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Determining the Transfer Functions from the Signal Flow Graphs

LUDVÍK PROUZA

Two theorems useful for determining the transfer functions from the signal flow graphs of a linear discrete system are derived.

1. INTRODUCTION

In an interesting article ([1], p. 70) Ramamoorthy has been shown a new method for computing the transfer functions in a linear discrete system. Although the basic idea of the method is clear, the derivation thereof is obscure and the results are given in a poorly applicable form being not stated explicitly. Moreover, many superfluous mutually cancelling terms appear in the resulting expression.

It is the purpose of this article to remove these inconveniences.

2. BASIC EQUATIONS OF A LINEAR DISCRETE SYSTEM

A linear discrete system is described by a system of linear first-order difference equations

$$(1)\quad \begin{aligned} x_1(t+1) &= a_{11}x_1(t) + a_{21}x_2(t) + \ldots + a_{n1}x_n(t) + b_{11}y_1(t) + \ldots + b_{m1}y_m(t) \\ x_2(t+1) &= a_{12}x_1(t) + a_{22}x_2(t) + \ldots + a_{n2}x_n(t) + b_{12}y_1(t) + \ldots + b_{m2}y_m(t), \\ &\vdots \\ x_n(t+1) &= a_{1n}x_1(t) + a_{2n}x_2(t) + \ldots + a_{nn}x_n(t) + b_{1n}y_1(t) + \ldots + b_{mn}y_m(t), \end{aligned}$$

where $x_1(t), x_2(t), \ldots, x_n(t)$ are the system variables, $y_1(t), y_2(t), \ldots, y_m(t)$ are the input variables, $t = 0, 1, 2, \ldots$ and a_{ij}, b_{lk} $(i, j, k = 1, 2, \ldots, n,\ l = 1, 2, \ldots, m)$ are constants. A unique solution of the equation system results being given the initial conditions $x_1(0), x_2(0), \ldots, x_n(0)$.

Taking the Z-transform of the system (1), one obtains the vector (matrix) equation

(2) $$X(z) = A \, . \, z^{-1} \, . \, X(z) + B \, . \, z^{-1} \, . \, Y(z) - x(0) \, .$$

Solving (2) formally, one gets with zero initial conditions (I being the unit matrix)

(3) $$X(z) = (I - A \, . \, z^{-1})^{-1} \, . \, B \, . \, z^{-1} \, . \, Y(z) \, .$$

The solution exists for z distinct from the eigenvalues of the matrix A, i.e. the roots of the characteristic equation of the system. Denoting the characteristic determinant

(4) $$|I - A \, . \, z^{-1}| = C \, ,$$

one may write more explicitly

(5) $$X(z) = \begin{bmatrix} \frac{C_{11}}{C}, \ldots, \frac{C_{1n}}{C} \\ \frac{C_{21}}{C}, \ldots, \frac{C_{2n}}{C} \\ \ldots\ldots\ldots\ldots \\ \frac{C_{n1}}{C}, \ldots, \frac{C_{nn}}{C} \end{bmatrix} . \, z^{-1} \, . \, B \, . \, Y(z) \, .$$

Now, one is interested in the relation between $X_i(z)$ and $Y_k(z)$, the remaining $Y_j(z) = 0$ for $j \neq k$.

From (5)

(6) $$\frac{X_i(z)}{Y_k(z)} = \frac{z^{-1}}{C} \, . \, (C_{i1} b_{k1} + \ldots + C_{in} b_{kn})$$

follows and this is the desired transfer function from $Y_k(z)$ to $X_i(z)$.

Thus, one needs compute C and the cofactors thereof.

3. THE COMPUTATION OF C AND C_{ij} FROM THE GRAPHS

In Fig. 1, one sees the construction of the signal flow graph to the matrix C (for a system of the third order, the graph is complete). The determinant C is given by the first Mason rule

(7) $$C = 1 - \sum P_1 + \sum P_2 + \ldots + (-1)^n \sum P_n$$

where $\sum P_j$ is the sum of products of the branch transfers corresponding to the j-tuples of non-touching loops.

The cofactor C_{ij} may be computed by the second Mason rule.
But,

$$
(8) \qquad C_{ij} = \begin{vmatrix} & O_i & \\ & \vdots & \\ & O_i & \\ O_j, \ldots, O_j, & (-1)^{i+j}, & O_j, \ldots, O_j \\ & O_i & \\ & \vdots & \\ & O_i & \end{vmatrix}
$$

where the row j and the column i are explicitly given, the elements O_j being all 0 and the elements O_i being arbitrary or contrarily O_i being 0 and O_j being arbitrary. All other elements of (8) are the same as in (4).

