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THE STRUCTURAL DESCRIPTIONS OF SELF-EMBEDDING NATURE OF WAVEFORM PEAKS

JIRÍ KEPKA

In the paper the structural approach to recognition of self-embedding nature of waveform peaks is proposed. POL systems are shown to be a convenient tool to generate the structural descriptions of nested peaks. The recursive analytical algorithm for obtaining such structural descriptions is described. The utilization of the proposed approach for waveform recognition is discussed.

1. INTRODUCTION

The problem of waveform analysis and recognition is an important one in pattern recognition because experimental waveforms (curves) are a typical way of representing the results of many scientific and technical experiments. The requirements of objective conclusions of the analysis of their results call for an automatic data processing. The many different mathematical techniques used to solve pattern recognition problems are grouped into two general approaches: the decision-theoretic approach and the syntactic (structural) approach.

At the beginning of any task of waveform analysis one must solve the problem how to choose suitable features, primitives and relationships. Their appropriate choice is problem dependent and first of all there is no general solution of it in the case of the decision-theoretic approach. In spite of this fact there are structural techniques general enough to be applied to most types of waveforms, e.g. [2, 5, 8].

One of the serious problems connected with the structural approach is the one of segmentation which should determine waveform primitives. Segmentation of waveforms can be as simple as a fixed-interval sampling or can be performed during structural (syntax) analysis and controlled by syntax rules. For primitive recognition template matching with error tolerance and/or decision-theoretic methods can be used then. Piecewise polynomial (often only linear) approximation is also used for waveform segmentation and following primitive recognition.

The structural approach itself is usually not sufficient and must be appropriately combined with the decision-theoretic one. The effort for removing the known drawbacks of the structural approach (especially sensitivity to noise) has resulted in the use of stochastic grammars, attributed grammars and various deformation models.

For more details see e.g. [3].

Without any controversy one of the most important tasks in waveform analysis is peak recognition because much of useful information can be obtained by locating peaks, measuring their amplitudes, durations, and determining their directions and shapes.

The well-known method for waveform peak recognition was proposed by Horowitz [5]. First, let us recall its main principle. Let a waveform be represented by a discrete set of points $\{(x_i, y_i), i = 1, \dots, n\}$ representing the analog function $y = f(x)$, with $x_i < x_{i+1}$ for all i . Denote the first difference of a waveform by $d_1, d_2, \dots, d_i, \dots, d_{n-1}$, where

$$d_i = (y_{i+1} - y_i)/(x_{i+1} - x_i)$$

corresponds to f' , the first derivative (slope) of analog function. Obviously, the first difference reverses sign (possibly through zero) if and only if a local extremum has been encountered. Assign to the i -th pair of waveform points $[(x_i, y_i), (x_{i+1}, y_{i+1})]$ the symbol ω_i encoding the slope characteristic of the line segment joining the two points in this manner:

$$\begin{aligned} \omega_i = p &\longleftrightarrow d_i > 0, \\ \omega_i = n &\longleftrightarrow d_i < 0, \\ \omega_i = 0 &\longleftrightarrow d_i = 0, \end{aligned}$$

where p denotes positive slope, n denotes negative slope and 0 denotes zero slope. Regular expressions or a finite-state language W over the alphabet $\{p, n, 0\}$ may be constructed to denote infinite sets of substrings representing positive and negative peaks.

The string $w = \omega_1 \dots \omega_i \dots \omega_{n-1}$, $w \in \{p, n, 0\}^*$, is the resultant string encoding of the waveform with respect to the first difference. The left side of a positive peak is given (in agreement with usual terminology) by the regular expression

$$L = p + p(p + 0^*)p. \quad (1)$$

Similarly, the right side of a positive peak is described by the regular expression

$$R = n + n(n + 0^*)n. \quad (2)$$

A complete positive peak is given combining (1) and (2) by

$$K = L0^*R. \quad (3)$$

Likewise, a complete negative peak is given by

$$Z = R0^*L. \quad (4)$$

Either a waveform contains either one peak or no peaks, or the peaks must form an uninterrupted alternating sequence, i.e.

$$w \in W, \quad W = 0^* \dots K0^*Z0^*K0^*Z \dots 0^*. \quad (5)$$

As Horowitz showed in [5] the deterministic context-free grammar can be constructed by means of (1)–(5), which recognizes both positive and negative peaks if any exist in a waveform represented by a string of the form (5).

