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SELFTUNING LQ CONTROLLERS WITH PRESPECIFIED STATE

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A design of simple selftuning LQ controllers (e.g. equivalent to a digital PID regulator) is proposed, based on a more complex model than corresponds to a prespecified controller structure. The core of the approach consists in the optimal transformation of the model to the form needed for the optimal design of a controller working on the prespecified part of a measurable system state. The standard selftuning setting is assumed, i.e. recursively gained point estimates substitute unknown parameters. In this way an attempt is done to make up for mismodelling errors and to gain a basis for comparing the performance of different strategies for simple digital control. The presented specialization to LQG problems with a multistep criterion is proposed for practical use in fixed controller adjustment.

1. INTRODUCTION

The power and price of hardware in conjunction with the need for more sophisticated controllers have stimulated growing interest in industrial application of adaptive controllers. The continuity of development naturally results in attempts to design three-term adaptive controllers.

A variety of approaches have been tried. A lot of them are based on explicit identification of a simple model combined with some control design through the enforced separation. As a typical example simple LQ-selftuners of Böhm and coworkers [1] can be taken. The limiting factor for this plausible way is mismodelling. Due to it instability may occur even for systems which can be successfully controlled by an ordinary three-term controller. The danger caused by mismodelling is now intensively studied, see e.g. [2]. Experience as well as theoretical analysis in the vein of [2] have shown that the problem can be resolved by a sufficiently rare sampling. This safe way is, however, paid (sometimes substantially) by the control quality achieved.

Our aim when designing the adaptive controller described in the paper has been two-fold:

– to propose reference algorithm for robustness studies of different simple adaptive controllers,
– to obtain the tool for automatic commission of standard three-term controllers.

To this purpose suits the following straightforward formulation of the low order controller design: the objective function is optimized under the restriction that the optimal controller has to work on a prespecified state (for related idea see [3]).

In contrast with former solutions, the system model used in our approach is obtained by the optimal reduction of a more complex model which is believed to fit system input – output behaviour. The complex model itself is gained within the standard selftuning setting, i.e. recursively evaluated point estimates substitute unknown parameters. The form of the controller design, which formally coincides with dynamic programming, implies that any suboptimal design described in [4] can be used with the model reduction proposed.

2. CONTROL DESIGN WITH PRESPECIFIED STATE

Let the quality of the closed-loop behaviour of the system with the finite-dimensional measurable state $x(t)$ and the system input $u(t)$ be evaluated through the additive criterion

$$K(1..N) = \frac{1}{N} \mathsf{E} \sum_{t=1}^{N} q(a(t), u(t)) \tag{1}$$

where $\mathsf{E}(\cdot)$ denotes expectation, $q(\cdot, \cdot)$ a non-negative loss function, N a (finite) horizon and $a(t)$ is a fixed part of the state $x(t)$. Without loss of generality, we can assume that

$$x(t) = \begin{bmatrix} a(t) \\ b(t) \end{bmatrix} \begin{matrix} \} \, i_a \\ \} \, i_d \end{matrix} \tag{2}$$

In the sequel we shall adopt the following notation:

$p(a \mid b)$ = conditional probability density function (abbreviated c.p.d.f.) of a random variable a (at the point a) conditioned on b;

$\mathsf{E}(a \mid b)$ = conditional expectation corresponding to c.p.d.f. $p(a \mid b)$;

$x(k..l) = (x(k), x(k+1), .., x(l))$ for $l \geqq k$.

The criterion (1) has to be minimized in the class of nonanticipative randomized controllers under the restriction

$$p(u(t) \mid x(1..t-1), u(1..t-1)) = p(u(t) \mid a(t-1)) \tag{3}$$

The requirement (3) expresses the specificity of the formulated problem.

The definition of state implies that

$$p(x(t) \mid x(1..t-1), u(1..t)) = p(x(t) \mid x(t-1), u(t)) \tag{4}$$

The system description (4) is assumed to be known. A possible imbedding of the results into selftuning context will be commented in the next section.

