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FROM HISTORY OF GEOMETRY IN EDUCATION

JOSEF MOLNÁR

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If the teaching of geometry is in public opinion considered to be one of the educational aims, then the development of space imagination is certainly one of them, too. This paper deals with a position of geometry and stereometry in teaching mathematics from a historical point of view. The aim of this paper is to compare the scope, contents and conception of geometry instruction in our curriculum of mathematics, technical drawing, descriptive geometry after 1948 and in textbooks of mathematics after 1933 at General Schools.

Before we begin to compare, we have to outline briefly the development of Czechoslovak system of education since 1948.

The first act forming the unified educational system of Czech and Slovak schooling was approved by National Assembly on April 21, 1948. Five-year Primary School (národní škola) and four-year Unified Secondary School (jednotná střední ško-
la) were followed by four-year Grammar School (gymnázium) or by Vocational Secondary School (škola odborná).

The reform of April 24, 1953 established through the new School Act the basic Eight-Year Schools and Eleven-Year Schools (osmiletá a jedenáctiletá škola - OSŠ a JSŠ). Obligatory eight-year school attendance was fixed. The shortening of total teaching period resulted in the introducing of linear curriculum. However, Eleven-Year School had not been fully introduced into practice and a new experimental curriculum and textbooks, which were to solve the problems of JSŠ, were tested since 1957. This conception was abandoned before the first undergraduates achieved the ninth year of the experimental schools and the conception of twelve-year School attendance was being introduced.

In April 1959 the Central Committee of the Czechoslovak Communist Party passed a resolution that school should be closely linked with life. Elementary education was ensured by means of obligatory Basic Nine-Year Schools (ZDŠ) which were to become gradually the schools of general, vocational and polytechnic education.

They were called Primary Schools and were divided into two stages. Primary Schools are followed by Secondary Schools, among which Secondary General School (SVVŠ) provides subsequent general education. The authors of the curriculum and textbooks tried to harmonize requirements of "scientism" in education with mental abilities of pupils. It was enabled by means of renewal of cyclic sequence of the curriculum.

The 1968 Education Act modified and extended the three-year Secondary General School into four-year Secondary Grammar School (SVVŠ a G).

The conclusions of the 14th Communist Party Congress and of the Communist Party session, held in January 1973, served as an impetus for the edition of "Further development of Czechoslovak educational system". This system is based on a unique network of schools and educational facilities laying accent on working and polytechnic education. The obligatory school atten-
dance became ten years long. The educational process was fo-
cused on the development of activity, creativity and individual
abilities of students.

Tables 1, 2 and 3 comprise only approximate data, because
of the continuous modification of the number of lessons pres-
ccribed by respective teaching programmes. Nevertheless, from
the tables mentioned above, we can see certain trends of de-
velopment in teaching stereometry.

Tables 1, 2 and 3

The development in teaching geometry, descriptive geometry
and technical drawing at General Schools is also interesting.

After 1933 reform, technical drawing was ranged in 3rd
and 4th year of all types of Grammar Schools and Technical Se-
condary Schools (gymnázia a réály). Descriptive geometry was
optional in 7th and 8th year of Grammar Schools and Reform
Grammar Schools, another one was conversation in a living fo-
 reign language. At Technical Secondary School descriptive geo-
metry was taught during the final three years of study, e.g.
from 5th to 8th year.

In 4th form of Basic School (JŠ) technical drawing was
taught in two lessons a week, at Grammar School in 4, 3, 2, 2
lessons a week in respective forms as an optional subject.
(Table 1.)

In the beginning, the technical drawing at JŠŠ, which was
taught 1 lesson a week in 9 - 11th form, followed the techni-
cal drawing of 8th form of OSS. But it was soon replaced by
descriptive geometry and, besides that, in 10th form the number
of periods was increased by 2 and a new non-compulsory subject
- technical drawing - was introduced.