Most of the structural techniques for waveform representation have been based on the use of the relationship of concatenation, which results in syntactic descriptions of the form of a string of primitives. This entirely holds about the method mentioned above, too. Consequently, the result of syntax analysis is a derivation tree that unfortunately contains no information about self-embedding nature of nested peaks, see Figure 1. Nesting of peaks is induced by sequences of valleys with increasing heights whose extents are enclosed by the extent of the next lower valley.

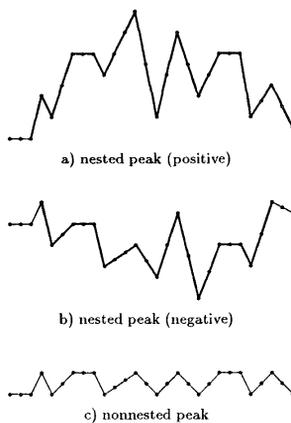


Fig. 1. Examples of waveforms with the same structural description
00pnp00npppnppnpp00ppnn.

2. RECOGNITION OF SELF-EMBEDDING NATURE OF WAVEFORM PEAKS

In the following subsection P0L systems are shown as a very convenient tool for generating the structure of nested peaks.

2.1. P0L Systems and Their Use for the Description of Self-Embedding Nature of Peaks

0L systems were introduced to describe the development of filamentous organism in which no interaction between cells takes place, i.e. what is happening to a cell depends only on the state of the cell itself, see e.g. [4].

First, basic definitions are briefly reviewed (according to usual conventions of formal language theory).

Definition 1. A 0L system is a triple $G = (V, P, \omega)$, where V is a finite, nonempty alphabet, P is a finite, nonempty set of productions of the form $a \rightarrow \alpha$; $a \in V, \alpha \in V^*$; and $\omega \in V^+$ is the axiom of G . G is said to be propagating if there is no erasing production in P of the form $a \rightarrow \lambda$ (λ is an empty symbol). Likewise, G is said to be deterministic if for every $a \in V$ there exists exactly one $\alpha \in V^*$ such that $a \rightarrow \alpha$ in P . Otherwise, G is called nondeterministic.

Definition 2. Let $G = (V, P, \omega)$ be a 0L system. The language generated by G , denoted by $L(G)$, is defined as

$$L(G) = \{x \mid \omega \xrightarrow{*} x\}.$$

Definition 3. A language L is said to be a 0L language if and only if $L = L(G)$ for some 0L system G . If G is propagating, then L is said to be a propagating 0L language or P0L language.

So far L systems have not been used in the structural approach to waveform recognition with any considerable result. In further text in this subsection representation of waveforms, especially representation of self-embedding nature of peaks, by P0L systems will be shown. To make the approach clear let us consider the following example.

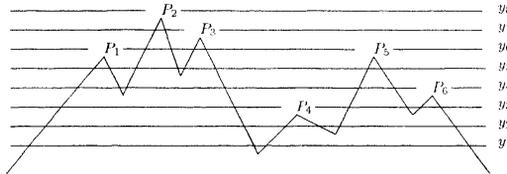


Fig. 2. The analyzed unipolar waveform.

Example 1. Consider the unipolar¹ waveform in Figure 2. Its structural description can be generated by P0L system in the following way. The unipolar waveform as a whole can be regarded as one compound peak. Mark this compound peak by the symbol, e.g. c , and examine its "behaviour" with increasing levels y_1, \dots, y_8 . On the level y_1 the split of this compound peak into two peaks is detected. This event can be expressed by the rule

$$c \rightarrow (cc), \quad (6)$$

¹The problem of unipolar and bipolar waveforms will be later discussed.

