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SIEGEL'S TEST FOR PERIODIC COMPONENTS IN MULTIPLE TIME SERIES AND ITS APPLICATION IN ENGINEERING PRACTICE

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Tests of periodicity in multivariate time series are studied. The distribution of multivariate version of Siegel's test statistic is found and the tables of critical values for some parameters are given. The multivariate process arising as the vibrations of building structure is investigated with the help of these tests.

1. INTRODUCTION

In several different positions the vibration of the building structure is measured so that we obtain several signals in the corresponding points. The goal of this measurement is to describe the behaviour of the structure as well as to reveal the changes caused by damage. We observe the multivariate process $X(t) = ({}^1X(t), \ldots, {}^rX(t))'$ which consists of the periodic component $m(t) = ({}^1m(t), \ldots, {}^rm(t))'$ and of the random noise $e(t)$ arising as a result of the errors of measurement as well as a result of the effects of another "inherent" random noise, i.e.

$$X(t) = m(t) + e(t) .$$

In digitizing a continuous record we are converting the original continuous multivariate process into a multiple time series $X(1) = ({}^1X(1), \ldots, {}^rX(1))', \ldots, X(N) = ({}^1X(N), \ldots, {}^rX(N))'$. The aim of the statistical inference is to estimate the periodic component $m(t)$ given N observations $X(1), \ldots, X(N)$.

2. MATHEMATICAL FORMULATION OF THE PROBLEM

We consider real, r-vector, time series

$$X(t) = m(t) + e(t) .$$

We suppose that $\{e(t)\}$ is a set of independent r-vector variables each distributed normally with zero mean and covariance matrix $\sigma^2 I_r$. Further we suppose that every coordinate of the periodic component can be expressed as a sum of cosine functions with the frequencies $\theta_1, \ldots, \theta_R$,

$$^j m(t) = \sum_{k=1}^{R} {}^j\varrho_k \cos(t\theta_k + {}^j\phi_k) \quad (j = 1, \ldots, r)\,.$$

We note that the system of the amplitudes $\{^j\varrho_k\}$ and the phases $\{^j\phi_k\}$ corresponding to the different coordinates may be different.

If we want to estimate $m(t)$ we have to solve two problems. First, we are to find Θ – the set of the frequencies describing the behaviour of $m(t)$, it means to find their number and values. Second, we are to find the amplitudes $^j\varrho_k$ and the phases $^j\phi_k$ corresponding to the different coordinates. For the latter problem the method of least squares can be used (see [1] and [5]). The first problem which is more complicated can be solved with the help of Fourier analysis.

If we denote by $v^x(\lambda) = 1/\sqrt{(2\pi N)} \sum_{t=1}^{N} X_t\, \mathrm{e}^{-\mathrm{i}t\lambda}$ the Fourier transform of the sequence $X(1), \ldots, X(N)$ then the matrix $I^x(\lambda) = v^x(\lambda)\,(v^x(\lambda))^*$ (the asterisk denoting complex conjugate transform) is called the periodogram matrix. If we consider the periodogram matrix of a non-random periodic component then it can be shown that all elements $\{I^m_{jk}(\lambda)\}$ are of order N if $\lambda \in \Theta$ and they are bounded if $\lambda \notin \Theta$.

We note that the Euclidean norm of the periodogram matrix $\|I^x(\lambda)\|$ is equivalent to the trace of $I^x(\lambda) = \sum_{j=1}^{r} I^x_{jj}(\lambda)$ since

$$\|I^x(\lambda)\|^2 = \sum_{j=1}^{r} \sum_{k=1}^{r} v_j(\lambda)\, v_k^*(\lambda)\, v_j^*(\lambda)\, v_k(\lambda) = \Big(\sum_{j=1}^{r} v_j(\lambda)\, v_j^*(\lambda)\Big)^2\,.$$

