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MODELS OF INFLATION

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A Phillips curve relating inflation and unemployment rates is a usual way how to study the development of inflation. Another approach is applied by modelling mutual influence between inflation and aggregate demand. Any observed relation among inflation and other factors should reflect the price expectations of firms and households which can be formalized by an adaptive or rational expectations hypothesis.

A stabilization program is performed by the Czech Republic government by a control of the inflation by an aggregate demand. Respecting this fact an attempt is made here to model inflation with domestic data.

1. INFLATION–UNEMPLOYMENT RELATION

The original idea of Phillips (see e.g. [8]) was based on the assumption of an inverse dependence of the rate of change of wages on the unemployment rate. In a steady state, the money wage W_t will grow at a rate that exceeds the price level P_t growth rate by the fixed amount λ which can be interpreted as a rate of technical progress. Given $\lambda = 0$ (the absence of technical progress) the money wage rate and the price level will each grow at the same rate as does the money stock. For any given λ, W_t and P_t are perfectly correlated across steady states. That is why the original relation between wage rate and unemployment rate is very similar to a most usual one between inflation rate and unemployment rate. Due to Phillips, a trade-off between steady state values of inflation rate and unemployment rate is established the existence of which brings serious consequences for the decisions of all policy makers.

Once formulated, the Phillips curve proved remarkably stable for some countries and some periods of time until the end of the 1960s. E.g. the curve plotted for inflation and unemployment in U.S.A. during the period 1961–69 is one of the best known examples of a stable Phillips curve [10]. Nevertheless, attempts made during the 1970s to predict inflation by the help of original Phillips curve were unsuccessful because other stimulant factors of an inflation rise appeared (e.g. a large expansion of money supply in many countries accompanied by the worldwide break down of fixed exchange rate).

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In fact, it is a real wage both firms and workers care about and two factors are crucial in determining the wage bargain: the rate of unemployment at the time of negotiations and the expected rate of inflation between the current year and the next year.

A conclusion was made by Friedman and Phelps (see e. g. [3]) that any observed relationship between inflation and unemployment should reflect the price expectations of firms and households; only stable expectations can imply a stable Phillips curve.

Under this assumptions and using $w_t = \log W_t$ and $p_t = \log P_t$, the real wage being W_t/P_t, the original Phillips relation should be reformulated as

$$\Delta w_t - \Delta p_t = f(un_{t-1}) \tag{1}$$

where un is for the unemployment rate. The long-term wage contracts are bargained under the exact knowledge of actual unemployment rate but with future value of inflation only expected. That is why (1) should rather be – after rearranging – reformulated as

$$\Delta w_t = f(un_{t-1}) + \Delta p_t^e \tag{2}$$

with Δp_t^e for the expected value of Δp_t.

The most important feature of (2) is that the idea of steady state trade-off between inflation and unemployment is not more valid here.

As many economists suppose the influence of expected price level to be only a partial one, this version was changed in favour of the form

$$\Delta w_t = f(un_{t-1}) + \alpha \Delta p_t^e + u_t, \quad u_t \sim N(0, \sigma_u^2) \tag{3}$$

parameter α and parameters of $f(un_{t-1})$ part to be estimated by the help of econometric methods. Besides, a substitution has to be made for the unknown Δp_t^e value.

2. EXPECTATIONS ABOUT ECONOMIC VARIABLES

The modelling of expectations is said to be one of the most important and the most difficult task of the applied econometrics [5].

There are different ways how to formulate and formalize economic expectations. Leaving apart naive static expectations, two principles became a part of standard econometric apparatus: adaptive expectations and rational expectations hypothesis.

Adaptive expectations

Adaptive expectations process involves a simple learning process in which in each time period the actual value of the variable is compared with the value that had been expected. If the actual value is greater (lower) the expected value is adjusted upwards (downwards) for the next period.