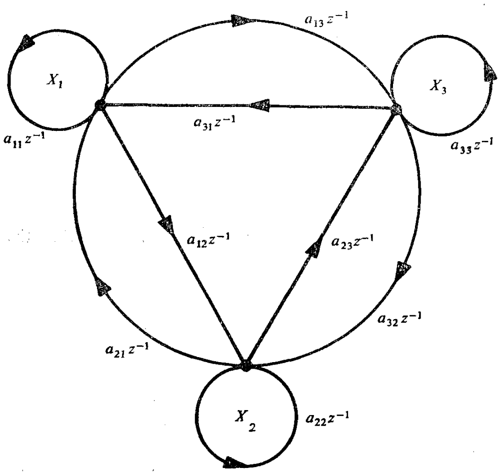

Fig. 1.

Thus, there is no unique graph corresponding to (8). Ramamoorthy chooses that with minimum number of changes compared with the graph for C. But, it is better to choose that with the maximum number of suppressed branches. Comparing (8) with (4) one will distinguish two cases: C_1) $i \neq j$, C_2) $i = j$.

Case C_1): This is the same as to put in C: $a_{ij} = z \, . \, (-1)^{i+j+1}$, $a_{ii} = z$ or $a_{jj} = z$ (but not both) and all remaining $a_{kj} = 0$ and $a_{ik} = 0$.

Case C_2): This is the same as to put in C all $a_{ki} = 0$ and all $a_{ik} = 0$, $k \neq i$.

Thus, following two theorems result.

Theorem 1: *Let $i \neq j$. To compute C_{ij}, suppress in the graph for C all branches ending in X_j and all branches beginning in X_i, with exception of the branch from X_i to X_j, which has now the transfer $(-1)^{i+j+1}$, and of the self-loop from X_i to X_i or from X_j to X_j (but not both), which has now the transfer 1. Then, use the first Mason rule.*

Theorem 2: *Let $i = j$. To compute C_{ii}, suppress in the graph for C all branches ending and beginning in X_i. Then, use the first Mason rule.*

4. ILLUSTRATIVE EXAMPLES

To compute C_{13}, one constructs from the graph in Fig. 1 with the aid of Theorem 1 the graph in Fig. 2. The first Mason rule gives

(9)
$$C_{13} = 1 - (1 + a_{22}z^{-1} - a_{21}a_{32}z^{-2} - a_{31}z^{-1}) + \\ + (a_{22}z^{-1} - a_{31}a_{22}z^{-2}) = a_{31}z^{-1} + a_{21}a_{32}z^{-2} - a_{31}a_{22}z^{-2}\,.$$

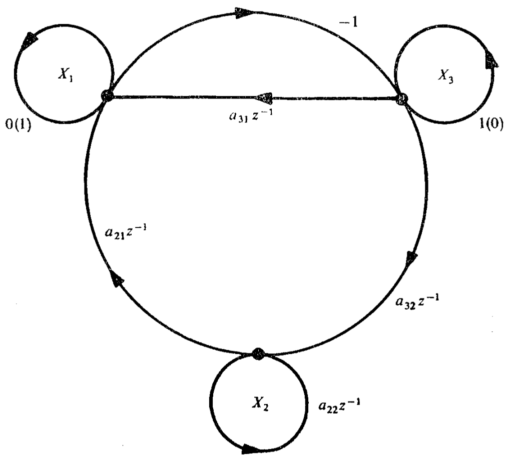

Fig. 2.

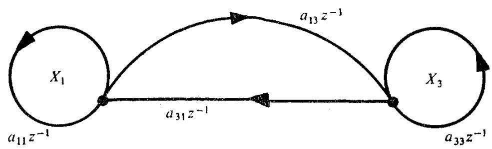

Fig. 3.

To compute C_{22}, one constructs from the graph in Fig. 1 with the aid of Theorem 2 the graph in Fig. 3. The first Mason rule gives

$$C_{22} = 1 - (a_{11}z^{-1} + a_{33}z^{-1} + a_{31}a_{13}z^{-2}) + a_{11}a_{33}z^{-2} . \tag{10}$$

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[1] Ramamoorthy, C. V.: Discrete system representation and analysis by generating functions of abstract graphs. IEEE Int. Conv. Rec. (1965), Pt 6, 68—77.

VÝTAH

Určení přenosových funkcí z grafů signálových toků

LUDVÍK PROUZA

V článku [1] odvodil Ramamoorthy novou metodu pro výpočet přenosových funkcí v lineárním diskrétním systému. Metoda není popsána explicitním návodem a kromě toho při její aplikaci vzniká řada zbytečných vzájemně se rušících sčítanců ve výsledné formuli. V tomto článku se odvozují dvě věty, které odstraňují uvedené nevýhody.

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