Further

$$\mathsf{E}\|I^x(\lambda)\|^2 = \mathsf{E}(\operatorname{tr} I^x(\lambda))^2 = \mathsf{E}\Big(\sum_{j=1}^{r} (I^x_{jj}(\lambda))^2 + \sum_{j \neq k}^{r} \sum^{r} I^x_{jj}(\lambda)\, I^x_{kk}(\lambda)\Big)$$

and for $N \to \infty$ and $\lambda \in (0, \pi)$

$$\mathsf{E}(I^x_{jj}(\lambda))^2 = \frac{1}{4\pi^2 N^2}\, \mathsf{E}\Big(\sum_{t=1}^{N} ({}^j m(t) + {}^j e(t))\, \mathrm{e}^{-\mathrm{i}t\lambda}\Big)\Big(\sum_{t'=1}^{N} ({}^j m(t') + {}^j e(t'))\, \mathrm{e}^{\mathrm{i}t'\lambda}\Big)$$

$$\Big(\sum_{s=1}^{N} ({}^j m(s) + {}^j e(s))\, \mathrm{e}^{-\mathrm{i}s\lambda}\Big)\Big(\sum_{s'=1}^{N} ({}^j m(s') + {}^j e(s'))\, \mathrm{e}^{\mathrm{i}s'\lambda}\Big) \cong (I^m_{jj}(\lambda))^2 + 4\,\frac{\sigma^2}{2\pi}\, I^m_{jj}(\lambda) + 2\,\frac{\sigma^4}{4\pi^2},$$

$$\mathsf{E}(I^x_{jj}(\lambda)\, I^x_{kk}(\lambda)) = \frac{1}{4\pi^2 N^2}\, \mathsf{E}\Big(\sum_{t=1}^{N} ({}^j m(t) + {}^j e(t))\, \mathrm{e}^{-\mathrm{i}t\lambda}\Big)$$

$$\Big(\sum_{t'=1}^{N} ({}^j m(t') + {}^j e(t'))\, \mathrm{e}^{\mathrm{i}t'\lambda}\Big)\Big(\sum_{s=1}^{N} ({}^k m(s) + {}^k e(s))\, \mathrm{e}^{-\mathrm{i}s\lambda}\Big)\Big(\sum_{s'=1}^{N} ({}^k m(s') + {}^k e(s'))\, \mathrm{e}^{\mathrm{i}s'\lambda}\Big) \simeq$$

$$\simeq I^m_{jj}(\lambda)\,\frac{\sigma^2}{2\pi} + I^m_{kk}(\lambda)\,\frac{\sigma^2}{2\pi} + I^m_{jj}(\lambda)\, I^m_{kk}(\lambda) + \frac{\sigma^4}{4\pi^2}$$

and therefore

$$\mathsf{E}\|I^x(\lambda)\|^2 \cong \frac{(r + r^2)\,\sigma^4}{4\pi^2} + (2 + 2r)\frac{\sigma^2}{2\pi}\left(\operatorname{tr} I^m(\lambda)\right) + \left(\operatorname{tr} I^m(\lambda)\right)^2 .$$

We conclude that the Euclidean norm of the periodogram matrix $\|I^x(\lambda)\|$ is a good indicator of periodicities.

3. MULTIDIMENSIONAL FISHER'S TEST

For finding the set Θ we can use the multidimensional Fisher's test. The null hypothesis is that there is no periodic activity. For testing the null hypothesis Mac Neil [4] found a multiple time series analogue of Fisher's test for periodicities. The test is based upon max $\{2\pi\|I^x(\lambda_p)\|/\sigma^2,\ p = 1, \ldots, n = [(N - 1)/2]\}$ where $\|I^x(\lambda_p)\|$ is the Euclidean norm of the periodogram matrix computed at $\lambda_p = 2\pi p/N$. The statistics $\{2\pi\|I^x(\lambda_p)\|/\sigma^2,\ p = 1, \ldots, n\}$ are independent identically distributed according to the gamma law with the density function $f(x) = e^{-x}x^{r-1}/\Gamma(r)$. If σ^2 is unknown then the statistics $2\pi\|I^x(\lambda_p)\|/\sigma^2$ are replaced by $Y_p = \|I^x(\lambda_p)\| / \sum_{v=1}^{n} \|I^x(\lambda_v)\|$. The statistics $\{Y_p,\ p = 1, \ldots, n - 1\}$ have the Dirichlet distribution with the density function

$$f(y_1, \ldots, y_{n-1}) = \frac{(nr - 1)!}{[(r - 1)!]^n} \prod_{p=1}^{n-1} y_p^{r-1} \left(1 - \sum_{v=1}^{n-1} y_v\right)^{r-1} \tag{1}$$

