Daniela Jarušková
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SIEGEL'S TEST FOR PERIODIC COMPONENTS IN MULTIPLE TIME SERIES AND ITS APPLICATION IN ENGINEERING PRACTICE

DANIELA JARUŠKOVÁ

Tests of periodicity in multivariate time series are studied. The distribution of multivariate version of Siegel's test statistic is found and the tables of critical values for some parameters are given. The multivariate process arising as the vibrations of building structure is investigated with the help of these tests.

1. INTRODUCTION

In several different positions the vibration of the building structure is measured so that we obtain several signals in the corresponding points. The goal of this measurement is to describe the behaviour of the structure as well as to reveal the changes caused by damage. We observe the multivariate process \( X(t) = (X(1), \ldots, X(t))' \) which consists of the periodic component \( m(t) = (m(1), \ldots, m(t))' \) and of the random noise \( e(t) \) arising as a result of the errors of measurement as well as a result of the effects of another "inherent" random noise, i.e.

\[
X(t) = m(t) + e(t).
\]

In digitizing a continuous record we are converting the original continuous multivariate process into a multiple time series \( X(1) = (X(1), \ldots, X(1))', \ldots, X(N) = (X(N), \ldots, X(N))' \). The aim of the statistical inference is to estimate the periodic component \( m(t) \) given \( N \) observations \( X(1), \ldots, X(N) \).

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider real, \( r \)-vector, time series

\[
X(t) = m(t) + e(t).
\]

We suppose that \( \{e(t)\} \) is a set of independent \( r \)-vector variables each distributed
normally with zero mean and covariance matrix $\sigma^2 I_r$. Further we suppose that every coordinate of the periodic component can be expressed as a sum of cosine functions with the frequencies $\theta_1, \ldots, \theta_k$,

$$f_m(t) = \sum_{k=1}^{r} \phi_k \cos (\theta_k + j \phi_k) \quad (j = 1, \ldots, r).$$

We note that the system of the amplitudes $\{\phi_k\}$ and the phases $\{\theta_k\}$ corresponding to the different coordinates may be different.

If we want to estimate $m(t)$ we have to solve two problems. First, we are to find $\Theta$ — the set of the frequencies describing the behaviour of $m(t)$, it means to find their number and values. Second, we are to find the amplitudes $\phi_k$ and the phases $\theta_k$ corresponding to the different coordinates. For the latter problem the method of least squares can be used (see [1] and [5]). The first problem which is more complicated can be solved with the help of Fourier analysis.

If we denote by $v(\lambda) = \sqrt{2\pi N} \sum_{i=1}^{N} X_i e^{-i\lambda t}$ the Fourier transform of the sequence $X(1), \ldots, X(N)$ then the matrix $P(\lambda) = v(\lambda) v(\lambda)^*$ (the asterisk denoting complex conjugate transform) is called the periodogram matrix. If we consider the periodogram matrix of a non-random periodic component then it can be shown that all elements $\{P_{m,n}(\lambda)\}$ are of order $N$ if $\lambda \in \Theta$ and they are bounded if $\lambda \notin \Theta$.

We note that the Euclidean norm of the periodogram matrix $\|P(\lambda)\|$ is equivalent to the trace of $P(\lambda) = \sum_{j=1}^{r} P_{j}(\lambda)$ since

$$\|P(\lambda)\|^2 = \sum_{j=1}^{r} \sum_{k=1}^{r} P_{j}(\lambda) P_{k}(\lambda) = (\sum_{j=1}^{r} v_{j}(\lambda) v_{j}^*(\lambda) )^2.$$ 