Should a value x_{t+1} be expected in period t, we believe that the size of the adjustment is proportional to the discrepancy between the actual and expected value according to the formula

$$x_{t+1}^e - x_t^e = k(x_t - x_t^e), \quad 0 \le k \le 1 \tag{4}$$

and after rearranging

$$x^e_{t+1} = kx_t + (1-k)\,x^e_t, \quad 0 \le k \le 0 \tag{5}$$

what means that the expected value the next time period is a weighted average of the expected and actual values this time period. Under assumption that (5) is valid it is also

$$x^e_t = kx_{t-1} + (1-k)\,x^e_{t-1}. \tag{6}$$

Substituting (6) into (5) we have

$$x^e_{t+1} = kx_t + k(1-k)\,x_{t-1} + (1-k)^2 x^e_{t-1} \tag{7}$$

and repeating indefinitely

$$x^e_{t+1} = k[x_t + (1-k)\,x_{t-1} + (1-k)^2 x_{t-2} + \ldots]. \tag{8}$$

The adaptive expectation process is so characterised by the help of past values of the variable in question weighted by geometrically declining weights.

Going back to the equation (2) and expressing the expected Δp^e_t by its past we have

$$\Delta w_t = f(un_{t-1}) + k[\Delta p_{t-1} + (1-k)\,\Delta p_{t-2} + (1-k)^2 \Delta p_{t-3} + \ldots] + u_t. \tag{9}$$

Due to the value of k, (9) represents a Koyck distribution form. As for the purpose of econometric estimation, it can be treated in the manner described e. g. by Hušek [7].

The adaptive expectation schema as well as the Koyck distribution one is an autoregressive process. The adaptive expectations are based on the assumption of the declining influence of the past values on a future one.

Rational expectations

The expected adjustment of future value of variable to its past development might be seen as a weak point of such a formalization as it is a pure mechanic process and other relevant information is ignored here. When expectations of economic agents formed in previous periods using all available information are assumed to have an impact on the system, a rational expectations hypothesis (REH) is formulated.

Suppose a value of a variable x in the period t be sufficiently explained by the values of y, z and x in period $t-1$ so, that

$$x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 y_{t-1} + \beta_3 z_{t-1} + u_t, \quad u_t \sim N(0, \sigma^2_u). \tag{10}$$

Knowing the estimates of parameters and the observations related to period $t-1$, we can express the expected value of x_t based on the $t-1$ period information like

$$E_{t-1}x_t = \beta_0 + \beta_1 x_{t-1} + \beta_2 y_{t-1} + \beta_3 z_{t-1} \tag{11}$$

when $E_{t-1}u_t = 0$ by

$$u_t = x_t - E_{t-1}x_t. \tag{12}$$

This is the formulation of REH hypothesis the validation of which has to be tested. Relevant tests are based on the assumption of certain relations among the variables in question. Tests known as weak, resp. strong, test can be found e.g. in Barro [2].

Another test based on the relations among the parameters can be performed as follows. Let the expected value of x_t be expressed by (10), resp. (11). Besides, suppose a variable q_t to be explained by a variable r_t and expected value $E_{t-1}x_t$ according to the formula

$$q_t = \gamma_0 + \gamma_1 r_t + \gamma_2 E_{t-1} x_t + \eta_t \tag{13}$$

with $\eta_t \sim N(0, \sigma_\eta^2)$. Substituting (11) to (13) we have

$$\begin{aligned} q_t &= \gamma_0 + \gamma_1 r_t + \gamma_2[\beta_0 + \beta_1 x_{t-1} + \beta_2 y_{t-1} + \beta_3 z_{t-1}] + \eta_t \\ &= \gamma_0 + \gamma_2\beta_0 + \gamma_1 r_t + \gamma_2\beta_1 x_{t-1} + \gamma_2\beta_2 y_{t-1} + \gamma_2\beta_3 z_{t-1} + \eta_t. \end{aligned} \tag{14}$$

(14) can also be seen as

$$q_t = \delta_0 + \delta_1 r_t + \delta_2 x_{t-1} + \delta_3 y_{t-1} + \delta_4 z_{t-1} + \eta_t \tag{15}$$

subject to constraints

$$\delta_2 = \gamma_2\beta_1, \quad \delta_3 = \gamma_2\beta_2, \quad \delta_4 = \gamma_2\beta_3. \tag{16}$$

The validation of REH so become a question of the validation of parameter constraints (16). It can be tested like in Ghosh [6] with respect to the fact that the relations among the parameters are non-linear.

Testing the validity of restrictions (16) we contemporary test the REH. Respecting the cross-equation nature of relations (16) an analogy of the likelihood ratio (LR) statistic is computed by constructing the estimated variance-covariance matrix $V1$ for (10) and (13) on the base of their estimates and similarly the matrix $V2$ for (10) and

$$q_t = \phi_0 + \phi_1 r_t + \phi_2 x_{t-1} + \phi_3 y_{t-1} + \phi_4 z_{t-1} + \varepsilon_t \tag{17}$$

which is completely free of restrictions. When the restrictions are true, the generalized variances from the restricted and the unrestricted versions should be about equal.