The following proposition justifies the application of dynamic programming even under the restriction (3), i.e. even when controllers cannot use the full state. Notice that a usual filtering is not permitted; past data $a(1\,.\,.\,t-2)$ are not included in the description of the admissible controllers.

Proposition 1. Optimal controller minimizing the criterion (1) under the restriction (3) can be chosen as deterministic feedback. Optimal inputs are minimizing arguments in the sequence of tasks

$$K^*(a(t-1)) = \min_{u(t)} \mathsf{E}(q(a(t), u(t)) + K^*(a(t)) \mid a(t-1), u(t)) \tag{6}$$

$$t = N, N-1, \ldots, 1\,; \quad K^*(a(N)) = 0$$

The minimum achieved is

$$K^*(1\,.\,.N) = \frac{1}{N}\,\mathsf{E}(K^*(a(0))) \tag{7}$$

Proof. Apparently,

$$N\,K(1\,.\,.N) = \mathsf{E}\sum_{t=1}^{N} q(a(t), u(t)) = \tag{8}$$

$$= \sum_{t=1}^{N} \iint q(a(t), u(t))\, p(a(t), u(t))\, \mathrm{d}a(t)\, \mathrm{d}u(t)$$

The p.d.f. $p(x(1\,.\,.\,t-1), u(1\,.\,.\,t-1))$ is fully determined by the model (4), $p(x(0))$ and $\{p(u(\tau) \mid a(\tau-1))\}_{\tau=1}^{t-1}$, thus its marginal p.d.f.'s $\{p(a(\tau), u(\tau))\}_{\tau=1}^{t-1}$ are not influenced by the rest of the control strategy.
Thus it holds

$$\min_{1\ldots N} N\,K(1\,.\,.N) = \min_{1\ldots t-1} \Big(\sum_{\tau=1}^{t-1} \mathsf{E}(q(a(\tau), u(\tau))) + \tag{9}$$

$$+ \min_{t\ldots N} \mathsf{E}\Big(\sum_{\tau=t}^{N} q(a(\tau), u(\tau))\Big)\Big)$$

where $\min_{j\ldots k}(\cdot)$ stands for minimization over $\{p(u(t), a(t-1))\}_{t=j}^{k}$.

For $t = N$ we have to minimize the second term of (9)

$$\mathsf{E}(q(a(N), u(N))) = \iint q(a(N), u(N))\, p(a(N), u(N))\, \mathrm{d}a(N)\, \mathrm{d}u(N) = \tag{10}$$

$$= \iiint q(a(N), u(N))\, p(a(N) \mid a(N-1), u(N))\, p(u(N) \mid a(N-1))\,.$$

$$.\, p(a(N-1))\, \mathrm{d}a(N-1\,.\,.N)\, \mathrm{d}u(N)\,.$$

Notice that the p.d.f. $p(a(N-1))$ is the marginal p.d.f. of $p(x(1\,.\,.N-1), u(1\,.\,.\,.N-1))$, thus, it is not influenced by the choice of $p(u(N) \mid a(N-1))$. If we define

$$K^*(a(N-1)) = \min_{u(N)} \int q(a(N), u(N))\, p(a(N) \mid a(N-1), u(N))\, \mathrm{d}a(N)\,, \tag{11}$$

then for minimizing $u^*(N)$ and any admissible controller $p(u(N) \mid a(N-1))$ we have

$$(12) \qquad K^*(a(N-1)) = \int K^*(a(N-1))\, p(u(N) \mid a(N-1))\, \mathrm{d}u(N) \leqq$$

$$\leqq \iint q(a(N), u(N))\, p(a(N) \mid a(N-1), u(N))\, p(u(N) \mid a(N-1))\, \mathrm{d}a(N)\, \mathrm{d}u(N)$$

