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Adaptive Closed Loop Control of Some Special Plants by means of the Gradient Model - without Plant Identification

JAROSLAV MARŠÍK

In some special cases, if the cascade control is applicable, it is possible to design an adaptive closed loop control circuit using the gradient model, without special test signals and plant identification. Instead of the plant model directly a part of the real plant can be used.

1. INTRODUCTION

Recently, the gradient model (or sensitivity model, respectively) found many applications in the domain of adaptive circuits. However, some difficulties with the realization can arise, if it is used in any closed loop control circuit with an unknown plant. In such cases, the plant transfer function must be identified to enable the modelling of the gradient. For this purpose, first the plant model must be found (what may be done by means of the gradient model method again). Therefore the complete system will be very complicated.

In [1] a simple device is described which solves the problem of identification and adaptive control in a closed loop simultaneously, using one model only. In some special cases, however, the identification problem can be avoided (generally, it is possible in open loop circuits only — cf. [2]).

The aim of this paper is to show when and how it may be done.

2. GENERAL PRINCIPLE OF THE ADAPTIVE CIRCUIT

The controller parameters are adjusted automatically to achieve the conditions of an optimal performance. We will take notice of the simplest performance criterion the mean square error. The error is minimized by means of the gradient method
according to equations (in the time domain):

\[
\frac{dx_i}{dt} = -\lambda_i \frac{\partial \phi}{\partial x_i} \phi^2 = -2\lambda_i \frac{\partial \phi}{\partial x_i} (ui = 1, 2, 3, \ldots),
\]

where we denote by \( x_i \) the controller parameters, by \( \phi \) the control error, and by \( \lambda_i \) the proportionality constant.

In accordance with Fig. 1 we have (in the Laplace transform notation)

\[
\phi = \frac{DP + W}{1 + PC},
\]

where: \( D \) is the input disturbance, \( P \) is the plant transfer function, \( C \) is the controller transfer function, and \( W \) is the reference signal.

The variable controller parameters \( x_i \) are regarded as constants, of course, therefore they must change rather slowly as compared with the system response (otherwise the gradient would be wrong). Then, the components of the gradient model are obtained by differentiation of the Eq. (2) with respect to the corresponding parameters \( x_i \):

\[
\frac{\partial \phi}{\partial x_i} = -\frac{DP + W}{1 + PC} \frac{\partial C}{\partial x_i},
\]

Making use of (2) one may write:

\[
\frac{\partial \phi}{\partial x_i} = -\frac{\phi P}{1 + PC} \frac{\partial C}{\partial x_i}.
\]

Eq. (4) shows that the gradient model is, in fact, the model of the control loop with the control error \( \phi \) at its input, completed by \( \partial C/\partial x_i \) at the output (see Fig. 1a).

The terms \( \partial C/\partial x_i \) need not be simulated separately, for they can be taken from the controller itself, as can readily be demonstrated on a \( P-I-D \) controller:

\[
C = x_1 + \frac{x_2}{p} + x_3 p.
\]

From (5) it follows:

\[
\frac{\partial C}{\partial x_1} = 1; \quad \frac{\partial C}{\partial x_2} = \frac{1}{p}; \quad \frac{\partial C}{\partial x_3} = p;
\]

(see also Fig. 1b).

From Fig. 1 we see that the complete adaptive control system is not too complicated if the plant model is known. (Otherwise it is not worth while- there exist simpler
methods, for example [1], [2]). As mentioned above there, are some special cases, when the identification is not necessary even if the plant model is not known. In this cases, the control system can be divided into two identical parts in a cascade, the second part performing the desirable function of the model besides the normal control action.

![Diagram](image.png)
3. THE CASCADE CONTROL

In practice, the cascade control — if applicable at all — is often the only effective method to overcome poor dynamic properties of a plant. For instance in the chemical industry, there are many cases, such as concentration or temperature control of fluids, where this way is available. As a further example can serve the steam superheater control in any power station; the superheater is divided into two or more sections in series, each of them being controlled separately.

