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ON AN ALMOST PERIODICITY CRITERION OF SOLUTIONS FOR SYSTEMS OF NONHOMOGENEOUS LINEAR DIFFERENTIAL EQUATIONS WITH ALMOST PERIODIC COEFFICIENTS

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1. Introduction

We consider a system of nonhomogeneous linear differential equations

$$y' = A(t)y + f(t) \tag{1}$$

with $A:R \longrightarrow R^{n \times n}$ an almost periodic square matrix function of order n and $f:R \longrightarrow R^n$ an almost periodic vector function of order n, $n \geq 2$. Besides (1) we consider system

$$x' = A(t)x . \tag{2}$$

As is well-known from by Favard [2] (see e.g. also [7] Theorem 18.2, p.207, [6] Theorem 4.2.2, p.180, [3] Theorem 4, p.218) there exists an almost periodic solution of (1) if has a bounded solution (on R) and every nontrivial bounded solution x of the system $x' = B(t)x$ satisfies $\inf_{t\in R} \|x(t)\| > 0$ for every $B \in H(A)$.

This paper presents sufficient conditions for even every bounded solution of (1) to be almost periodic. This result is then used to establish sufficient conditions for almost periodicity of every "bounded" solution of nonhomogeneous n-th order linear differential equations.

2. Basic concepts, notations, lemmas

We assume the matrix function $A:R \longrightarrow R^{n\times n}$ ($n \geqq 2$) to be almost periodic (i.e. $A \in C(R,R^{n\times n})$ and for any sequence $\{h_n\}$, $h_n \in R$, there exists a subsequence $\{h_{n_k}\}$ such that $\{A(t+h_{n_k})\}$ is uniformly convergent on R) and $f:R \longrightarrow R^n$ to be the almost periodic vector function (see e.g. [1], [3], [5] - [7]).

Let H(A) be the hull generated by A and H(f) be the hull generated by f (i.e. $B \in H(A)$ ($g \in H(f)$) if and only if there exists a sequence $\{h_n\}$, $h_n \in R$, such that $\lim_{n\to\infty} A(t+h_n) = B(t)$ ($\lim_{n\to\infty} f(t+h_n) = g(t)$) uniformly on R). It holds: if $B \in H(A)$ ($g \in H(f)$), then $H(B) = H(A)$ ($H(g) = H(f)$).

Next we assume:

(i) the space of bounded solutions of (2) has dimension m, $1 \leqq m < n$;

(ii) the space of bounded solutions of system

$$x' = B(t)x, \qquad (B \in H(A)) \tag{3}$$

has dimension m, for every $B \in H(A)$;

(iii) every nontrivial bounded solution x of (2) satisfies $\inf_{t\in R} \|x(t)\| > 0$.

By $x_1,\ldots,x_m$ we understand linearly independent bounded solutions of (2).

<u>Lemma 1</u>. Let $B \in H(A)$. Then, there exists a sequence $\{h_n\}$, $h_n \in R$, such that

$$\lim_{n\to\infty} A(t+h_n) = B(t) \tag{4}$$

uniformly on R.

$$\lim_{n\to\infty} x_i(h_n) = a_i, \quad i=1,2,\dots,m \tag{5}$$

and

$$\lim_{n\to\infty} x_i(t+h_n) = x_i^*(t), \quad i=1,2,\dots,m \tag{6}$$

local-uniformly on R, where x_i^* $(i=1,2,\dots,m)$ are linearly independent bounded solution of (3), $x_i^*(0) = a_i$, and every nontrivial solution x^* of (3) satisfies $\inf_{t\in R} \|x^*(t)\| > 0$.