In 9th form of Basic Nine-Year School there were at first
[/47/, 1960] 2 lessons of technical drawing, later [/48/, 1962]
only 1 a week. At Secondary General Schools and Grammar Schools,
directed at the study of mathematics and physics (from the
school year 1965,66), it was called "the branch of natural
sciences". Descriptive geometry was ranged as a compulsory subject with 2 lessons a week in the last two forms of mathematical and physical branches and as an optional one for the "basic branch" and later (since 1965,66) for the humanities. It was introduced as a non-compulsory subject also for humanities at Grammar Schools (1969,70), as well as technical drawing.

In the curriculum for Basic Schools [51], the technical drawing is not an independent subject. It is a part of mathematics for 8th form, where is does not serve its purpose. Descriptive geometry is obligatory only at the mathematical branch of Grammar Schools and is taught 0,0,1,2 lessons a week. At Grammar Schools of other types it is taught either as an optional subject for 3 lessons a week in 4th form or as a non-compulsory course with 2 lessons a week.

After a modification of curriculum in 1933 were published the adjusted textbooks of J.Vináš for the age corresponding to the second stage of Czech schools. They are interesting for us from the point of view of teaching geometry at Basic Schools, because it begins here by spatial solids.

In the introduction of the first part, geometry is explained as a branch of science dealing with investigation of the properties of solids concerning their formation, largeness and position, motivated by means of subjects of daily use. A cube and a cuboid are the first discussed concepts. A straight line arises by lengthening of the edge of a cube through both vertices, a square is introduced as a side of a cube. The pupils are currently acquainted in the textbook with the rules of technical drawing and with the mutual positions of a straight line and a plane. However the theorems are deduced from opinions and the textbooks are descriptive. The continuity of planimetry to stereometry is consistent. From stereometry concepts, for 1st form are further ranged the volumes of a cube and a cuboid, regular prisms and pyramids, plane symmetry. Prisms and pyramids (including tetrahedron, octahedron, nets, simple cuts and symmetry) are incorporated in
2nd form, there are commonly taught parallelograms and parallelepipeds, trapeziums and frustrum of pyramids, polygons with notation about regular prisms and pyramids. A cylinder, a cone and a sphere are introduced at the end. In 3rd form there are gradually modified the articles for computation of circumferences and areas of plane formations with computations of surface areas and volumes of solids.

The textbooks of J.Vojtěch adjusted to the new curriculum were used in the higher forms of Secondary Schools. In the textbook for 4th form [32], our attention will be drawn by the delimitation of contents (and conception) of geometry: "Geometry investigates those properties of the formations which do not modify themselves by virtue of their movement but by transformation into the formation symmetrical with them and by transformation into the formations similar to them". The analyse of teaching matter is then adopted to it and ordered to the system of theorems. Further we choose only a few exhibits from the contests: 5th form - proportional to the fourth, Ptolemy’s theorem, Kocański’s approximate construction for the circumference of a circle, Euler’s theorem. 6th form [34] - Tangent law. 7th and 8th form [35] - the common definition of conics, lemniscate, Quetelet-Dandelin theorem, subtangents, the course of a function. The analytical geometry is rather a studium of geometrical solids than the application of algebraical scheme. Stereometry for 5th form is explained in deductive system, but the proving of theorems and principles is not consistent.