where brackets are used to describe the branching structure of peaks. On the level y_2 the split of the second compound peak into two peaks is detected, i.e. $c \rightarrow (cc)$, while the structure of the first compound peak remains without any change which can be formally expressed by the rule

$$c \rightarrow c. \quad (7)$$

The application of these two rules and of the rules

$$(\rightarrow (, \quad (8)$$

$$) \rightarrow) \quad (9)$$

leads on the level y_2 to the description $(c(cc))$. On the level y_3 the split of the third compound peak into two peaks is detected. The structure of the first peak remains without any change, while the second peak does not reach the level y_3 , which can be formally expressed by the rule

$$c \rightarrow d. \quad (10)$$

The application of the above described rules and of the rule

$$d \rightarrow d \quad (11)$$

leads to the description $(c(d(cc)))$ on the level y_3 and to the description $((cc)(d(cd)))$ on the level y_4 . Likewise, on the levels y_5 , y_6 , y_7 and y_8 the following structural descriptions

$$\begin{aligned} &((c(cc))(d(cd))), \\ &((d(cc))(d(dd))), \\ &((d(cd))(d(dd))), \\ &((d(dd))(d(dd))). \end{aligned}$$

are obtained, respectively. The last resultant description remains unchanged because only the rules

$$(\rightarrow (,) \rightarrow), \quad d \rightarrow d$$

can be used.

The structure of every unipolar waveform can be generated by P0L system G , where $V = \{c, d, (,)\}$, the axiom $\omega = c$, and the set P is formed by the rules (6) – (11). Thus, self-embedding nature of waveform peaks is described by means of brackets. For the case of splitting a compound peak into three, four or more ones on the same level the corresponding rules $c \rightarrow (ccc)$, $c \rightarrow (cccc)$, etc. can be added to the set of rules.

Bipolar waveforms may be considered to be a series of alternating unipolar segments structural descriptions of which can be generated by the P0L system given above. Alternatively, the structural description of all positive (negative) unipolar segments may be generated by P0L system G' with the set of rules P and the alphabet V described above and with the axiom $\omega' = c\dots c$, where the number of symbols in ω' corresponds to the number of positive (negative) unipolar segments.

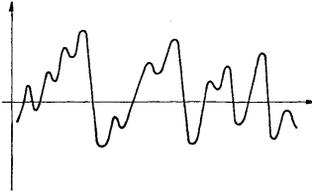


Fig. 3. The analyzed bipolar waveform.

Example 2. Consider the bipolar waveform in Figure 3. The structural description of its positive unipolar segments obtained by the proposed method is

$$d (d(dd)) (dd) (dd) d,$$

while the structural description of its negative unipolar segments is

$$d d (dd) d d (dd).$$

Both results can be combined into the resultant structural description of the bipolar waveform

$$d' dd'(d(dd))(d'd')(dd)d'(dd)d'd(d'd'),$$

where the apostrophes mark simple negative peaks.

The considerable advantage of the proposed method is the possibility of parallel implementation which results from the essence of OL systems. Thus, a structural description of a bipolar waveform can be quickly found. The additional semantic information about the amplitudes and durations of both individual and compound peaks, which is usually necessary for further analysis and interpretation, can be extracted during the search for the structural representation, too. The same holds for the levels, where the splits of compound peaks are detected.

It should be noticed here that certain qualitative information about the mutual positions of the levels, where the split was detected, results directly from the mutual positions of outside brackets and of inside ones in the structural description obtained.

Example 3. From the mutual positions of outside brackets and of inside ones in the structural description

$$((dd)d)$$

it can be easily seen that the split of the compound peak $((dd)d)$ into the left one (dd) and the right one d must forego the split of the left compound peak (dd) into two simple ones.

The described method is on the higher qualitative level than the one proposed by Horowitz. The former method, unlike the latter one, results in the structural description of self-embedding nature of waveform peaks. On the contrary, the latter method, unlike the former one, enables to intercept the finer structure of peaks, e.g. by means of the split and merge algorithm [5]. The combination of the proposed method and the method similar to Horowitz's one seems to be able to result in the very fitted structural descriptions of analyzed waveforms.