However, for a controller with $p(u(N) \mid a(N-1))$ concentrated on $u^*(N)$ the lower bound in (12) is achieved. Multiplication of (12) by a nonnegative $p(a(N-1))$ and integration over $a(N-1)$ imply that

$$(13) \qquad \min_{p(u(N) \mid a(N-1))} \mathsf{E}(q(a(N), u(N))) = \mathsf{E}(K^*(a(N-1))) =$$

$$= \int K^*(a(N-1))\, p(a(N-1))\, \mathrm{d}a(N-1) =$$

$$= \iint K^*(a(N-1))\, p(a(N-1), u(N-1))\, \mathrm{d}a(N-1)\, \mathrm{d}u(N-1).$$

Adding $K^*(a(N-1))$ to $q(a(N-1), u(N-1))$ we are, for $t = N - 1$, in the same situation as for $t = N$ (with the sum $K^*(a(N-1)) + q(a(N-1), u(n-1))$ playing the role of $q(a(N), u(N))$). The procedure can be repeated up to time $t = 1$. The starting step coincides with the rest for $K^*(a(N)) = 0$. □

The next proposition links the full state model (4) with the model required in dynamic programming (6) and for a set of controllers to which the optimal one belongs.

Proposition 2. For a deterministic-feedback admissible controller (3), the full and the partial state models $p(x(t) \mid x(t-1), u(t))$ and $p(a(t) \mid a(t-1), u(t))$ are related by

$$(14) \qquad p(a(t) \mid a(t-1), u(t)) =$$

$$= \iint p(x(t) \mid x(t-1), u(t))\, p(b(t-1) \mid a(t-1), u(t))\, \mathrm{d}b(t-1..t)$$

where for controllers (3) it holds

$$(15) \qquad p(b(t-1) \mid a(t-1), u(t)) = p(b(t-1) \mid a(t-1)) =$$

$$= p(x(t-1)/\int p(x(t-1))\, \mathrm{d}b(t-1)$$

and the p.d.f. $p(x(t))$ evolves according to the recursion

$$(16) \qquad p(x(t)) = \int p(x(t) \mid x(t-1), u(t))\, p(x(t-1))\, \mathrm{d}x(t-1)$$

$p(x(0))$ known.

Proof. Eq. (14), (15), (16) are direct consequences of the standard rules for c.p.d.f.'s and of the form of feedback. □

3. APPLICATION TO THE LQG PROBLEMS

The required state space description with measurable state is assumed to result from recursive estimation of the linear normal regression model, see [5] for details. The completeness of the state can be (practically) guaranteed by using the theory of model comparison described in [5] and elaborated to algorithmic details in [6]. Feasibility of the resulting selftuning controller is reached by substituting point parameter estimates for unknown parameters of the system model. Consequently, the theory is elaborated for the case of the normal regression model with known parameters. The implications of a state transformation needed for the decomposition (2) will form the only link to unknown parameter case.

Let the system with the m_y-dimensional output $y(t)$ and m_u-dimensional input $u(t)$ be described by the linear normal regression model of the "order" l

$$p(y(t) \mid y(1\,..\,t-1), u(1\,..\,t)) = \mathrm{N}(y(t) \mid \tilde{P}'\,\tilde{z}(t), RR') \tag{17}$$

where $\mathrm{N}(y \mid \hat{y}, S)$ denotes the normal p.d.f. of y with mean $\hat{y}$ and covariance matrix. S. The assumed regression vector $z(t)$ is of the form

$$\tilde{z}(t) = \begin{bmatrix} u(t) \\ \hline \tilde{x}(t-1) \end{bmatrix} = \left.\begin{bmatrix} u(t) \\ \hline y(t-1) \\ u(t-1) \\ \cdot \\ y(t-l) \\ u(t-l) \\ 1 \end{bmatrix}\right\} i_z = m_u + 1 + l(m_u + m_y) \tag{18}$$

the matrix $\tilde{P}$ contains corresponding regression coefficients and R is the Cholesky square root of covariance matrix (lower triangular). The system is Markovian with the state $\tilde{x}(t-1)$ and the transition p.d.f.