On the basic of the comments to the achieved state, mathematics was to be built on the scientific basis. After a new school law, which was passed in 1948, new concepts were to be exactly defined, theorems proved. The pupils were led to express themselves exactly, to make analyses, proofs and discussion in solving the exercises. The pupils came to 1st form of Secondary School with knowledge of a few concepts from 5th form of Basic School but geometry itself was a new subject for them. In the textbook [7] of E.Cech and collective there are published the introductory motivations from
neighbouring world, geometry concepts are derived from it by abstraction and idealization. After the "spatial" introduction follows planimetry, geometry is here an exemplary pro-pedeutics. Teachers taught the basic concepts, exercised exact technical drawing, which is an organic constituent of the whole subject matter in geometry, and developed the spatial imagination of their pupils. In stereometry curriculum the pupils were acquainted with properties of a cuboid and a cube, with their pictures and nets, with computations of surface areas and volumes. In 2nd form of Secondary School follows the deductive system of processing and teaching the subject matter. The textbook [8] contains many theorems and is used to promote the striving for a scientific character of the reform. Axioms are deduced by abstraction from practice, theorems are proved. The subject matter in stereometry is limited in 2nd and 3rd form to the calculation of areas and volumes of further solids and to construction of their lateral surfaces. The focus of stereometry lies in 4th form [10], where system- 
matization of the spatial geometry in deductive way on the basis of several basis theorems is applied. Along with training the pupils' inference, there is time enough for development of spatial imagination and mapping. The text is completed by a great number of pictures. On the whole, the textbooks are pretentious (exception 1st form), deductive processes are inadequate to the age of pupils.

A component part of the textbooks for Grammar Schools [11] - [14], the main author of which is again E.Cech, is always an annual tematic plan. The subject matter of geometry is the studium of space which consists of points and the most important parts of which are straight lines. The starting point in the introduction is motivationally space but further is in 1st form [11] ranged only planimetry, where the system of knowledge from Secondary School is built and the subject matter is by using the concept of congruence put on a unified basis. We meet here for example such concepts as the power of a point with respect to a circle and Euler's straight line, the emphasis is put on deduction, but the textbook is little
The same spirit can be found also in the higher forms. The systematic course of stereometry is included in 2nd form [12]. The strictly deductive system, which goes out of axioms incidence, properties of parallelism, ordering and metrics, contains among others such contents as a trihedra; it studies the mutual position of a plane with the plane of a cylinder and a cone, etc. The formulae for volume of solids are derived by means of Cavalieri's principle and there is given the proof of its more general modification. The textbook does not contain solved examples, there is very little space for independent work of students.

The teaching of geometry at Vocational Secondary School began by propedeutics in 5th form and was ended in 6th form. The pupils were to gain a certain quantity geometrical imaginations and knowledge, they learned to compute volumes of basic solids. Geometry in 6th form was absolute, axiom of parallelism was not used. The basic geometry concept was a point; the pupils had to imagine all other geometrical formations as composed from points. A systematic and closed course of planimetry was ranged in 7th and 8th form to give the school-leavers from Vocational Secondary School at least a partly comprehensive geometrical education. This course was however opened precociously in 7th form, inadequately to the age of pupils. There was only little stereometry, the developing of spatial imagination and increasing of capacity of knowledge in geometry was not the main aim of technical drawing in 8th form. Situation from the point of view of stereometry and developing of spatial imagination was essentially better in 9th - 11th form of Eleven-Year Secondary School. In the textbook of Vyšín for 9th form [37] there is compiled planimetry with emphasis on congruence, similarity and trigonometry. Stereometry is taught in 10th form and is divided into three parts: the positional and metric properties, polygons and solids. The spatial geometry is again built deductively on the basis of axioms of incidence, there are also included constructional tasks in space. Further are used parallelism and arrangement. Orthogonality begins by axioms of
congruence, distance and deviation are determined above all graphically. The plane symmetry is understood as a mapping specified by certain regulation, the plane is a set of points and also self-adjoint elements are studied. Every congruence in space is a collection of a final number of plane symmetries, the direct and indirect congruence are demonstrated on exhibits from the usual practice. The pupils start to study solids after they have learned the common parts of space. The teaching matter in 11th form is directed at computation of areas, volumes and surface which are introduced in accordance with functional conception of measure. A limit is used frequently for derivation of formulae. Euler’s theorem appears in the tasks. Stereometry is compiled profoundly, with pretentious proofs, and the author ranges here also tasks and examples from life.