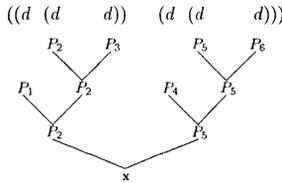


Fig. 4. The transition to the relational tree for the waveform from Figure 1.

The rather different “nongrammatical” approach to the representation of self-embedding nature of waveform peaks was proposed by Ehrich and Foith [2]. They introduced the concept of a relational tree which is a graph whose topological structure reflects structures of nested peaks. The nodes on the frontier of the tree correspond to peaks that have no further substructure and are labeled with those peaks. A parent node corresponds to a valley and is labeled with the dominant peak of the valley, i.e. the highest peak of the first descendants. Recursively, a parent node and another frontier or parent node are linked to the parent corresponding to the next lower valley. Recursion ends when no deeper valley is found [2]. In this approach only unipolar waveforms are considered. Although the authors marked their approach as nongrammatical it is not quite true, which can be easily seen from Figure 4. The transition from the structural description obtained by the proposed P0L system to the corresponding structure of the relational tree is evident.

2.2. The Analysis of Recursive Patterns – Compound Peaks

It is known that if all the pattern features which are effective for pattern analysis can be obtained at once, the classification can be reducible to a combinatorial decision making. But this approach comes to trouble for the patterns which require a lot of features for recognition, because the pattern space in such cases is not distributed densely and uniformly, but sparsely and locally and then it is better to choose the sequential approach. But there is a class of patterns which can not be analyzed well by the above two methods. They are the patterns which contain recursion, either the intrinsic recursion or the repetition, see Figure 5. Intrinsic recursion has a portion of embedded structures of arbitrary depth. For these structures analysis processes which are recursive have to be developed [9].

In the foregoing subsection it was shown that the unipolar waveform as a whole can be viewed as one compound peak. Each of compound peaks can be further formed by simpler (compound) peaks, etc. Therefore, compound peaks, i.e. their nested structures, contain intrinsic recursion. The structural description of them can be generated by the POL system given in the previous subsection. Intrinsic recursion can be immediately seen from the rule $c \rightarrow (cc)$. The check whether or not the submitted string can be generated by the POL system can be performed by syntax analysis. In practice, of course, only several types of nesting will be usually possible for each of pattern classes.

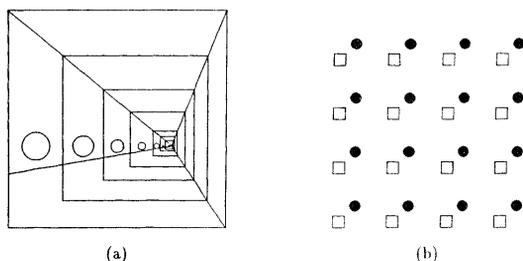


Fig. 5. Recursive patterns. (a) Intrinsic recursion. (b) Repetition.

In the set of rules there are no rules for the events of splitting a compound peak into three, four or more ones at the same level. As long as such a split exists in the analyzed waveform it is probable that it will not be detected because of digitization, noise and/or distortions. If in spite of this fact such a split (often false) is detected, let us conclude the agreement that it will be represented by means of successive application of the rule $c \rightarrow (cc)$ from left to right. That means that for example instead of the description (ccc) the description $(c(cc))$ will be obtained. The fact that there is such a split into more than two peaks can be represented by means of additional semantic information.

The following demands should be taken into consideration.

1. During the search for the structural description of the analyzed waveform semantic information about both simple peaks and compound substructures (their amplitudes, durations, etc.) should be extracted.
2. Both the structural description and semantic information should be obtained by one-pass through the data.
3. Nonrelevant peaks should be as much as possible removed from the structural description already during the search for it.