The estimated variance-covariance matrices of errors are

$$V1 = [\hat{u}\hat{\eta}]'\,[\hat{u}\hat{\eta}], \quad V2 = [\hat{u}\hat{\varepsilon}]'\,[\hat{u}\hat{\varepsilon}] \tag{18}$$

the determinant of each being the generalized variance of the equation system in question.

The statistic

$$\mathrm{LR} = n|\log(\det V1) - \log(\det V2)| \tag{19}$$

where n is the number of observations, is approximately distributed as $\chi^2(l)$, l being the number of restrictions tested.

3. INFLATION–AGGREGATE DEMAND RELATION

The IS–LM dynamic shows that today's level of aggregate demand not only has an effect on the current level of output and the price level, but also on future output and prices through the effects on wage negotiations. Policy makers then have several management options at their disposal. A mutual dependence between inflation and aggregate demand is described by the following Taylor's model (see e. g. [3]).

For aggregate demand one has

$$y_t = \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 (m-p)_t + \beta_4 (m-p)_{t-1} + \beta_5 E_{t-1}\pi_t + \beta_6 t + u_t \quad (20)$$

with $E_{t-1}\pi_t$ being inflation rate as expected in period $t-1$ and u_t is a disturbance term. Two lags of y involve multiplier and accelerator effects. Money balance actual and lagged variables $m-p$ affect real output, the lagged value also reflects a partial adjustment of money balance to changes of interest rate and income. As a consequence of the partial adjustment process, an absolute value of β_4 is likely to be smaller than that of β_3 and will have an opposite sign. All variables are expressed in their natural logarithms again.

The values of m_t and p_t are supposed to be predetermined in the period t. Expected inflation rate should involve transient events as are the changes in current expenses due to expected higher prices. That is why the parameter β_5 is supposed to have a plus sign. As for the time variable, it comprises long run trends of money sup^ly and aggregate demand.

The inflation equation is

$$\pi_t = \gamma_0 + \pi_{t-1} + \gamma_1 E_{t-1} y_t + v_t \quad (21)$$

with $E_{t-1}y_t$ being the value of y_t as expected in period $t-1$ and v_t is a disturbance term. The equation is formulated under the assumption that prices and wages are known because o: the existence of various contractions. The coefficient of π_{t-1} is a priori specified as unity because the firms are supposed to respect last decisions concerning prices and wages.

For the disturbance of (21) the following relation is supposed

$$v_t = \epsilon_t - \theta_2 \epsilon_{t-1} \quad (22)$$

where θ_2 part relates to transitory shocks and remaining $(1-\theta_2)$ part to influences of overlapping character of contracts.

Actual aggregate demand is explained by lagged real money balance and so the u_t error splits to an η_t-part and the lagged shock from the price equation ϵ_{t-1}. That is why

$$u_t = \eta_t - \theta_1 \epsilon_{t-1}. \quad (23)$$

The vector $e_t \equiv (\epsilon_t, \eta_t)'$ is supposed to be distributed independently and identically with zero expected value and constant nonsingular covariance matrix.

After substitution of (22) and (23) into (20) and (21) we have

$$\begin{aligned} E_{t-1}y_t &= \beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 (m-p)_t \\ &\quad + \beta_4 (m-p)_{t-1} + \beta_5 E_{t-1}\pi_t + \beta_6 t - \theta_1 \epsilon_{t-1} \end{aligned} \quad (24)$$

and

$$E_{t-1}\pi_t = \gamma_0 + \pi_{t-1} + \gamma_1 E_{t-1} y_t - \theta_2 \epsilon_{t-1} \tag{25}$$

and substituting for expected values the reduced form of the model is

$$\begin{aligned} y_t &= a[\beta_0 + \beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 (m-p)_t + \beta_4 (m-p)_{t-1} \\ &\quad + \beta_5 \pi_{t-1} + \beta_6 t + \beta_5 \gamma_0 - (\beta_5 \theta_2 + \theta_1)\epsilon_{t-1}] + \eta_t \end{aligned} \tag{26}$$