$(y_p \geqq 0, \sum y_p \leqq 1)$ and the distribution of the test statistics max Y_p is the following

$$\mathsf{P}\left(\max_{p=1,\ldots,n} Y_p > a\right) = \sum_{i=1}^{n} (-1)^{i-1} \binom{n}{i} \pi(i, a)\,, \tag{2}$$

where

$$\pi(i, a) = \mathsf{P}(Y_1 > a, \ldots, Y_i > a) = \tag{3}$$

$$= \sum_{j_1=0}^{r-1} \ldots \sum_{j_i=0}^{r-1} \frac{(nr - 1)!\, a^{\Sigma j_k} (1 - ia)_+^{nr - \Sigma j_k - 1}}{\prod j_k!\, (nr - \sum j_k - 1)!}$$

($(t)_+$ denotes max $(t, 0)$).

In one dimensional case ($r = 1$) is known that if there is an activity at several frequencies in the alternative hypothesis, the over-estimation of σ^2 in Fisher's test will occur reducing the power of the test (see [2] and [7]). In order to remedy this situation Siegel suggested to use a test statistic based on all large Y_p instead of only on their maximum, i.e. to use the statsitic $T_\lambda = \sum(Y_p - \lambda u_g)_+$, where u_g is the corresponding critical value of Fisher's test and λ is a parameter chosen between 0 and 1. In one dimensional case Siegel [6] found the distribution of the statistic T_λ. Because the situation concerning multiple time series ($r > 1$) is similar it can be useful in the case of compound periodicities to apply the same procedure using the analogous statistics.

4. DISTRIBUTION OF SIEGEL'S TEST STATISTIC FOR MULTIPLE TIME SERIES

Theorem. The statistic $S = \sum_{p=1}^{n} (Y_p - a)_+$ is a mixture of a degenerate and continuous random variable:

$$S = \left\langle \begin{matrix} A \\ B \end{matrix} \right. \quad \text{with probability} \quad \begin{matrix} p \\ (1-p) \end{matrix}$$

where $A = (1 - na)_+$ is degenerate and B is continuous with the density

$$f(t) = \frac{1}{1-p} \sum_{k=1}^{n} \binom{n}{k} (nr-1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \cdot \tag{4}$$

$$\cdot \sum_{l=1}^{\min(k,n-1)} \binom{k}{l} (-1)^{k+l} \frac{t^{lr-j_1-\dots-j_l-1}(1-ka-t)_+^{nr-lr-j_{l+1}-\dots-j_k-1}}{(lr-j_1-\dots-j_l-1)!\,(nr-lr-j_{l+1}-\dots-j_k-1)!}$$

The mixing probability is

$$p = 1 + \sum_{i=1}^{n} (-1)^i \binom{n}{i} \pi(i, a), \quad na > 1, \tag{5}$$

$$p = \pi(n, a), \qquad na \leqq 1.$$

The distribution function of S is

$$F(t) = \mathsf{P}(S \leqq t) = 1 + \sum_{k=1}^{n} \binom{n}{k} (nr-1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \cdot \tag{6}$$

$$\cdot \sum_{l=1}^{k} \binom{k}{l} (-1)^{k+l+1} \sum_{v=0}^{lr-j_1-\dots-j_l-1} \frac{t^v(1-ka-t)_+^{nr-j_1-\dots-j_k-v-1}}{v!\,(nr-j_1-\dots-j_k-v-1)!}$$

The proof of the theorem will follow from the following lemmas.

Lemma 1. The variable S has probability mass at least p at $(1 - na)_+$.

Proof. We consider three cases. If $a > 1/n$, then $(1 - an)_+ = 0$ and in this case in virtue of (2) $p = \mathsf{P}(Y_1 < a, \dots, Y_n < a) = 1 - \mathsf{P}(\max Y_i > a) = 1 + \sum_{i=1}^{n} (-1)^i \cdot \binom{n}{i} \pi(i, a)$. If $a < 1/n$ then $(1 - na)_+ = 1 - na$ and $p = \mathsf{P}(Y_1 > a, \dots, Y_n > a) = \pi(n, a)$. If $a = 1/n$ then $p = 0$. □