With respect to the fact that compound peaks belong to the class of recursive patterns it will be certainly advantageous to use such a programming language which

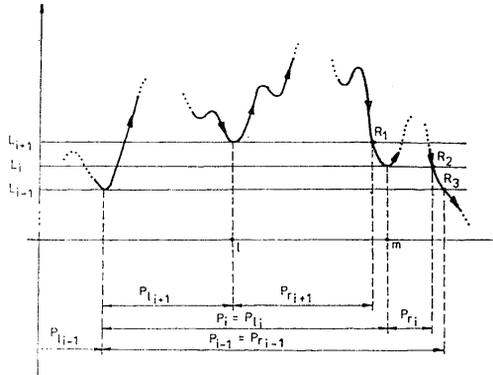


Fig. 6. The part of the analyzed waveform.

has a built in pattern recognition facility, as well as a simple and efficient way of handling recursive structures to obtain the desired structural description. That was the reason why Turbo Prolog was chosen to test the further described algorithm. Turbo Prolog is a typed Prolog compiler, which means that while it contains virtually all the features described in [1] it is much faster than interpreted Prolog [10].

To make the principle of the algorithm clear let us consider e.g. the part of the waveform in Figure 6. The recursive algorithm goes through the waveform points from left to right. Let us suppose the algorithm to be at the point $[l, L_{i+1}]$ in which the local extremum (minimum) has been encountered. That means that some compound peak P_i is splitted into the left simpler peak $P_{l_{i+1}}$ and the right simpler peak $P_{r_{i+1}}$, i.e.

$$P_i \rightarrow (P_{l_{i+1}} P_{r_{i+1}}).$$

The depth of recursion is determined by the index i . The next step of the algorithm is to find the description of $P_{r_{i+1}}$. After the course of the waveform has dropped below the level L_{i+1} (see the point R_1 in the figure) the description of $P_{r_{i+1}}$ is obtained and the one of P_i

$$P_i \leftarrow (P_{l_{i+1}} P_{r_{i+1}})$$

is completed.

At the point $[m, L_i]$ the split of some peak P_{i-1} into the left simpler peak P_{l_i} and the right simpler peak P_{r_i} occurs. While the left peak $P_{l_i} = P_i = (P_{l_{i+1}} P_{r_{i+1}})$, the description of P_{r_i} is still unknown and has to be obtained.

$$P_{i-1} \rightarrow (P_{l_i} P_{r_i}) \rightarrow ((P_{l_{i+1}} P_{r_{i+1}}) P_{r_i})$$

After the course of the waveform has dropped below the level L_i (see the point R_2) the description of P_{r_i} is obtained (P_{r_i} may be either a simple peak or a compound one) and the one of P_{i-1}

$$P_{i-1} \leftarrow (P_i, P_{r_i}) \leftarrow ((P_{i+1}, P_{r_{i+1}}), P_{r_i})$$

is completed.

After the course of the waveform has dropped below the level L_{i-1} (see the point R_3) the description of the peak $P_{r_{i-1}}$ is completed; formally $P_{r_{i-1}} = P_{i-1}$. That means that the obtained description of $P_{r_{i-1}}$ represents only the solution of one subtask at the recursion depth $i-2$.

The detailed description of the algorithm and its programming realization in Prolog can be found elsewhere, see [6]. Both the structural description of the analyzed waveform and the attributes of both simple and compound peaks are obtained by one pass through the data and at the same time the built-in mechanism of filtering removes from the structural descriptions the peaks which attributes (amplitude, duration) fail to satisfy the conditions set on them.

In the previous subsection it was mentioned that every bipolar waveform can be considered as a series of alternating unipolar segments structural descriptions of which can be computed in parallel.² The bipolar waveform is divided into unipolar segments by its baseline and hence the resultant structural description depends on it. But what will happen when the structural description is extracted with respect to a different "baseline"? There are at least two interesting possibilities. When the structural description is computed with respect to the artificially chosen baseline crossing the analyzed unipolar waveform, the task of the analysis of the unipolar waveform changes to the one of the bipolar waveform. In this case the structural descriptions obtained in parallel manner can be easily combined to get the original structural description of the analyzed unipolar waveform. In the latter case, when the structural description is determined with respect to the artificially chosen baseline passing below (above) the whole course of a bipolar waveform, then this waveform can be analyzed in the same manner as it would be unipolar.