Further

$$E[\|P(\lambda)\|^2] = E(\text{tr} P(\lambda))^2 = E(\sum_{j=1}^{r} (P_{j}(\lambda))^2 + \sum_{j<k} P_{j}(\lambda) P_{k}(\lambda))$$

and for $N \to \infty$ and $\lambda \in (0, \pi)$

$$E(P_{j}(\lambda) P_{k}(\lambda)) = \frac{1}{4\pi^2 N^2} E(\sum_{i=1}^{N} (f_m(t) + k e(t))^2 (\sum_{i=1}^{N} (f_m(t) + k e(t))^2 + \frac{\sigma^2}{2N} P_{j}(\lambda) + \frac{\sigma^4}{4\pi^2}. $$

$$E(\sum_{i=1}^{N} (f_m(t) + k e(t))^2 (\sum_{i=1}^{N} (f_m(t) + k e(t))^2 + \frac{\sigma^2}{2N} P_{j}(\lambda) + \frac{\sigma^4}{4\pi^2}. $$

\[= \frac{1}{4\pi^2 N^2} E(\sum_{i=1}^{N} (f_m(t) + k e(t))^2 (\sum_{i=1}^{N} (f_m(t) + k e(t))^2 + \frac{\sigma^2}{2N} P_{j}(\lambda) + \frac{\sigma^4}{4\pi^2}. $$
\]
and therefore
\[ E[\|P(\lambda)\|^2] \approx \frac{(r + r^2)\sigma^4}{4\pi^2} + (2 + 2r)\frac{\sigma^2}{2\pi} (\text{tr} P(\lambda)) + (\text{tr} P(\lambda))^2. \]

We conclude that the Euclidean norm of the periodogram matrix \( \|P(\lambda)\| \) is a good indicator of periodicities.

3. MULTIDIMENSIONAL FISHER'S TEST

For finding the set \( \Theta \) we can use the multidimensional Fisher's test. The null hypothesis is that there is no periodic activity. For testing the null hypothesis Mac Neil [4] found a multiple time series analogue of Fisher's test for periodicities. The test is based upon \( \max \{2\pi \|P(\lambda_p)\|/\sigma^2, \ p = 1, \ldots, n = \lceil (N - 1)/2 \rceil \} \) where \( \|P(\lambda_p)\| \) is the Euclidean norm of the periodogram matrix computed at \( \lambda_p = 2np/N \). The statistics \( \{2\pi \|P(\lambda_p)\|/\sigma^2, \ p = 1, \ldots, n \} \) are independent identically distributed according to the gamma law with the density function \( f(x) = e^{-x}x^{r-1}/\Gamma(r) \). If \( \sigma^2 \) is unknown then the statistics \( 2\pi \|P(\lambda_p)\|^2/\sigma^2 \) are replaced by \( Y_p = \|P(\lambda_p)\|^2/\sum_{p=1}^n \|P(\lambda_p)\|^2 \). The statistics \( \{Y_p, p = 1, \ldots, n-1\} \) have the Dirichlet distribution with the density function
\[
\pi(i, a) = \frac{(nr - 1)!}{(r - 1)!} \prod_{p=1}^n y_p^{r-1}(1 - \sum_{p=1}^{n-1} y_p)^{r-1}, \quad (y_p \geq 0, \sum y_p \leq 1) \]
and the distribution of the test statistics max \( Y_p \) is the following
\[
P(\max_{p=1}^n Y_p > a) = \sum_{i=1}^n (-1)^{n-1} \binom{n}{i} \pi(i, a),
\]
where
\[
\pi(i, a) = P(Y_1 > a, \ldots, Y_i > a) = -\sum_{j=0}^{r-1} \sum_{\lambda=0}^{r-1} (nr - 1)! a^j(1 - i a)^{j-1} \prod_{k=1}^i (nr - \sum_{k=1}^{j-1} k - 1)!
\]
\((i, a) \) denotes \( \max (i, a) \).

In one dimensional case \( (r = 1) \) is known that if there is an activity at several frequencies in the alternative hypothesis, the over-estimation of \( \sigma^2 \) in Fisher's test will occur reducing the power of the test (see [2] and [7]). In order to remedy this situation Siegel suggested to use a test statistic based on all large \( Y_p \) instead of only on their maximum, i.e., to use the statistic \( T_{n} = \sum (Y_p - \lambda_{W})^2 \) where \( \lambda_{W} \) is the corresponding critical value of Fisher's test and \( \lambda \) is a parameter chosen between 0 and 1. In one dimensional case Siegel [6] found the distribution of the statistic \( T_{n} \). Because the situation concerning multiple time series \( (r > 1) \) is similar it can be useful in the case of compound periodicities to apply the same procedure using the analogous statistics.
4. DISTRIBUTION OF SIEGEL'S TEST STATISTIC FOR MULTIPLE TIME SERIES