Lemma 2. The moments of S are given by

$$\mathsf{E}S^m = \sum_{k=1}^{n} \binom{n}{k} (nr-1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \cdot \tag{7}$$

$$\cdot \sum_{l=1}^{k} \binom{k}{l} (1-ka)_+^{nr-j_1-\dots-j_k+m-1} (-1)^{k+l} \cdot$$

$$\cdot \frac{(lr-j_1-\dots-j_l+m-1)!}{(nr-j_1-\dots-j_k+m-1)!\,(lr-j_1-\dots-j_l-1)!}$$

Proof. The moment of S can be expressed as the sum of the mixed moments of $\{Y_p, p = 1, \ldots, n-1\}$ which have Dirichlet distribution $\left(Y_n = 1 - \sum_{p=1}^{n-1} Y_p,\ y_n = 1 - \sum_{p=1}^{n-1} y_p\right)$.

$$\mathsf{E}S^m = \mathsf{E}[(Y_1 - a)_+ + (Y_2 - a)_+ + \ldots + (Y_n - a)_+]^m = \tag{8}$$

$$= \sum_{k=1}^{n} \binom{n}{k} \sum_{\substack{l_1+\ldots+l_k=m \\ l_1 \geq 1,\ldots,l_k \geq 1}} \mathsf{E}(Y_1 - a)_+^{l_1} (Y_2 - a)_+^{l_2} \ldots (Y_k - a)_+^{l_k} \frac{m!}{l_1! \ldots l_k!}$$

Further

$$\mathsf{E}(Y_1 - a)_+^{l_1} (Y_2 - a)_+^{l_2} \ldots (Y_k - a)_+^{l_k} = \frac{(nr-1)!}{(nr-kr-1)!\,[(r-1)!]^k} \,.$$

$$. \int \ldots \int (y_1 - a)_+^{l_1} \ldots (y_k - a)_+^{l_k}\, y_1^{r-1} \ldots y_k^{r-1} (1 - \sum_{i=1}^{k} y_i)^{nr-kr-1}\, \mathrm{d}y_1 \ldots \mathrm{d}y_k =$$

$$= \frac{(nr-1)!}{(nr-kr-1)!\,[(r-1)!]^k} \int \ldots \int (y_1)_+^{l_1} \ldots (y_k)_+^{l_k} (y_1 + a)^{r-1} \ldots (y_k + a)^{r-1} \,.$$

$$. (1 - ka - \sum y_i)_+^{nr-kr-1}\, \mathrm{d}y_1 \ldots \mathrm{d}y_k = \frac{(nr-1)!}{(nr-kr-1)!\,[(r-1)!]^k} \,.$$

$$. \sum_{j_1=0}^{r-1} \ldots \sum_{j_k=0}^{r-1} \frac{[(r-1)!]^k}{\prod j_i! \prod (r-1-j_i)!} a^{j_1+\ldots+j_k} \int \ldots \int y_1^{r-1-j_1+l_1} \ldots$$

$$\ldots y_k^{r-1-j_k+l_k} (1 - ka - \sum y_i)_+^{nr-kr-1} = (nr-1)! \sum_{j_1=0}^{r-1} \ldots \sum_{j_k=0}^{r-1}$$

$$\frac{a^{j_1+\ldots+j_k}}{\prod j_i! \prod (r-1-j_i)!} (1 - ka)_+^{nr-j_1-\ldots-j_k+m-1} \frac{(r-1-j_1+l_1)! \ldots (r-1-j_k+l_k)!}{(nr-j_1-\ldots-j_k+m-1)!}$$

Substituting this expression into (8) we get

$$\mathsf{E}S^m = \sum_{k=1}^{n} \binom{n}{k} \sum_{\substack{l_1+\ldots+l_k=m \\ l_1 \geq 1\ldots l_k \geq 1}} \frac{m!}{l_1! \ldots l_k!} (nr-1)! \sum_{j_1=0}^{r-1} \ldots \sum_{j_k=0}^{r-1}$$

$$\frac{a^{j_1+\ldots+j_k}}{\prod j_i! \prod (r-1-j_i)!} (1 - ka)_+^{nr-j_1-\ldots-j_k+m-1} \frac{(r-1-j_1+l_1)! \ldots (r-1-j_k+l_k)!}{(nr-j_1-\ldots-j_k+m-1)!} =$$

$$= \sum_{k=1}^{n} \binom{n}{k} (nr-1)! \sum_{j_1=0}^{r-1} \ldots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\ldots+j_k}}{\prod j_i!} \sum_{l=1}^{k} \binom{k}{l} .$$

$$. (1 - ka)_+^{nr-j_1-\ldots-j_k+m-1} (-1)^{k+l} \frac{(lr-j_1-\ldots-j_l+m-1)!}{(nr-j_1-\ldots-j_k+m-1)!\,(l_r-j_1-\ldots-j_l-1)!}$$

□

Lemma 3. The density function of B is $f(t)$.