Especially, the latter case deserves our attention. Suppose that the "baseline", which the structural description is computed with respect to, passes below (above) the whole course of a bipolar waveform. The interesting results can be obtained by the analysis of its structural description as it will be shown now. The structural descriptions of all original positive (negative) unipolar segments are kept in the structural description obtained. The left boundary of such a segment is determined by the last negative (positive) level of splitting before the positive (negative) one. The right boundary of it is determined by the first negative (positive) level of splitting which occurs after an uninterrupted sequence of positive ones immediately following the left boundary. The structural descriptions of all original negative (positive) unipolar segments are transformed because the amplitudes of negative (positive) peaks have become the levels of splitting and the negative (positive) levels of splitting have become the amplitudes of simple positive (negative) peaks. But their original descriptions can be easily extracted in the above described manner if

²Note the parallelism of OL systems.

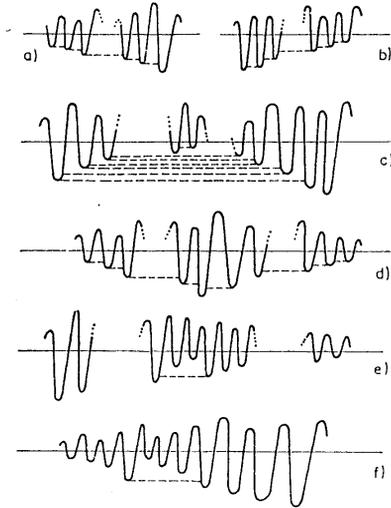


Fig. 7. Examples of analyzed waveforms.

the bipolar waveform is analyzed this time with respect to the “baseline” passing above (below) its whole course. Note that the description with respect to the “baseline” passing below and the one with respect to the “baseline” passing above the whole course of an analyzed waveform can be in principle computed also in parallel.

From the obtained description in the form of a string,³ the qualitative information whether or not the amplitudes of negative peaks increase from left to right can be easily obtained. The increase of amplitudes results in the string of the form (Fig. 7a):

$$(((...(((cc)c)...))c)c).$$

This type of nesting is due to the fact that the amplitudes of negative peaks have become the levels of splitting. Likewise, the decrease of amplitudes of negative peaks from left to right results in the string of the form (Fig. 7b):

$$(c(c(c(...(c(c(cc)))...))))).$$

³The introduction of brackets makes always possible to transform the obtained description in the form of a string into the form of a tree.

The analysis of a waveform, where first the amplitudes of negative peaks decrease and then they increase, results in the string of the form (Fig. 7c):

$$(((c((c((c(c(c...c(cc)...c))c))c))c)c).$$

Likewise, the analysis of a waveform, where first the amplitudes of negative peaks increase and then they decrease, results in the string of the form (Fig. 7d):

$$((((...(((cc)c)c)...c)c)(c(c(...(c(cc))...))))).$$

The analysis of a waveform, where the decrease of amplitudes of negative peaks from left to right is interrupted by only one increase of an amplitude of one negative peak (followed by the next decrease of amplitudes), results in the string of the form (Fig. 7e):

$$(c(c(...(c((cc)))(c(c(...(c(cc))...)))))).$$

Likewise, the analysis of a waveform, where the increase of amplitudes of negative peaks from left to right is interrupted by only one decrease of an amplitude of one negative peak (followed by the next increase of amplitudes), results in the string of the form (Fig. 7f):

$$(((((((cc)c)c)(((cc)c)c)c)c)c).$$

From such strings further information can be easily extracted. If there is a part of such a string, where only the symbols "c", "(" appear, then the amplitudes of negative (positive) peaks decrease in it. On the contrary, if there is a part of such a string where only the symbols "c", ")" appear, then the amplitudes of negative (positive) peaks increase. Likewise, the distortion, i.e. the interruption in the increase (decrease) of amplitudes of peaks by the decrease (increase) of an amplitude of one peak (followed by the next increase (decrease) of amplitudes), can be determined by finding out the sequence of symbols ")(". The certain qualitative information about to which extent the distortion exerted in the structure of the analyzed waveform can be determined from the number of left (right) brackets in their sequence immediately placed after (in front of) the sequence of symbols ")(". In the case of the distortion caused by the unexpected decrease (increase). Likewise, it is very easy to find out from the analyzed strings the compound peaks of certain type, e.g. (cc), (c(cc)), ((cc)c), etc.