**Theorem.** The statistic \( S = \sum_{p=1}^{n} (Y_p - a)_+ \) is a mixture of a degenerate and continuous random variable:

\[
S = \frac{A}{B} \quad \text{with probability} \quad p \quad \left(1 - p\right)
\]

where \( A = (1 - na)_+ \) is degenerate and \( B \) is continuous with the density

\[
f(t) = \frac{1}{1 - p} - \sum_{i=1}^{n} \binom{n}{i} (nr - 1)! \prod_{j=0}^{r-1} a_{i+j} \quad \text{min}(n, s-1) \sum_{j=0}^{s-1} \binom{k}{j} (-1)^{r-1} j^{r-j-i-1} (1 - ka - t)^{s-j-1} \prod_{j=0}^{s-1} j!
\]

The mixing probability is

\[
p = 1 + \sum_{i=1}^{n} (-1)^{r} \binom{n}{i} \pi(i, a), \quad na > 1,
\]

\[
p = \pi(n, a), \quad na \leq 1.
\]

The proof of the theorem will follow from the following lemmas.

**Lemma 1.** The variable \( S \) has probability mass at least \( p \) at \( (1 - na)_+ \).

**Proof.** We consider three cases. If \( a > 1/n \), then \((1 - na)_+ = 0\) and in this case in virtue of (2) \( p = P(Y_1 < a, ..., Y_n < a) = 1 - P(\max Y_i > a) = 1 + \sum_{i=1}^{n} (-1)^{r} \pi(i, a)\).

If \( a < 1/n \) then \((1 - na)_+ = 1 - na\) and \( p = P(Y_1 > a, ..., Y_n > a) = \pi(n, a)\). If \( a = 1/n \) then \( p = 0\). \( \square \)

**Lemma 2.** The moments of \( S \) are given by

\[
E S^n = \sum_{k=1}^{r-1} \binom{n}{k} (nr - 1)! \prod_{j=0}^{r-1} a_{i+j} \prod_{j=0}^{r-1} j!
\]

\[
\left(1 - ka\right)^{n-j-i-1} (1 - ka - t)^{s-j-1} \prod_{j=0}^{s-1} j!
\]

\[
\frac{1}{(nr - j_1 - ...) - j_k + m - 1)!} \frac{(nr - j_1 - ...) + m - 1)!}{(nr - j_1 - ...) - j_k + m - 1)!} \frac{(lr - j_1 - ...) + m - 1)!}{(lr - j_1 - ...) + m - 1)!}
\]

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Proof. The moment of \( S \) can be expressed as the sum of the mixed moments of \( \{Y_p, p = 1, \ldots, n - 1\} \) which have Dirichlet distribution \( (Y_p = 1 - \sum_{p=1}^{n-1} Y_p, \; y_n = 1 - \sum_{p=1}^{n-1} y_p) \).

\[
\text{ES}^m = E[(Y_1 - a)_+ \cdot (Y_2 - a)_+ \cdot \ldots \cdot (Y_n - a)_+]^m = \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} E(Y_1 - a)^l_1 \cdot (Y_2 - a)^l_2 \cdot \ldots \cdot (Y_n - a)^l_n.
\]

Further

\[
E(Y_1 - a)^l_1 \cdot (Y_2 - a)^l_2 \cdot \ldots \cdot (Y_n - a)^l_n = \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n = \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

Substituting this expression into (8) we get

\[
\text{ES}^m = \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

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\[
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\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
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\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]

\[
= \frac{1}{\prod_{j=1}^{r} (r - 1 - j)!} \sum_{k=1}^{n} \binom{n}{k} \sum_{l_1 + \ldots + l_n = m} \frac{m!}{l_1! \ldots l_k!} \cdot \frac{(nr - 1)!}{(nr - kr - 1)! \cdot [(r - 1)!]^k} \cdot \int \ldots \int \prod_{j=1}^{r} y_j^{l_j - 1} \prod_{j=1}^{nr} dy_1 \ldots dy_n
\]
Lemma 3. The density function of $B$ is $f(t)$.