Proof. From the fact that $B \in H(A)$ there exists a sequence $\{h_n\}$, $h_n \in R$, such that (4) is uniformly on R. It follows from the boundedness of $x_1,\dots,x_m$ that we may without loss of generality assume the validity of (5) (refining the sequence $\{h_n\}$ if necessary). From Theorem 2.4 [4], p.15, we obtain (6) local-uniformly on R, where x_i^* are bounded solutions of (3), $x_i^*(0) = a_i$ $(i=1,2,\dots,m)$. We now prove x_i^* $(i=1,2,\dots,m)$ being linearly independent solutions of (3). In the contrary case there exist $c_i \in R$ $(i=1,2,\dots,m)$, $\sum_{i=1}^m c_i^2 > 0$, such that

$$\sum_{i=1}^m c_i x_i^*(t) = 0 \text{ for } t \in R.$$

Set $\bar{x} := \sum_{i=1}^m c_i x_i$. Then $\bar{x}$ is a nontrivial bounded solution of (2). Then assumption (iii) yields $\inf_{t\in R} \|\bar{x}(t)\| > 0$ contrary to $\lim_{n\to\infty} \bar{x}(t+h_n) = 0$ local-uniformly on R. It remains to prove that every nontrivial bounded solution x^* of (3) satisfies $\inf_{t\in R} \|x^*(t)\| > 0$. With respect to assumption (ii) we see that x_i^* $(i=1,2,\dots,m)$ form a base of the space of bounded solutions of (3) wherefore we have for every nontrivial bounded solution x^* of (3)

$$x^* = \sum_{i=1}^{m} d_i x_i^* \quad ,$$

with $d_i \in R$, $\sum_{i=1}^{m} d_i^2 > 0$. Then $\tilde{x} := \sum_{i=1}^{m} d_i x_i$ is a nontrivial bounded solution of (2) and $\inf_{t \in R} \|\tilde{x}(t)\| > 0$. In view of the fact that $\lim_{n\to\infty} \tilde{x}(t+h_n) = x^*(t)$ local-uniformly on R we obtain $\inf_{t\in R} \|x^*(t)\| > 0$.

Definition 1. A set $\Omega \subset R^{n+1}$ is called the integral set of bounded solutions (ISBS) of (2) ((1)) if:

a) $(t,x(t)) \in \Omega$, $t \in R$, for every bounded solution x of (2) ((1));

b) $(t_o,x_o) \in \Omega$ then there exists the bounded solution x of (2) ((1)), $x(t_o) = x_o$ and $(t,x(t)) \in \Omega$ for $t \in R$.

Convention. We say that $h:R \to R^n$ lies in a set $\Omega \subset R^{n+1}$ if $(t,h(t)) \in \Omega$ for $t \in R$.

Remark 1. Let Ω be the ISBS of (2). The a function $h: R \to R^n$ lies in Ω if and only if there exists functions $c_i:R \to R$ $(i=1,2,\dots,m)$ such that

$$h(t) = \sum_{i=1}^{m} c_i(t)x_i(t) \quad \text{for } t \in R.$$

Lemma 2. Let Ω be the ISBS of (1), y a solution of (3) lying in Ω and let for a sequence $\{h_n\}$, $h_n \in R$,

$$\lim_{n\to\infty} A(t+h_n) = B(t), \qquad \lim_{n\to\infty} f(t+h_n) = g(t)$$

uniformly on R and $\lim_{n\to\infty} y(h_n) = a$, $a \in R^n$. Then

$$\lim_{n\to\infty} y(t+h_n) = y^*(t) \qquad (7)$$

local-uniformly on R, where y^* is a solution of the system

$$y' = B(t)y + g(t), \qquad (8)$$

$y^*(0) = a$.

If Ω^* is the ISBS of (3), then y^* is lying in Ω^*.

Proof. The first part of Lemma 1 follows from Theorem 2.4 [4], p.15.