and

$$\begin{aligned} \pi_t &= a[\gamma_1(\beta_1 y_{t-1} + \beta_2 y_{t-2} + \beta_3 (m-p)_t + \beta_4 (m-p)_{t-1}) \\ &\quad + \pi_{t-1} + \gamma_1 \beta_6 t + \gamma_1 \beta_0 + \gamma_0 - (\gamma_1 \theta_1 + \theta_2)\epsilon_{t-1}] + \epsilon_t \end{aligned} \tag{27}$$

where $a \equiv 1/(1-\beta_5\gamma_1)$ and the rational assumption is

$$E_{t-1} y_t = y_t, \quad E_{t-1}\pi_t = \pi_t. \tag{28}$$

A method for estimation of (26) and (27) is chosen according to the character of disturbance terms. In case of $\theta_1 = \theta_2 = 0$ the two-stage least squares method gives consistent estimates.

4. INFLATION MODELLING WITH THE CZECH REPUBLIC DATA

The establishment of a realistic structure in the Czech economy provoked a one-time burst of inflation (about 20 % in 1990). At the same time, a stabilization program started to be performed by the government. This program could be classified as a heterodox one; its main feature is a control of the inflation by an aggregate demand. Greater one-time changes in inflation rate usually are consequences of further reform steps (e. g. tax system reform). Now, there is no serious disproportion between the inflation rate announced by the government and that measured by statisticians ex post. From the people's point of view, the inflation can be seen as anticipated.

To model relations of inflation rate to another economic variables in the Czech Republic the data representing its possible sources are: inflation rate INF, gross national product Y (measured in bil. Kč = Czech Crown), money supply M2 (bil. Kč), and foreign trade deficit S (bil. Kč). All variables were measured quarterly from the 1990 first quarter to the 1996 third quarter inclusive and their transformation to 1992 second quarter prices was done. As for the data concerning the federal Czechoslovak economy, their analogs for the Czech Republic were approximized by the Czech Statistic Bureau (ČSÚ).

The insufficient length of time-series in question makes the using of the apparatus for analysing the data generating process a little doubtful, that is why some standard procedures were not applied here. An eventual Granger causality was studied related to all possible pairs of variables using four lags. The only positive answer established that INF is Granger caused by the Y. Nevertheless, an autoregressive time-series model explaining the behavior of any variable by its own past development is not a proper tool to handle a relatively short time-series. In such a situation an appropriate theory can be of a very great importance. Unfortunately, even the theories use to fail when meeting the data collected during the period of economic transformation.

Because of a low and stable unemployment rate of 3–4% a formulation of a Phillips curve is out of the question. An output–inflation trade-off most probably seems to exist in the Czech Republic rather than a direct unemployment–inflation one. Nevertheless, the attempt to formulate and estimate an analogy to Taylor's model comprising two simultaneous equations with Y and INF being endogenous was unsuccessful, too. Performing the Hausman test, only the endogeneity of INF was proved, but not that of Y.

By formulating an econometric model the question whether to use measured values or its logarithms may arise. The reason might be to achieve a stationarity (see e. g. [9]) or to adapt the data to linear relations, the choice between logarithmic and non-logarithmic version being resolved by the help of Box–Cox test (see e. g. [5]). In case of studying the rate of a change of a variable, logarithmic form corresponds as a discrete version to the relevant continuous time concept. Unfortunately, the existing data do not respect any logic and do not match theories neither in logarithmic nor in non-logarithmic form. Further, no logarithms are used. The estimates presented here resulted from experimenting with different data and model forms and are the only not contradicting elementary statistical and economical conditions; the author is conscious of their limited validity.

Respecting the fact that the stabilization program performed by the government of the Czech Republic to control inflation is based on regulation of aggregate demand, a RLH assumption was formulated here relating the inflation rate to the expected value of GNP. With the Y_{t-1} being nonsignificant, the relation describing Y was established as

$$Y_t = \alpha_0 + \alpha_1 \mathrm{INF}_{t-1} + \alpha_2\, \mathrm{M2}_{t-1} + \alpha_3 S_{t-1} + u_t. \tag{29}$$

For the inflation respecting expectations about GNP the equation, after some experiments, was found

$$\mathrm{INF}_t = \beta_1 E_{t-1} Y_t + \beta_2 Y_{t-1} + \beta_3\, \mathrm{M2}_t + \beta_4\, \mathrm{M2}_{t-1} + v_t. \tag{30}$$

The assumption is comprised in (30) that inflation rate in period t depends on economic operations influenced by the in period $t-1$ expected GNP development in period t, e. g. $E_{t-1}Y_t$.