Proof. Let us denote

$$f(t; k, l, f_1, \ldots, f_k) =$$

$$= \frac{(nr - j_1 - \ldots - j_k - 1)! \prod_j (1 - \frac{1}{n} - \frac{1}{l} - \frac{1}{f_j})}{(lr - j_1 - \ldots - j_k - 1)! (nr - lr - f_{k+1} - \ldots - j_k - 1)!}$$

for $m = 0, 1, \ldots$

$$\int_0^t t^m f(t; k, l, f_1, \ldots, f_k) \, dt = (1 - ka)^{nr - j_1 - \ldots - j_k + m - 1}.$$

and therefore for $m = 0, 1, \ldots$

$$\int_0^t t^m f(t) \, dt = \frac{1}{1 - p} \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \prod_{j=0}^{r-1} \prod_{j=0}^{r-1} \frac{a^{j_1 + \ldots + j_k}}{j_1! (nr - j_1 - \ldots - j_k + m - 1)! (lr - j_1 - \ldots - j_k - 1)!}.$$

For $na \geq 1$

$$1 - p = \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \prod_{j=0}^{r-1} \prod_{j=0}^{r-1} \frac{a^{j_1 + \ldots + j_k}}{j_1! (nr - j_1 - \ldots - j_k + m - 1)! (lr - j_1 - \ldots - j_k - 1)!}.$$

and $(1 - na)_{+} = 0$,

for $na < 1$

$$\sum_{k=1}^{n} (-1)^{k-1} \binom{n}{k} a^{j_1 + \ldots + j_k} = 1.$$

and $1 - p = 1 - \pi(n, a)$,

therefore

$$\int f(t) \, dt = 1.$$

From the decomposition we have $EB^m = (1/(1 - p)) ESm^m - (p/(1 - p))(1 - na)^m$.

In all cases the moments $EB^m$ can be expressed in the following way

$$EB^m = \frac{1}{1 - p} \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \prod_{j=0}^{r-1} \prod_{j=0}^{r-1} \frac{a^{j_1 + \ldots + j_k}}{j_1! (nr - j_1 - \ldots - j_k + m - 1)! (lr - j_1 - \ldots - j_k - 1)!}.$$

and so $EB^m = \int t^m f(t) \, dt.$

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Similarly as in Siegel’s paper (6) \( f(t) \) is bounded from below and because the probability distribution on \( (0, 1) \) is uniquely characterized by its moments it follows that \( f(t) \) is the density function of the variable \( B \). We remark that \( p \) is given by (5) because for another value of mixing probability \( B \) are not the moments of an absolutely continuous distribution.

**Lemma 4.** The cumulative distribution of \( S \) is \( F(t) \) given by

\[
F(t) = 1 + \sum_{i=1}^{n} \binom{n}{k} (nr - 1)! \sum_{j=0}^{r-1} \sum_{k=0}^{r-1} \frac{a^{j+k+...+k}}{i!}.
\]

\[
\times \sum_{l=1}^{\min(n-1)} \binom{k}{l} (-1)^{k+l+1} \frac{\sum_{\sum_{j=0}^{r-1} \sum_{k=0}^{r-1} (nr - j)_{l+1}!}}{v!(nr - j_1 - ... - j_k - v - 1)!}
\]

**Proof.** Since the function

\[
F(t; k, l, j_1, ..., j_k) = - \sum_{v=0}^{r-1} \sum_{l=0}^{r-1} \frac{a^{j+k+...+k}}{v!(nr - j_1 - ... - j_k - v - 1)!}
\]

is the primitive function of \( f(t; k, l, j_1, ..., j_k) \) then the function

\[
F_d(t) = \frac{1}{1 - p} \sum_{l=1}^{\min(n-1)} \binom{k}{l} (-1)^{k+l+1} F(t; k, l, j_1, ..., j_k)
\]

is the primitive function of \( f(t) \). Therefore the distribution function of \( B \) is \( F_B(t) = 1 + F_d(t) \). From the decomposition of \( S \) it follows that \( F(t) = p_{x(t_1(1-\alpha))} + (1 - p) F_d(t) \). If \( t \geq \alpha(1 - \alpha)_a \) then \( (1 - na - t)_+ = 0 \) and

\[
F(t) = p + (1 - p) \left( 1 + \sum_{l=1}^{\min(n-1)} \binom{k}{l} (-1)^{k+l+1} \frac{\sum_{\sum_{j=0}^{r-1} \sum_{k=0}^{r-1} (nr - j)_{l+1}!}}{v!(nr - j_1 - ... - j_k - v - 1)!} \right) = (6)
\]