For utilizing the cascade in the adaptive circuit the following conditions must be fulfilled:
- Two stages of the cascade must be identical (the second stage being model of the first one).
- The plant and the controller must be linear, the incidental changes of the plant parameters being sufficiently slow.
- To prevent the instability, the selfadjustment of the controller parameters must be slow, too.
- Inner disturbances of the second stage must be either negligible or uncorrelated with those of the first stage.

The general view of such an adaptive system, is shown in Fig. 2. It is easy to see that this system differs only slightly from that shown in Fig. 1.

If the reference values $W_1$ and $W_2$ were zero then Fig. 2 and Fig. 1 were identical. Nevertheless, the identity can also be achieved if those values are constant, the controller having an integrating component. Thus, the mean value of both control deviations is zero too as if $W_1$ and $W_2$ were zero.
It must be stated here that only the parameters of the first stage are adjusted to optimal values according to the given performance criterion. Due to the fact that the input signal of the second stage has another character than that of the first one, the parameters of the second stage ought to be somewhat different. Nevertheless, it does not matter — the adaptability is conserved even though not optimal for both stages.

Generally, the cascade could be extended to three or more stages, the scheme being analogous. One additional stage only for modelling the gradient is necessary. For \((k + 1)\) stages we have (see Fig. 3)

\[
\Phi = \frac{DP^k}{(1 + PC)^k}
\]

\[
\frac{\partial \Phi}{\partial x_i} = - Dk \frac{P^{k+1}}{(1 + PC)^{k+1}} \frac{\partial C}{\partial x_i}
\]

Using (7) and (8) we may write

\[
\frac{\partial \Phi}{\partial x_i} = - k \frac{P}{1 + PC} \frac{\partial C}{\partial x_i}
\]

what corresponds, in fact, to Eq. (4) (the difference consisting in the constant \(k\) only); \(k\) stages are optimized, the \((k + 1)\) stage working as the “model”.

As mentioned above, the adjustment of the parameters must be relatively slow otherwise the adaptive loops become unstable (as consequence of the incorrect gradient).
4. CONCLUSION

It has been shown that the cascade control is convenient for utilizing in a simple adaptive circuit. Neither special test signals nor plant identification are necessary.

The basic condition of realizability is the division of the plant into two identical stages, (or more), each of them being controlled by an identical controller. The last stage represents the additional model which is necessary for simulation of the performance-criterion gradient (merely one stage for an arbitrary number of preceding stages is sufficient). It must be pointed out that the gradient of the performance criterion is not valid in a rigorous mathematical sense, because the optimized controller parameters cannot be constant.

Therefore the adjustment speed of these parameters ought to be low (the more stages — the lower) for the sake of stability. It is evident that the adjustment equations are nonlinear (because of the adjusted parameters $a_i$ in the denominator of the function $\partial \phi / \partial x_i$, see Eqs. (4) and (5)). For that reason, the occurrence of ambiguous adjustments cannot be excluded and stability cannot be solved generally. As for noise influence, it has been said that the noise in the last stage must be either negligible or uncorrelated with the noise in the preceding stages.

This condition follows from Eq. (1):

$$\frac{dx_i}{dt} = - 2\lambda_i \phi \frac{\partial \phi}{\partial x_i}$$

Integration of this equation yields

$$x_i = - 2\lambda_i \int \phi \frac{\partial \phi}{\partial x} dt$$

and analogous for the mean values:

$$\bar{x}_i = - 2\lambda_i \int \bar{\phi} \frac{\partial \phi}{\partial x_1} dt .$$

The noise of the last stage is comprised in $\partial \phi / \partial x_i$, only so that the mean product $\bar{\phi}(\partial \phi / \partial x)$ does not depend on this noise (see also Fig. 3). Consequently, the steady-state values of the parameters $a_i$ are independent of the noise in the last stage.

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Adaptivní regulace některých speciálních soustav pomocí modelu gradientu — bez identifikace soustavy

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V některých speciálních případech, kdy můžeme použít kaskádové regulace, lze navrhnout adaptivní regulační obvod s modelem gradientu, a to bez zvláštního zkušebního signálu a bez identifikace soustavy. Místo modelu soustavy můžeme použít přímo části soustavy samé.

Tento způsob je možno aplikovat u soustav, které lze rozdělit na dva nebo více stejných článků zapojených za sebou, přičemž každý článek má i vlastní stejný regulátor.

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