Suppose $y = \mathrm{col}(y_1, y_2, \dots, y_m)$ to lie in Ω. Then

$$y(t) = \sum_{i=1}^{m} c_i(t)x_i(t), \quad t \in R, \qquad (9)$$

with $c_i : R \to R$ $(i=1,2,\dots,m)$. Let $t_o \in R$. It follows from the boundedness of $x_i = \mathrm{col}(x_{1i}, x_{2i}, \dots, x_{ni})$ $(i=1,2,\dots,m)$ that there exists a sequence $\{h_{n_k}\}$ selected from $\{h_n\}$ such that

$$\lim_{n\to\infty} x_i(t_o+h_{n_k}), \quad i=1,2,\dots,m,$$

exist. Of course, we then have from Lemma 1

$$\lim_{k\to\infty} x_i(t+h_{n_k}) = x_i^*(t), \quad i=1,2,\dots,m,$$

local-uniformly on R, where $x_i^* = \mathrm{col}(x_{1i}^*, x_{2i}^*, \dots, x_{ni}^*)$, $i=1,2,\dots,m$, are linearly independent bounded solutions of (3). Therefore the rank of the $n \times m$ matrix

$$(x_1^*(t_o),\ x_2^*(t_o), \dots, x_m^*(t_o))$$

is equal to m. From this matrix may then be selected m rows so that the obtained $m \times m$ matrix S is regular. For simplicity let S be formed by the first m rows

$$S = (x_{ji}^*(t_o))_{j,i=1}^{m} \quad (\det S \neq 0).$$

Setting $s_k := \det (x_{ji}(t_o+h_{n_k}))_{j,i=1}^{m}$ yields $\lim_{k\to\infty} s_k = \det S \neq 0$ and therefore $s_k \neq 0$ for sufficiently large k. It is possible for there k from the system of linear equation

$$y_j(t_o+h_{n_k}) = \sum_{i=1}^{m} c_i(t_o+h_{n_k})x_{ji}(t_o+h_{n_k}) \quad j=1,2,\dots,m, \qquad (10)$$

(as obtained from (9)) to express $c_i(t_o+h_{n_k})$ by Cramer's rule using $y_i(t_o+h_{n_k})$ and $x_{ji}(t_o+h_{n_k})$. From this expression it is apparent that there exist $\lim_{k\to\infty} c_i(t_o+h_{n_k})$ $(=: c_i(t_o))$ (i=1,2,.. ..,m) and passing to the limit as $k \to \infty$ in (10) we get

$$y^*(t_o) = \sum_{i=1}^{m} c_i^*(t_o)x_i^*(t_o).$$

Consequently $(t_o,y^*(t_o)) \in \Omega^*$ for $t_o \in R$, thus y^* lies in Ω^*.

Corollary 1. Let Ω be the ISBS of (2). If a bounded solution y of (1) is not lying in Ω, then no bounded solution of (1) is lying there.

Proof. Every bounded solution w of (1) is of the form

$$w = y + \sum_{i=1}^{m} c_i x_i,$$

with $c_i \in R$ (i=1,2,...,m). If for a $t_o \in R$ we have $(t_o,y(t_o)) \notin \Omega$, then $(t_o,w(t_o)) \notin \Omega$. Contraryvise if $(t_o,w(t_o)) \in \Omega$, we get $(t_o,w(t_o) - \sum_{i=1}^{m} c_i x_i(t_o)) \in \Omega$. Hence $(t_o,y(t_o)) \in \Omega$. This contradiction proves Corollary 1.

Corollary 2. Let Ω be the ISBS of (2) and let for a sequence $\{h_n\}$, $h_n \in R$,

$$\lim_{n\to\infty} A(t+h_n) = B(t), \quad \lim_{n\to\infty} f(t+h_n) = g(t)$$

uniformly on R. Let Ω^* be the ISBS of (3).

If a (and then every) bounded solution of (1) does not lie in Ω, then no bounded solution of (8) lies in Ω^*.

Proof. Assume a (and by Corollary 1 every) bounded solution y^* of (8) be lying in Ω^*. Then there exists a subsequence $\{h_{n_k}\}$ of $\{h_n\}$ such that

$$\lim_{k\to\infty} y^*(t-h_{n_k}) = y(t)$$

local-uniformly on R, where y is a bounded solution of (1). By Lemma 2, y lies in Ω, which is (with respect to Corollary 1) contrary to our assumptions of Corollary 2.