$$p(\tilde{x}(t) \mid \tilde{x}(t-1), u(t)) = \mathrm{N}\left(\tilde{x}(t) \mid \tilde{A}\, z(t)\,,\ \begin{bmatrix} RR' & 0 \\ 0 & 0 \end{bmatrix}\right) \tag{19}$$

with

$$\tilde{A} = \begin{bmatrix} \tilde{P}' \\ \hline 1\ 0 \ldots 0\ \ 0\ 0 \ldots 0\ 0 \\ 0\ 1 \ldots 0\ \ 0\ 0 \ldots 0\ 0 \\ \cdot\ \cdots\cdots\ \ \cdot\ \cdots\cdots\ \cdot \\ 0\ 0 \ldots 1\ \ 0\ 0 \ldots 0\ 0 \\ 0\ 0 \ldots 0\ \ 0\ 0 \ldots 0\ 1 \end{bmatrix} \begin{matrix} \}\ m_y \\ \\ \left.\begin{matrix} \\ \\ \\ \\ \end{matrix}\right\} (l-1)(m_y + m_u) + m_u \\ \\ \}\ 1 \end{matrix} \ \Bigg\}\ i_z - m_u \tag{20}$$

$$\underbrace{\qquad\qquad}_{m_u + (l-1)(m_u + m_y)}\ \underbrace{\qquad\quad}_{m_u + m_y}\ \underbrace{\ }_{1}$$

The state $x(t)$ is assumed to be such a linear regular transformation of the state

$\tilde{x}(t)$ (18) that $y(t)$ forms the first m_y entries of $x(t)$, i.e.

$$x(t) = \underbrace{\begin{bmatrix} I & 0 \\ 0 & F \end{bmatrix}}_{m_y} \tilde{x}(t) \tag{21}$$

Thus for

$$z(t) = \begin{bmatrix} u(t) \\ x(t-1) \end{bmatrix} \tag{22}$$

we have

$$p(x(t) \mid x(t-1), u(t)) = \mathrm{N}\left(x(t) \mid A\, z(t), \begin{bmatrix} RR' & 0 \\ 0 & 0 \end{bmatrix} \right) \tag{23}$$

with

$$A = \underbrace{\begin{bmatrix} P' \\ \hline D \end{bmatrix}}_{i_z} \begin{matrix} \} \, m_y \\ \} \, i_z - m_y - m_u \end{matrix} \tag{24}$$

$$D = F \begin{bmatrix} I & 0 \ldots 0 & 0 \\ \text{-------} & \ldots\ldots & \vdots \\ \underbrace{0 \ldots 0}_{m_u + (l-1)(m_y + m_u)} & \underbrace{0 \ldots 0}_{m_y + m_u} & \underbrace{1}_{1} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & F^{-1} \end{bmatrix} \} \, m_y + m_u \tag{25}$$

where P is the appropriate linear function of $\tilde{P}$. The above simple algebra demonstrates that for any transformation (21) parameters P of the regression model $N(y \mid P' z(t), RR')$ can be identified and used for construction of a Markov model (23) according to (24), (25). The matrix D (25) can be computed beforehand.

We shall restrict ourselves to the quadratic form of the criterion (1), i.e.

$$q(a(t), u(t)) = a'(t)\, Q_a(t)\, a(t) + u'(t)\, Q_u(t)\, u(t) = z_a'(t)\, Q(t)\, z_a(t) \tag{26}$$

with

$$Q_a(t) \geqq 0\,, \quad Q_u(t) \geqq 0\,, \quad Q(t) = \operatorname{diag}\left(Q_u(t), A_a(t)\right) \tag{27}$$

The kernel $Q(t)$ is time-dependent in order to cover presence of the stabilizing term in criterion (1), i.e. usually

$$Q(N) = Q_0 > 0\,, \quad Q(t) = Q \geqq 0 \quad \text{for} \quad t < N\,. \tag{28}$$

By the choice (27) the regulation problem is formulated. It can be extended to other to the control problems in a straightforward manner. Such extensions are not pursued here to focus attention to the main topic of the paper. Also penalization of $u(t)$ is used for the same reasons; the technically more sound penalization of $\Delta u(t) = u(t) - u(t-1)$ can be achieved by a suitable definition of $x(t)$ with $\Delta u(t)$ in the role of $u(t)$.

