c_1 y_1' - c_0 y_1 - d_2 y_2'' - d_1 y_2' - d_0 y_2) + a_1 y_1' +$$

$$+ a_0 y_1 + b_2 y_2'' + b_1 y_2' + b_0 y_2 = 0 \qquad (12)$$

$$c_2 y_1'' + c_1 y_1' + c_0 y_1 + d_2 y_2'' + d_1 y_2' + d_0 y_2 = 0 \ ,$$

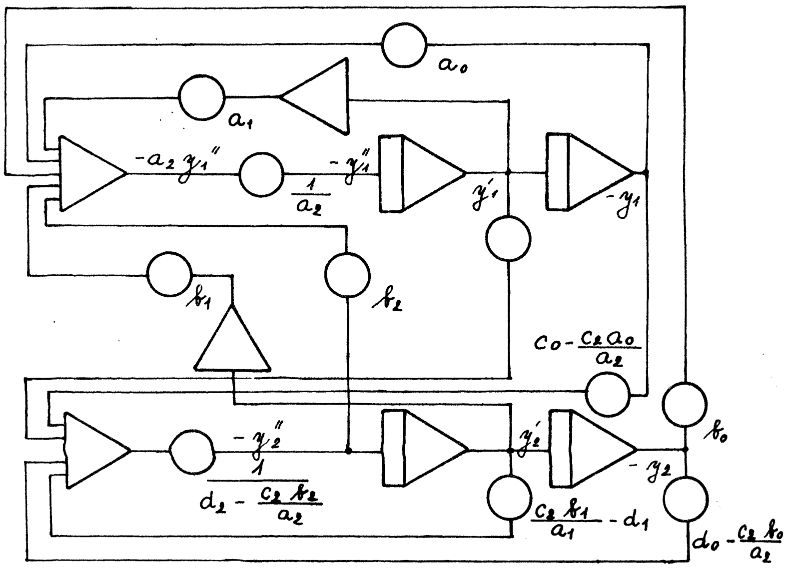
after modification

$$(a_1 - \frac{a_2 c_1}{c_2}) y_1' + (a_0 - \frac{a_2 c_0}{c_2}) y_1 + (b_2 - \frac{a_2 d_2}{c_2}) y_2'' +$$

$$+ (b_1 - \frac{a_2 d_1}{c_2}) y_2' + (b_0 - \frac{a_2 d_0}{c_2}) y_2 = 0 \qquad (13)$$

$$c_2 y_1'' + c_1 y_1' + c_0 y_1 + d_2 y_2'' + d_1 y_2' + d_0 y_2 = 0 \ .$$

The system (13) has except coefficients identical form as the system (11), its program diagram correspond to in an formal way the program diagram in the Fig.5, where algebraical loop doesn't occur.



Summary

In the work there is inquired into a possibility of removing of disadvantegenous amplification of loop with the transformation of variables.

Souhrn

ALGEBRAICKÉ SMYČKY A TRANSFORMACE PROMĚNNÝCH

V práci je zkoumána možnost odstranění nevýhodného zesílení smyčky transformací proměnných.

REFERENCES

[1] N o v á k, V.: Programování pro analogové počítače I. ČVUT Praha, 1970.

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АЛГЕБРАИЧЕСКИЕ ЦИКЛЫ И ПРЕОБРАЗОВАНИЕ ПЕРЕМЕННЫХ

В работе исследована возможность устранения невыгодного цикла преобразований переменных.

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