The pupils at Basic Nine Year School gained geometrical knowledge by inductive processes. In cases appropriate to their understanding the theorems were proved. Stereometrical knowledge was ranged into individual parts and was linked up with subject matter about planimetric solids familiar to them. The formulae for computation of volumes and surfaces result from generalization of concrete computations. First stereometrical knowledge was already brought to pupils of the first stage. The propedeutic course of geometry was opened in 4th form and continued in the following forms. Its contents was limited only to elementary subject matter, avoided the theoretical material and was based on opinion and geometrical experience of pupils. Technical drawing was an independent subject matter in 9th form, but its basic rules were taught already since 6th form. The textbooks [29], [25], [26], [30] are coloured, completed by photographs, a great emphasis is put on connection with practical life. Stereometry is concentrated on solids and computations of their areas and volumes.

Textbooks of mathematics for Secondary Grammar Schools appeared in 1964 [41], [17], [24]. Curriculum of geometrical character to which a great attention is paid, goes out of practice and polytechnization of General Secondary School is
emphasized. Stereometry is ranged in 1st and 3rd form of Secondary Grammar School; it is based on adequately deductive way, yet some theorems are not purposefully proved. Analogy with planimetry is here used and the authors's aim at the development of spatial imagination. Curriculum is supplemented especially in 1st form by a great number of well-arranged and objective pictures. In 3rd form the formulae for computation of volumes and surface of basic solids are not proved, with the note that the proof is possible; in case of round solids with reference to integral calculus.

The need for new textbook appeared in 1969, when Grammar Schools came into being. For the time being in so called commentary era, books for Secondary Grammar Schools were used. The teaching matter in geometry was however as a consequence of the global trends to modernization in the 1960s reduced.

The modernization in mathematics at Grammar School resulted in a set of exercise books M1 - M8; their co-ordinator was M.Zedek. In a consistently set-logical conception is here compiled a great quantity of subject matter along with voluntary complements and problems for inquisitive pupils. Stereometry is concentrated in a separate exercise book M 4/2 [5]. Its processing differs expressively from the proceeding text books. The text consists above all of exercises and inductive processes and local deduction are used. There is space for independence of pupils and teachers, too. Free parallel projection and symbolic registrations are though used rather in stereotypes. The textbook contains mutual positions of straight lines and planes including orthogonality and accumulation factor, coincident mapping in space and solids with computation of areas and volumes. Among the solids of rotation is taught annulus, in the complement is Cavalieri's principle. Endeavour to modernization appears also in conception of textbooks for Basic Schools [18], [19], [20], [21], [22], [27], [23]. Geometry is introduced as an elementary Euclidian geometry of three-dimensional space, in the framework of which are given the corresponding plane specifications. The unified
construction of knowledge of geometrical and arithmetical character is ensured by set access. The school geometry is based on the system of axioms (Kabele, Janků) in which the basic concepts are the point and the line segment. New concepts are taught to pupils in various activities, for example by modelling. An important part of stereometry at Basic Schools are computations, the measures are usually introduced by using "upper and lower limit". Little attention is focused on abilities in technical drawing. Too great emphasis is concentrated on one method of modelling, other forms of developing spatial imagination are neglected. Curriculum at Basic Schools was nevertheless too demanding and that is why it was reduced and reorganized by a methodical act of Ministry of Education of the CSR [56].

The actual textbooks for 2nd and 3rd form of Basic Schools [15], [4] are aimed at acquiring basic knowledge about natural number and geometrical solids in space with help of instructive set methods and in a rich mathematical programme.

The new curriculum for 2nd stage of Basic Schools [54] holds from the school year 1988/89. The main difference in comparison with the former one [51] is the rearrangement of the technical drawing in to all forms and giving enough time to practise the teaching matter. The teaching matter in geometry has been slightly increased, which corresponds with trends in theoretical and applied mathematics. Placing of the passage Volumes and Surfaces of Solids at the end of 8th year is however questionable.