The following Lemma needed in the sequel is well-known, but not in the presented Cholesky-square-root version which suits well to obtain numerically stable implementation.

Lemma. For

$$p\left(\begin{bmatrix} c \\ d \end{bmatrix}\right) = \mathrm{N}\left(\begin{bmatrix} c \\ d \end{bmatrix} \middle| \begin{bmatrix} \bar{c} \\ \bar{d} \end{bmatrix}, \begin{bmatrix} G_c & 0 \\ G_{dc} & G_d \end{bmatrix} \begin{bmatrix} G'_c & G'_{dc} \\ 0 & G'_d \end{bmatrix}\right) \tag{29}$$

it holds

$$p(d \mid c) = \mathrm{N}(d \mid \bar{d} + G_{dc} G_c^{-1}(c - \bar{c}), \quad G_d G'_d) \tag{30}$$

$$p(c) = \mathrm{N}(c \mid \bar{c}, \; G_c G'_c) \tag{31}$$

Proof. It is simple and, therefore, omitted. □

Assuming initial conditions $y(t)$, $u(t)$ for $t < 1$ normally distributed, the state $x(0)$ is normally distributed. With the state decomposed according (2), the suboptimal controllers used will be of the linear form

$$u(t) = -L'_a(t)\, a(t-1) = -[\underbrace{L'_a(t)}_{i_a}, 0]\, x(t-1) = -L'(t)\, x(t-1) \tag{32}$$

This implies normality of p.d.f.s (14), (15), (16) thus only the evolution of their first and second moments is required. For this we shall denote

$$\mathsf{E}(c(t)) = \bar{c}(t) \quad \text{for} \quad c = u, x, b \dots \tag{33}$$

$$\operatorname{cov}(x(t)) = G(t)\, G'(t) = \begin{matrix} i_a \{ \\ i_b \{ \end{matrix} \begin{bmatrix} G_a(t) & 0 \\ \underbrace{G_{ba}(t)}_{i_a} & \underbrace{G_b(t)}_{i_b} \end{bmatrix} G'(t) \tag{34}$$

$$A = \underbrace{\begin{bmatrix} A_a \\ A_b \end{bmatrix}}_{i_z} \begin{matrix} \} \, i_a, \\ \} \, i_b, \end{matrix} \quad A_a = [\underbrace{A_{aa}}_{i_a}, \underbrace{A_{ab}}_{i_b}] \} \, i_a \tag{35}$$

Apparently,

$$\bar{x}(t) = A \begin{bmatrix} -L'(t) \\ I \end{bmatrix} \bar{x}(t-1) \tag{36}$$

$$G(t) = \left[A \begin{bmatrix} -L'(t) \\ I \end{bmatrix} G(t-1) \,\middle|\, \begin{matrix} R \\ 0 \end{matrix} \right]^{\mathrm{Ch}} \tag{37}$$

where $[\cdot]^{\mathrm{Ch}}$ denotes "the operation" of taking the Cholesky square root of the matrix product $[\cdot]\,[\cdot]'$. The straightforward application of the above Lemma to

$$p\left(\begin{bmatrix} a(t) \\ b(t) \end{bmatrix} \middle| x(t-1), u(t)\right) = \mathrm{N}\left(\begin{bmatrix} a(t) \\ b(t) \end{bmatrix} \middle| \begin{bmatrix} A_a\, z(t) \\ A_b\, z(t) \end{bmatrix}, \begin{bmatrix} \underbrace{R}_{m_y} & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} R' & 0 & 0 \\ 0 & 0 & 0 \\ \underbrace{0}_{i_a} & & \underbrace{0}_{i_b} \end{bmatrix}\right) \tag{38}$$

gives

$$(39) \qquad p(a(t) \mid x(t-1), u(t)) = \mathrm{N}\left((a(t) \mid A_a z(t), \underbrace{\begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix}}_{m_y} \underbrace{\begin{bmatrix} R' & 0 \\ 0 & 0 \end{bmatrix}}_{i_a}\right)$$