If \( na < 1 \) and \( t < (1 - na)_a \) then \( p = \pi(n, a) \)

\[
F(t) = 1 - \pi(n, a) + \sum_{l=1}^{\min(n-1)} \binom{k}{l} (-1)^{k+l+1} \frac{\sum_{\sum_{j=0}^{r-1} \sum_{k=0}^{r-1} (nr - j)_{l+1}!}}{v!(nr - j_1 - ... - j_k - v - 1)!} = (6)
\]
5. CRITICAL VALUES FOR $T^A = \sum(Y_p - \lambda u_g) + t$

Critical values are computed for $r = 2, 3, n = 20, 25, 30, \ldots, 50$ and significance levels $\alpha = 0.01, 0.05$. The threshold values are chosen $0.5$ and $0.7\mu_g$, where $\mu_g$ is the corresponding critical value of Fisher's test. The critical values were obtained from the approximation

\begin{equation}
P(T_k > t) = \sum_{i=1}^{k} \binom{n}{k} (nr - 1)! \sum_{I=0}^{r-1} \prod_{j=0}^{r-1} \binom{nr - \sum_{j=1}^{I} (nr - j - 1)!}{I! (-1)^{k-I} F(t; k, l, j, \ldots, j_l)}
\end{equation}

Table 1.

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<td>0.1001</td>
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<td>0.0804</td>
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<tr>
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<td>0.0736</td>
<td>0.0326</td>
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<tr>
<td>50</td>
<td>0.0849</td>
<td>0.0635</td>
<td>0.0270</td>
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</tbody>
</table>

6. EXAMPLE

In three points of a concrete fundament the vibrations were measured. The record was digitized so that we obtained 512 observations and the frequency interval $\Delta \omega = 4.883$ Hz. For every point we calculated the periodogram using FFT. As we looked for the frequencies within the interval $(0, \Delta \omega)$ we considered the values of periodogram for $\lambda_p = 2\pi p/N (p = 1, \ldots, 40)$ only. The results are given in Table 2.

Using Siegel's multidimensional test with $\lambda = 0.7$ we get that two values $Y_{10} = 0.6485$ (corresponding to frequency 48.83 Hz) and $Y_{40} = 0.174$ (corresponding
to frequency 195-31 Hz) are greater than $0.7 u_g = 0.0723$ and \((Y_{10} - 0.0723) + + (Y_{40} - 0.0723) = 0.679 > 0.0326$. Using the Siegel's test with $\lambda = 0.5$ we get that three values $Y_{10}$, $Y_{40}$ and $Y_{20} = 0.0644$ are greater than $0.5 u_g = 0.0516$ and $\sum_{i=10,20,40} (Y_i - 0.0516) = 0.733 > 0.0736$. Further we omit the frequencies 48-83 Hz, 97-66 Hz, 146-49 Hz, 195-31 Hz corresponding to the frequency of the electric current and its harmonics as well as the frequency 190-44 Hz. After normalizing the new inference shows that the values $Y_{2}$ (9-77 Hz), $Y_{3}$ (14-65 Hz), $Y_{13}$ (63-48 Hz), $Y_{15}$ (73-24 Hz), $Y_{26}$ (126-96 Hz) > 0.5 $u_g = 0.0581$ and $\sum_{i=2,3,13,15,26} (Y_i - 0.0581) =$ 0.375 > 0.0804 or the values $Y_{2}$, $Y_{3}$, $Y_{13}$, $Y_{26} > 0.7 u_g = 0.0813$ and $\sum_{i=2,3,13,26} (Y_i - 0.0813) = 0.279 > 0.0364$.

If we reject the null hypothesis because of some frequencies, we include these frequencies into the set $\Theta$. We conclude that if we omit the frequencies corresponding to the electric current and its harmonics the vibration frequencies are 9-77 Hz and 14-65 Hz (it might be only one between them), 63-48 Hz, 73-24 Hz and 126-96 Hz.

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REMARKS


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