Should Y_t be generated by (29), it is under REH

$$E_{t-1} Y_t = \alpha_0 + \alpha_1\, \mathrm{INF}_{t-1} + \alpha_2\, \mathrm{M2}_{t-1} + \alpha_3 S_{t-1} \tag{31}$$

what can be estimated as

$$\begin{aligned} \hat{E}_{t-1} Y_t \;=\; & \underset{(17.357)}{72.640} + \underset{(0.352)}{0.889\, \mathrm{INF}_{t-1}} + \underset{(0.028)}{0.213\, \mathrm{M2}_{t-1}} + \underset{(0.156)}{0.687\, S_{t-1}} \\ & R^2 = 0.729, \quad F(3,22) = 19.716, \quad DW = 2.352 \end{aligned} \tag{32}$$

using OLS.

Then, after replacing $E_{t-1}Y_t$ by $\hat{E}_{t-1}Y_t$ in (30) we estimate

$$\begin{aligned} \mathrm{INF}_t \;=\; & -0.168\hat{E}_{t-1}Y_t \;+\; 0.211Y_{t-1} \;-\; 0.0827\,\mathrm{M2}_t \;+\; 0.075\,\mathrm{M2}_{t-1} \\ & \quad (0.074) \qquad\qquad (0.065) \qquad\quad (0.017) \qquad\quad (0.020) \\ & R^2 = 0.840, \qquad F(4,22) = 28.901, \qquad DW = 1.721. \end{aligned} \tag{33}$$

Despite of the unfavored formulation comprising actual and lagged explaining variables, the ridge regression removing eventual multicollinearity gives the same result (differences on third event. fourth decimal place) as OLS which was used here.

The REH is tested as described above by the help of LR statistic comparing the restricted (4 constraints) and unrestricted versions.

Analogically to (18) and (19) relevant covariance matrices were estimated and LR statistic formulated as

$$\log(\det V1) = 13.26, \quad \log(\det V2) = 13.23, \quad n = 26, \quad l = 4$$

$$\mathrm{LR} = 0.78 \sim \chi^2(4) < \chi^2_{\mathrm{CRIT}}(4) = 9.48$$

what enables not to reject the REH at 5 % level of significance.

According to the declarations of the government of the Czech Republic, the hypothesis was formulated that the inflation in the CR is controlled by the level of aggregate demand. The hypothesis, formalised by the help of REH, was proved here as being valid during the period from the first quarter of 1990 to the third quarter of 1996.

To tell the truth, this application of REH, though widely used in literature, has its critiques, R. Lucas jr. (Nobel Prize 1995) being the pioneer. Econometric models are often constructed for the purposes of predictions and government policy simulations. But this roles cannot be fulfilled by the model presented above.

The essential can be formalized by the help of the concept of exogeneity. The problem of exogeneity arises if we intend to analyse a subset of variables of a model given the behaviour of the remaining variables. A common idea of a variable to be exogenous if it is determined outside the system in question does not entirely correctly corresponds to models involving expectations. The expectation of a variable may be exogenous but the accuracy of the expectations is determined inside the system. As a consequence, incorrect inferences about certain parameters result.

An unambiguous concept of exogeneity (see e. g. [1]) given by the definition of weak exogeneity is relating a variable in question to the parameters of interest. Nevertheless, weak exogeneity of a variable z_t by the help of which we try to explain a variable y_t may occur as an insufficient characteristic if z_t is affected by past values of y, more exactly if z is Granger caused by y. In such a case, it is hardly possible to accept conditional predictions more than one period ahead because z_t may vary with y_{t-1}.

For the purposes of conditional prediction the strong exogeneity defined as weak exogeneity plus the absence of relevant Granger causality should be fulfilled. If an economic policy analysis is of our prior interest, it is necessary to avoid the changes in z_t to induce an instability of parameters. In such a case implementing a policy decision through the z_t variables would change the model on which that policy was based. Hence, relevant parameters of the model should be invariant to interventions like changes in financial policy, deregulations and similar governmental decisions. The matter is precised by the help of definition of super exogeneity. For exact definitions see e. g. Banerjee et al [1].

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