Similarly for $p\left(\begin{bmatrix} a(t-1) \\ b(t-1) \end{bmatrix}\right)$ we have

$$(40) \qquad p(b(t-1) \mid a(t-1)) =$$

$$= \mathrm{N}(b(t-1) \mid \bar{b}(t-1) + G_{ba}(t-1)\, G_a^{-1}(t-1)\,(a(t-1) - \bar{a}(t-1)),$$

$$G_b(t-1)\, G_b'(t-1))$$

Theorem. Let the system be described by the c.p.d.f. (23), the initial state be normally distributed and linear state controllers of the form (32) be used up to time $t-1$. Then the system model needed for minimizing the criterion (1) in the class of controllers (3), (2) takes the form

$$(41) \qquad p(a(t) \mid a(t-1), u(t)) =$$

$$= \mathrm{N}(a(t) \mid A_a \begin{bmatrix} u(t) \\ a(t-1) \\ b(t-1) + G_{ba}(t-1)^{-1}\, G_a^{-1}(t-1)\,(a(t-1) - \bar{a}(t-1)) \end{bmatrix},$$

$$A_{ab}\, G_b(t-1)\, G_b'(t-1)\, A_{ab}' + \underbrace{\begin{bmatrix} R \\ 0 \end{bmatrix}}_{m_y} \begin{bmatrix} R \\ 0 \end{bmatrix}' \Big)$$

Proof. Again the normality of c.p.d.f. $p(a(t) \mid a(t-1), u(t))$ can be directly verified thus only the first two moments are needed:

$$(42) \qquad \mathsf{E}(a(t) \mid a(t-1).\, u(t)) = \mathsf{E}(\mathsf{E}(a(t) \mid x(t-1), u(t)) \mid a(t-1), u(t)) =$$

$$= A_a \begin{bmatrix} u(t) \\ a(t-1) \\ \mathsf{E}(b(t-1) \mid a(t-1), u(t)) \end{bmatrix} =$$

$$= A_a \begin{bmatrix} u(t) \\ a(t-1) \\ \bar{b}(t-1) + G_{ba}(t-1)^{-1}\, G_a^{-1}(t-1)\,(a(t-1) - \bar{a}(t-1)) \end{bmatrix}$$

To compute the (conditional) covariance matrix, we shall evaluate the innovation $\delta(t) = a(t) - \mathsf{E}(a(t) \mid a(t-1), u(t))$, using (42) and the assumption that $u(t)$ is fully determined by $a(t-1)$,

(43)

$$\delta(t) = A_a \begin{bmatrix} 0 \\ 0 \\ b(t-1) - \mathsf{E}(b(t-1) \mid a(t-1), u(t)) \end{bmatrix} + a(t) - \mathsf{E}(a(t) \mid x(t-1, u(t))$$

(notice different conditions!). Relations (23), (40), (43), imply

$$(44) \qquad \mathsf{E}(\delta(t)\,\delta'(t) \mid a(t-1), u(t)) =$$

$$= A_{ab} \operatorname{cov}(b(t-1) \mid a(t-1))\, A'_{ab} + \begin{bmatrix} R & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} R' & 0 \\ 0 & 0 \end{bmatrix} +$$

$$+ A_{ab}\, \mathsf{E}((b(t-1) - \mathsf{E}(b(t-1)) \mid a(t-1), u(t)))\,.$$

$$.\,(a(t) - \mathsf{E}(a(t) \mid x(t-1), u(t)))' \mid a(t-1), u(t)) +$$

$$+ \text{ transposition of the foregoing term}$$

The last two terms in (44) equal zero as can be seen by using $\mathsf{E}(\cdot \mid a(t-1), u(t)) = \mathsf{E}(\mathsf{E}(\cdot \mid x(t-1), u(t)) \mid a(t-1), u(t))$ and by taking into account that $b(t-1)$ is a part of $x(t-1)$. □