Lemma 3. Let Ω be the ISBS of (2). If a (and then every) bounded solution of (1) lies in Ω, then there lies the function f also. Conversely also, if the function f lies in Ω, then there exists a solution of (1) lying there also.

Proof. Let y be a bounded solution of (1) lying in Ω. By Corollary 1 every bounded solution of (1) lies in Ω so that we may without any loss of generality assume $y(0) = 0$. Let Y be a fundamental matrix of (2) and the first m columns of Y be linearly independent bounded solutions $x_1, x_2, \dots, x_m$ of (2). Then

$$y(t) = Y(t) \int_0^t Y^{-1}(s) f(s) ds \quad \text{for} \quad t \in R.$$

Set $c(t) := \int_0^t Y^{-1}(s) f(s) ds$ for $t \in R$. Let $c = \text{col}\,(c_1, c_2, \dots, c_n)$ and $Y(t) = (x_{ji}(t))_{j,i=1}^n$. By our assumption y is lying in Ω and therefore

$$\sum_{i=m+1}^{n} x_{ji}(t) c_i(t) = 0 \text{ for } t \in R, \ j=1,2,\dots,n. \tag{11}$$

With a fixed t ($\in R$) we may consider (11) as a system of n linear equations having n-m unknowns $c_j(t)$, $j=m+1,\dots,n$. Since the rank of the matrix of system (11) is n-m, then necessarily $c_j(t) = 0$ for $t\in R$ and $j = m+1,\dots,m$. Consequently

$$\int_0^t Y^{-1}(s)f(s)ds = col(c_1(t),\dots,c_m(t),0,\dots,0),$$

whence

$$Y^{-1}(t)f(t) = col(c_1'(t),\dots,c_m'(t),0,\dots,0)$$

and

$$f(t) = Y(t)col(c_1'(t),\dots,c_m'(t),0,\dots,0).$$

From this ($f = col(f_1,f_2,\dots,f_n)$)

$$f_j(t) = \sum_{i=1}^{m} c_i'(t)x_{ji}(t), \quad t\in R, \quad j=1,2,\dots,n$$

and f is lying in Ω.

Letting $(t,f(t))\in\Omega$ for $t\in R$ yields $f(t) = Y(t)c(t)$ with $c:R \to R^n$, $c = col(c_1,c_2,\dots,c_m,0,\dots0)$. If $y(t) :=$ $:= Y(t)\int_0^t Y^{-1}(s)f(s)ds$ for $t\in R$, then y is a solution of (1) and from the equality $y(t) = Y(t)\int_0^t c(s)ds$, $t\in R$, we obtain $y(t) = \sum_{i=1}^{m} x_i(t)\int_0^t c_i(s)ds$. Hence y is lying in Ω.

Results

Theorem 1. Let $A:R \to R^{n\times n}$ be an almost periodic matrix function, $f:R \to R^n$ be an almost periodic vector function and the assumptions (i) - (iii) be satisfied. Let Ω be the ISBS of (2) and f does not lie in Ω.

Then every bounded solution of (1) is almost periodic.

Proof. Let y be a bounded solution of (1). By our assumption f is not lying in Ω. Therefore, by Lemma 3, y is also not lying there. Suppose y is not an almost periodic function. Then there exists a sequence $\{h_n\}$, $h_n \in R$, such that every subsequence of $\{y(t+h_n)\}$ is not uniformly converging on R. From the almost periodicity A and f and from the boundedness of the solution y then follows the existence of a subsequence of $\{h_n\}$ (using the same notation for simplification) such that

$$\lim_{n\to\infty} A(t+h_n) = B_2(t), \quad \lim_{n\to\infty} f(t+h_n) = g_2(t)$$