The plane and the space geometry at Grammar Schools are nowadays represented mainly by affine and metric geometry, also the fundamentals of vector algebra are given. The emphasis is put on forming geometrical structures on the one hand and on calculations and parallelism of linear algebra with geometry on the other hand. Stereometry is taught in 2nd year of Grammar School and treated in [28]. The passage Geometry in Space follows the textbook M 4/2 [5] but does not use so
much inductive procedures, focuses its attention on using local deduction within the framework of certain system of knowledge. In contradiction to Basic Schools, the point, the straight line and the plane are here the fundamental concepts. The relative positions of lines and planes in space are presented without the relative position of three planes.

In the contents of the textbook are also included parallelism, orthogonality, distance, deviation, sections of angular solids, basic solids of rotation, volume and surface of solids, symmetry in space. The endeavour to supply at least partly the knowledge of descriptive geometry was the cause for inserting free parallel projection in a few lessons. One of the disadvantages of free parallel projection is the complexity of metric exercises. Less symbolic expressions render the text more understandable.

The mentioned comparison of scope and contents of geometrical teaching matter in curriculums or textbooks demonstrate that our pupils have less opportunity to develop their space imagination. From our historical survey we may see a continuous decreasing of the number of teaching periods of descriptive geometry and technical drawing until the actual state: today, technical drawing as an independent subject does not exist at all, descriptive geometry and technical drawing at Grammar Schools are taught as optional or non-compulsory subjects (the same situation is in art education at all secondary schools [3]). In mathematics itself, the proportion of geometry and stereometry has been decreasing, the number of geometrical tasks and figures with regard to the number of pages in textbooks as well. The efforts to supply the missing descriptive geometry in lessons in stereometry is falling flat. (See [3]).

The teaching geometry has an old tradition in our country. In forming an optimal school system in present and in future we must follow this positive tradition and modify it to the contemporary requirements.

- 333 -
Table 1 / number of lessons of mathematics /M/, geometry /G/, technical drawing /TD/ in curriculum of Basic Schools

<table>
<thead>
<tr>
<th>type of school</th>
<th>NS + JS</th>
<th>OSŠ</th>
<th>ZDŠ</th>
<th>ŽŠ</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st stage</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>altogether M</td>
<td>627</td>
<td>957</td>
<td>792</td>
<td>665</td>
</tr>
<tr>
<td>Ø M for one form</td>
<td>125</td>
<td>191</td>
<td>158</td>
<td>166</td>
</tr>
<tr>
<td>2nd stage</td>
<td>M G TD</td>
<td>M G TD</td>
<td>M G TD</td>
<td>M G TD</td>
</tr>
<tr>
<td>5th stage</td>
<td></td>
<td></td>
<td>175 56</td>
<td></td>
</tr>
<tr>
<td>6th stage</td>
<td>165 66</td>
<td>231 50</td>
<td>165 75</td>
<td>175 76</td>
</tr>
<tr>
<td>7th stage</td>
<td>132 66</td>
<td>198 60</td>
<td>165 75</td>
<td>175 44</td>
</tr>
<tr>
<td>8th stage</td>
<td>132 66 33</td>
<td>181 66 50</td>
<td>165 66</td>
<td>175 28 35+</td>
</tr>
<tr>
<td>9th stage</td>
<td>136 66 33</td>
<td></td>
<td>165 66 33</td>
<td></td>
</tr>
<tr>
<td>Ø for one form</td>
<td>141 83</td>
<td>203 75</td>
<td>165 79</td>
<td>175 60</td>
</tr>
<tr>
<td>/G+TD//M+TD/</td>
<td>0,52</td>
<td>0,34</td>
<td>0,48</td>
<td>0,34</td>
</tr>
</tbody>
</table>

Notation: + Technical drawing is ranged in as the component part of mathematics
Table 2