The obtained structural descriptions augmented by attributes can be used for classification. It seems to be more advantageous to perform classification sequentially, step by step, on separate hierarchical levels, especially, when there are important differences between pattern classes. The classification process can go from the rough description level to the more detailed ones until only one class assignment is possible. Note, that if all compound peaks including e.g. less than five simple peaks are temporarily considered to be simple, the rougher description is easily obtained. The idea is in a very simple form illustrated in Figure 8. A pattern class as a possible result of classification can be refused by means of the criterion which evaluates both structural and semantic differences determined between the analyzed waveform and the pattern class (represented e.g. by the appropriate attributed POL system) on a current hierarchical level.

The parallel nature of transitions between adjacent hierarchical levels makes the implementation of parallel algorithms possible. This fact and the sequential nature of processing can essentially reduce the time necessary for waveform classification. For more details see [7].

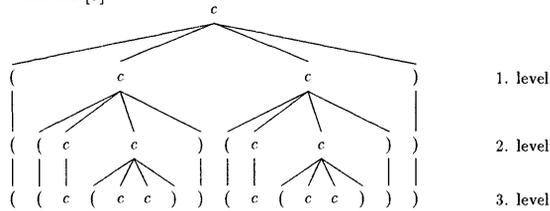


Fig. 8. The hierarchical description of the waveform from Figure 1.

3. CONCLUSIONS

In the paper the structural approach to recognition of self-embedding nature of waveform peaks is proposed. POL systems are shown to be able to describe the structure of nested peaks. They can also easily reflect the fact that every bipolar waveform may be considered as a series of alternating unipolar segments, structural descriptions of which can be computed in parallel.

The given method is on the higher qualitative level than the method proposed by Horowitz [5], which, on the contrary, can intercept the finer structure of simple peaks. This fact makes the idea of combining the proposed approach and the approach similar to the Horowitz's one very attractive because very comprehensive structural descriptions of analyzed waveforms can be obtained in this manner.

The "nongrammatical" approach introduced by Ehrich and Foith [2] and the proposed one were compared. It was found that the former approach is in principle the grammatical one and that there is the simple transition between the relational tree and the structural description generated by the corresponding POL system.

It was outlined that the compound peaks contain intrinsic recursion because each of them can be further formed by other simpler peaks. That was the reason, why Prolog was chosen to test the proposed method. The essence of the recursive algorithm which yields the desired structural descriptions augmented by attributes in one pass through the waveform data was briefly described.

The possibilities of the proposed approach for the analysis of bipolar waveforms were also treated. It was shown that every bipolar waveform can be considered to be one compound peak, positive or negative one, in dependence on whether the structural description is extracted with respect to the artificially chosen baseline either passing below or above its whole course. In this way all negative (positive - if the baseline is passing above the waveform) peaks exert on the structural descriptions, because their amplitudes become the levels of splitting.

The compound peaks of determined structure can be easily extracted from the structural description obtained. Moreover, the parts, where the amplitudes of negative (positive) peaks either increase or decrease, can be easily detected, too. The structural descriptions of all original positive (negative) unipolar segments are also kept in the structural description obtained. The detection of boundaries of these segments in the structural description can be performed by the analysis of semantic information about levels of splitting. It was also noted that the description with respect to the baseline passing below and the description with respect to the baseline passing above the course of the analyzed waveform can be computed in parallel. The comparison of these two descriptions can yield next interesting information. For example, the parts of the analyzed waveform, where amplitudes of peaks (oscillations) are either increasing or decreasing, can be easily detected.

The utilization of the proposed approach can be expected especially in the cases of waveform analysis when a priori information is lacking and as the result any specific structural model can not be constructed. The method can be used for efficient (hierarchical) data compression. The possibilities brought about by it for the solutions of the problems of waveform representation and of classification are further investigated.

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