uniformly on R and

$$\lim_{n\to\infty} y(h_n) = a(\in R^n).$$

On account of Lemma 2 we have

$$\lim_{n\to\infty} y(t+h_n) = y^*(t)$$

local-uniformly on R, where y^* is the solution of system

$$y' = B_2(t)y + g_2(t), \tag{12}$$

$y^*(0) = a$. Since $\{y(t+h_n)\}$ is not uniformly converging on R, there exist: an $\varepsilon > 0$, $\{t_n\}$($t_n \in R$, $\lim_{n\to\infty} |t_n| = \infty$) and subsequences $\{h_{k_n}\}$, $\{h_{r_n}\}$ of $\{h_n\}$ such that

$$\| y(t_n+h_{k_n}) - y(t_n+h_{r_n}) \| \geq \varepsilon, \quad n=1,2,\ldots \tag{13}$$

(see [5], p.156) and besides

$$\lim_{n\to\infty} A(t+t_n+h_{k_n}) = B(t), \quad \lim_{n\to\infty} f(t+t_n+h_{k_n}) = g(t),$$

$$\lim_{n\to\infty} A(t+t_n+h_{r_n}) = B_1(t), \quad \lim_{n\to\infty} f(t+t_n+h_{r_n}) = g_1(t),$$

uniformly on R. We may prove that $B = B_1$, $g = g_1$ analogous to [5] p.157. We may also assume (without any loss of generality)

$$\lim_{n\to\infty} y(t_n+h_{k_n}) = \alpha, \quad \lim_{n\to\infty} y(t_n+h_{r_n}) = \beta .$$

With respect to (13) then

$$\|\alpha - \beta\| \geqq \varepsilon . \tag{14}$$

Next by Lemma 2

$$\lim_{n\to\infty} y(t+t_n+h_{k_n}) = y_1^*(t), \quad \lim_{n\to\infty} y(t+t_n+h_{r_n}) = y_2^*(t)$$

local-uniformly on R, where y_1^* and y_2^* are bounded solutions of (8), $y_1^*(0) = \alpha$, $y_2^*(0) = \beta$. Let Ω^* be the ISBS of (3). From Corollary 2 then follows that y_1, y_2 are not lying in Ω^* and therefore $(t_o, y_1^*(t_o)) \notin \Omega^*$ for a $t_o \in R$. Since $y_1^* - y_2^*$ is a bounded solution of (3), we have $(t_o, y_1^*(t_o) - y_2^*(t_o)) \in \Omega^*$. If $(t_o, y_2^*(t_o)) \in \Omega^*$, then necessarily $(t_o, y_1(t_o)) \in \Omega^*$ which is a contradiction. Therefore $(t_o, y_2^*(t_o)) \notin \Omega^*$. Setting $X_1^* := \{x \in R^n;$ $(t_o, x) \in \Omega^*\}$, yields X_1^* ($\neq \{0\}$) being proper subspace in R^n. Let X_2^* be a complementary subspace to X_1^* in R^n, $y_1^*(t_o) \in X_2^*$, $X_1^* \oplus X_2^* = R^n$. From the fact that $y_1^*(t_o) - y_2^*(t_o) \in X_1^*$, there then exists a $b \in X_1$:

$$y_1^*(t_o) = y_2^*(t_o) + b. \tag{15}$$

We may express $y_1^*(t_o)$ being uniquely in the form $y_1^*(t_o) = u + v$ with $u \in X_1^*$, $v \in X_2^*$. Then, naturally, $u = 0$ and we get $b = 0$ in (15). Therefore $y_1(t_o) = y_2(t_o)$ whence $\alpha = \beta$ contrary to (14). Consequently $\{y(t+h_n)\}$ converges uniformly on R and y is an almost periodic solution of (1).

<u>Corollary 3</u>. Let $A: R \longrightarrow R^{n \times n}$ be an almost periodic matrix function and assumptions (i) - (iii) be satisfied. Let Ω be the ISBS of (2).