/number of lessons of mathematics /M/, geometry /G/ and stereometry /S/ in curriculum of Secondary General Schools

<table>
<thead>
<tr>
<th>type of school</th>
<th>JSS</th>
<th>SVVS branch of natural sciences</th>
<th>G</th>
<th>G</th>
</tr>
</thead>
<tbody>
<tr>
<td>curriculum from the year</td>
<td>45</td>
<td>1954</td>
<td>49</td>
<td>1968</td>
</tr>
<tr>
<td>teaching matter</td>
<td>M</td>
<td>G</td>
<td>S</td>
<td>M</td>
</tr>
<tr>
<td>9th form</td>
<td>190</td>
<td>82</td>
<td>0</td>
<td>136</td>
</tr>
<tr>
<td>10th form</td>
<td>198</td>
<td>124</td>
<td>74</td>
<td>165</td>
</tr>
<tr>
<td>11th form</td>
<td>198</td>
<td>95</td>
<td>20</td>
<td>132</td>
</tr>
<tr>
<td>12th form</td>
<td>198</td>
<td>124</td>
<td>74</td>
<td>165</td>
</tr>
<tr>
<td>13th form</td>
<td>198</td>
<td>124</td>
<td>74</td>
<td>165</td>
</tr>
<tr>
<td>altogether</td>
<td>594</td>
<td>301</td>
<td>94</td>
<td>447</td>
</tr>
<tr>
<td>Ø for one form</td>
<td>198</td>
<td>100</td>
<td>31</td>
<td>149</td>
</tr>
<tr>
<td>S / M</td>
<td>0.158</td>
<td>0.089</td>
<td>0.053</td>
<td>0.047</td>
</tr>
<tr>
<td>G / M</td>
<td>0.51</td>
<td>0.47</td>
<td>0.31</td>
<td>0.24</td>
</tr>
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</table>
Table 3

<table>
<thead>
<tr>
<th>type of school</th>
<th>JSŠ</th>
<th>ZDŠ + SVVŠ</th>
<th>ZDŠ + G</th>
<th>ZŠ + G</th>
</tr>
</thead>
<tbody>
<tr>
<td>number of stereometry lessons</td>
<td>105</td>
<td>101</td>
<td>87</td>
<td>74</td>
</tr>
</tbody>
</table>

NŠ - Primary School
JSŠ - Unified Secondary School
OSŠ - Vocational Secondary School
ZDŠ - Basic Nine Year School
ZŠ - Basic School
JSŠ - Unified Secondary School
SVVŠ - General Secondary School
G - Grammar School
SOUHRN

Z HISTORIE POSTAVENÍ GEOMETRIE VE VYUČOVÁNÍ

JOSEF MOLNÁR

V článku je v historickém přehledu porovnáno z hlediska možností rozvíjení prostorové představivosti postavení geometrie a speciálně stereometrie ve vyučování matematice i rozsah, obsah a pojetí učiva geometrického charakteru v osnovách matematiky, rýsování a deskriptivní geometrie po roce 1948 a v učebnicích matematiky po roce 1933 na všeobecně vzdělávacích školách v Československu.

V závěru je na základě provedených srovnání konstatován úbytek možností k rozvíjení prostorové představivosti žáků ve vyučování vyplývající ze snížování počtů vyučovacích hodin věnovaných výuce geometrie (včetně deskriptivní geometrie a rýsování) a změn v obsahu a pojetí školské geometrie.
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ročník, SPN, Praha 1960.
Цель преподавания геометрии представляет собой и развитие пространственного воображения школьников. В состав статьи входят анализ и сравнение разных учебников и программ математики, геометрии, черчения и начертательной геометрии с точки зрения развития пространственного воображения при обучении стереометрии. Результаты этого исследования показывают недостаточное формирование пространственного воображения в школе в настоящее время, вследствие прежде всего уменьшения времени, посвященного обучению геометрии.

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