4. ILLUSTRATIVE EXAMPLE

The optimal reduction of the full-state model described by the presented theorem has been coded and included into the subroutine library SIC [4]. The operation under the system KOS [4] admits (in interactive mode) to combine the resulting model with any suboptimal control strategy described in [4]. The simulation experience is encouraging and supports expectation related to the presented theory. As an illustrative example we have chosen a system described by Warwick in [3] which is given by third order ARMAX model

$$y(t) = u(t) - 1{\cdot}46\, u(t-1) + 0{\cdot}81\, u(t-2) + 2{\cdot}28\, y(t-1) - 1{\cdot}929\, y(t-2) + $$
$$+ 0{\cdot}558\, y(t-3) + 0{\cdot}1\, e(t) + 0{\cdot}0637\, e(t-1)$$

driven by white normal noise with the p.d.f. $\mathrm{N}(e(t) \mid 0, 1)$.

The table presented below shows values of quality criteria defined as sampling output variances with and without transient stage

$$k(j\,..\,200) = \frac{1}{200 - j + 1} \sum_{t=j}^{200} y^2(t) \quad \text{for} \quad j = 1{,}101$$

The following versions are compared in Table 1: R_0 – no control, R_1 – LQ adaptive

Table 1. Comparison of quality criteria for different controllers.

	$k(1\,..\,200)$	$k(101\,..\,200)$
R_1	$4{\cdot}376\,.\,10^{-2}$	$0{\cdot}985\,.\,10^{-2}$
R_2	$7{\cdot}397\,.\,10^{-2}$	$4{\cdot}242\,.\,10^{-2}$
R_3	$5{\cdot}451\,.\,10^{-2}$	$1{\cdot}754\,.\,10^{-2}$
R_0	$133{\cdot}909\,.\,10^{-2}$	$90{\cdot}610\,.\,10^{-2}$

controller based on the regression model of the third order with a certainty equivalence version of the IST strategy [4] R_2 – differing from R_1 by using the first order regression model and R_3 by using the optimally reduced first order model.

5. CONCLUSIONS

The adaptive controllers working on a prespecified state of a lower dimension that corresponds to an appropriate model of the controlled system have been so far designed in two essentially different ways:

– direct adjustment of controller parameters,
– optimization on the basis of an identified model intentionally simplified in structure to match the desired controller.

The first method has not yet been satisfactorily grounded in theory; the second has achieved limited control quality due to mismodelling errors. Better quality can be obtained in the latter case by using a simple model identified as some simplification of an adequate "full-state" system description [3]. Our approach is based on such a model reduction which is, however, optimal with respect to the control objective.

The theoretical and experimental results encourage further development of the method. Our evaluation of the present research stage and its perspectives can be summarized as follows:

– The presented solution relies heavily on availability of the true state model (4). The use of the structure determination algorithms [6] is believed to be the practical tool for it. Under this assumption a substantial step has been made in attacking the mismodelling problem. In the theory, however, the solution is by no means complete as an analysis of the influence of improper choice of the model (4) is missing.
– A practical use in controller commision will need broader simulation and test-case experience as well as more sophisticated algorithmization for the routine use.
– The comparative role of the design controller has not been fully exploited. Extensive comparisons of the type presented in Section 4 are necessary.
– A theoretical study of features of the optimal model reduction has remained open too. However, some general observations are immediately available, for instance,

  (a) a time-varying absolute term always occurs,

  (b) the state transitions matrix and covariance of innovations are time-varying, but the parameters weighting system inputs in state space models do not vary (cf. (41)),

  (c) the rate of model changes is essentially determined by that of full-state closed loops moments ((32), (36), (37)); it can be influenced partially by the quality criterion used (indirectly, through the control law, cf. (36), (37).

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