If (1) has bounded solution for an almost periodic function $f: R \longrightarrow R^n$, not lying in Ω, then every bounded solution of (2) is almost periodic.

Proof. Let y be a bounded solution of (1) with an almost periodic function f not lying in Ω . Since x + y is a bounded solution of (1) for every bounded solution x of (2), then y and x + y are almost periodic functions by Theorem 1. Consequently x (=(x+y)-x) is an almost periodic function.

Corollary 4. Let $A:R \to R^{n\times n}$ be an almost periodic matrix function and assumptions (i) - (iii) be satisfied. Let the system

$$y' = A(t)y + c$$

have a bounded solution for every $c \in R^n$. Then all bounded solutions of (2) are almost periodic functions.

Proof. Let Ω be the ISBS of (2) and $c_o \in R^n$, $(0,c_o) \notin \Omega$. The system $y' = A(t)y + c_o$ has a bounded solution and since the function $f(t):= c_o$ for $t \in R$ is not lying in Ω, then, every bounded solution of (2) is almost periodic with respect to Corollary 3.

Let us now consider a homogeneous n-th order ($n \geq 2$) linear differential equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \dots + a_n(t)x = 0 \qquad (16)$$

with $a_i:R \to R$ ($i=1,2,\dots,n$) being scalar almost periodic functions. Setting $a(t):= col(a_1(t),a_2(t),\dots,a_n(t))$, $t \in R$, then $a:R \to R^n$ is an almost periodic vector function. As it well-known, equation (16) can be converted into a system of type (2). We introduce the following definitions for transforming the above results, holding for systems of linear differential equations, to the n-th order linear differential equations.

Definition 3. We say, a function g has the property (BD) if

$$\text{(BD)} \qquad g:R \to R,\ g \in C^{n-1}(R) \text{ and } g^{(i)}(t) \text{ are bounded functions on } R\ (i=0,1,\dots,n-1).$$

Definition 4. We say, a function g has the property (BAD) if

$g:R \to R$, $g \in C^{n-1}(R)$ and $g^{(i)}(t)$ are almost periodic (BAD) functions $(i=0,1,\dots,n-1)$.

Definition 5. We say, a set $\Omega \subset R^{n+1}$ is the integral set of BD-bounded solutions (BD-ISBS) of (16) if $(t,x(t),\dots,x^{(n-1)}(t)) \in \Omega$ for $t \in R$, for every solution x of (16) with the property (BD), and to every point $(t_o,x_o,\dots,x_{n-1}) \in \Omega$ there exists the solution u of (16) having the property (BD), $u^{(i)}(t_o) = x_i$ $(i=0,1,\dots,n-1)$ and $(t,u(t),u'(t),\dots,u^{(n-1)}(t)) \in \Omega$ for $t \in R$.

Assumptions (i) - (iii) for system (2) may be formulated for equation (16) in this way:

(i') the space of solutions of (16) having the property (BD) has the dimension m, $1 \leq m < n$;

(ii') the space of solutions of the equation
$x^{(n)} + b_1(t)x^{(n-1)} + \dots + b_n(t)x = 0$
having the property (BD) has for every $b:=\mathrm{col}(b_1,b_2,\dots,b_n) \in H(a)$ the dimension m;

(iii') $\inf_{t \in R} \sum_{j=1}^{n-1} |x^{(j)}(t)| > 0$ for every solution x of (16) having the property (BD).

Theorem 2. Let $a = \mathrm{col}(a_1,a_2,\dots,a_n):R \to R^n$ be an almost periodic vector function and (i') - (iii') be satisfied. Let Ω be the BD-ISBS of (16), $p:R \to R$ be an almost periodic function and the vector function $(0,\dots,0,p):R \to R^n$ be not lying in Ω. Then, every solution of equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \dots + a_n(t)x = p(t) \qquad (17)$$

having the property (BD) has the property (BAD) as well.

The proof follows immediately from Theorem 1.

Remark 2. If $a_1, a_2, \dots, a_n$ are ε-periodic and continuous (on R) functions and p is an almost periodic function, then (as is well-known form [1] p.423, [5] p.128) every bounded solution of (17) is almost periodic.

From Corollary 3 now immediately follows

Corollary 5. Let $a = \mathrm{col}(a_1, a_2, \dots, a_n) : R \longrightarrow R^n$ be an almost periodic vector function and assumptions (i') - (iii') be satisfied. Let Ω be the BD-ISBS of (16). If equation (17) has for an almost periodic function $(0, \dots 0, p) : R \longrightarrow R^n$ not lying in Ω a solution with the property (BD), then every solution of (16) with the property (BD) has the property (BAD) as well.

Corollary 6. Let $a = \mathrm{col}(a_1, a_2, \dots, a_n) : R \longrightarrow R^n$ be an almost periodic vector function and assumptions (i') - (iii') be satisfied. If there exists a solution of the differential equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \dots + a_n(t)x = 1 \tag{18}$$

having the property (BD), then every solution of (16) having the property (BD) has the property (BAD) as well.

Proof. Let x_1 be a solution of (18) having the property (BD). Then $x_2 := cx_1$ is a solution of the differential equation

$$x^{(n)} + a_1(t)x^{(n-1)} + \dots + a_n(t)x = c$$

having the property (BD) for all $c \in R$. Therefore every solution of (16) having the property (BD) has also the property (BAD) as follows from Corollary 4.

Summary

Let $A : R \longrightarrow R^{n \times n}$ be an almost periodic matrix function and $f : R \longrightarrow R^n$ be an almost periodic vector function. In the paper are present sufficient conditions for every bounded solution of the system

$$y' = A(t)y + f(t)$$

to be almost periodic. One in conditions required that f is not lying in the integral set of bounded solutions of the system $y' = A(t)y$. The results are applied to a derivation of an almost periodicity criterion of solutions for a nonhomogeneous n-th order linear differential equation.

Souhrn

KRITERIUM SKOROPERIODIČNOSTI ŘEŠENÍ NEHOMOGENNÍHO SYSTÉMU LINEÁRNÍCH DIFERENCIÁLNÍCH ROVNIC SE SKOROPERIODICKÝMI KOEFICIENTY

Nechť $A:R \longrightarrow R^{n \times n}$ je skoroperiodická maticová funkce a $f:R \longrightarrow R^n$ skoroperiodická vektorová funkce. V práci jsou uvedeny podmínky, které jsou postačující k tomu, aby každé ohraničené řešení systému

$$y' = A(t)y + f(t)$$

byla skoroperiodická funkce. Jeden z předpokladů je, aby funkce f neležela v integrální množině ohraničených řešení systému $y' = A(t)y$. Výsledky jsou použity k odvození kriteria skoroperiodičnosti řešení nehomogenní lineární diferenciální rovnice n-tého řádu.

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ПРИЗНАК ПОЧТИ-ПЕРИОДИЧНОСТИ РЕШЕНИЙ НЕОДНОРОДНОЙ СИСТЕМЫ ЛИНЕЙНЫХ ДИФФЕРЕНЦИАЛЬНЫХ УРАВНЕНИЙ С ПОЧТИ-ПЕРИОДИЧЕСКИМИ КОЭФФИЦИЕНТАМИ

Пусть $A: R \to R^{nxn}$ - почти - периодическая матричная функция, $f: R \to R^n$ - почти - периодическая векторная функция. В работе приводятся условия, которые достеточные для того, чтобы все ограниченные решения системы

$$y' = A(t)y + f(t)$$

были почти - периодическими функциями. Одно из условий предпологает что f лежит в интегральном множестве ограниченных решений системы $y' = A(t)y$. Результаты использованы при выводу признака почти - периодичености решений неоднородного линейного дифференциального уравнения n-го порядка.

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