Proof. Let us denote

$$f(t;k,l,j_1,\dots,j_k)=$$
$$=\frac{(nr-j_1-\dots-j_k-1)!\,t^{lr-j_1-\dots-j_l-1}(1-ka-t)_+^{nr-lr-j_{l+1}-\dots-j_k-1}}{(lr-j_1-\dots-j_l-1)!\,(nr-lr-j_{l+1}-\dots-j_k-1)!}$$

for $m=0,1,\dots$

$$\int_0^{1-ka}t^m f(t;k,l,j_1,\dots,j_k)\,\mathrm{d}t=(1-ka)_+^{nr-j_1-\dots-j_k+m-1}\,.$$
$$\cdot\frac{(nr-j_1-\dots-j_k-1)!\,(lr-j_1-\dots-j_l+m-1)!}{(nr-j_1-\dots-j_k+m-1)!\,(lr-j_1-\dots-j_l-1)!}$$

and therefore for $m=0,1,\dots$

$$\int_0^1 t^m f(t)\,\mathrm{d}t=\frac{1}{1-p}\sum_{k=1}^{n}\binom{n}{k}(nr-1)!\sum_{j_1=0}^{r-1}\dots\sum_{j_k=0}^{r-1}\frac{a^{j_1+\dots+j_k}}{\prod j_i!\,(nr-\sum j_i-1)!}\,.$$
$$\cdot\sum_{l=1}^{\min(k,n-1)}\binom{k}{l}(1-ka)_+^{nr-j_1-\dots-j_k+m-1}(-1)^{k+l}\,.$$
$$\cdot\frac{(nr-\sum j_i-1)!\,(lr-j_1-\dots-j_l+m-1)!}{(nr-j_1-\dots-j_k+m-1)!\,(lr-j_1-\dots-j_l-1)!}$$

For $na\geqq 1$

$$1-p=\sum_{k=1}^{n}\binom{n}{k}(-1)^{k-1}\pi(k,a)=$$
$$=\sum_{k=1}^{n}\binom{n}{k}(-1)^{k-1}\sum_{j_1=0}^{r-1}\dots\sum_{j_k=0}^{r-1}(nr-1)!\frac{a^{\Sigma j_i}(1-ka)_+^{nr-\Sigma j_i-1}}{\prod j_i!\,(nr-\sum j_i-1)!}$$

and $(1-na)_+=0$,

for $na<1$

$$\sum_{k=1}^{n}(-1)^{k-1}\binom{n}{k}\pi(k,a)=1$$

and $1-p=1-\pi(n,a)$,

therefore

$$\int f(t)\,\mathrm{d}t=1\,.$$

From the decomposition we have $\mathsf{E}B^m=(1/(1-p))\,\mathsf{E}S^m-(p/(1-p))\,(1-na)_+^m$. In all cases the moments $\mathsf{E}B^m$ can be expressed in the following way

$$\mathsf{E}B^m=\frac{1}{1-p}\sum_{k=1}^{n}\binom{n}{k}(nr-1)!\sum_{j_1=0}^{r-1}\dots\sum_{j_k=0}^{r-1}\frac{a^{j_1+\dots+j_k}}{\prod j_i!}\,.$$
$$\cdot\sum_{l=1}^{\min(k,n-1)}\binom{k}{l}(1-ka)_+^{nr-j_1-\dots-j_k+m-1}(-1)^{k+l}\,.$$
$$\cdot\frac{(lr-j_1-\dots-j_l+m-1)!}{(nr-j_1-\dots-j_k+m-1)!\,(lr-j_1-\dots-j_l-1)!}$$

and so $\mathsf{E}B^m=\int t^m f(t)\,\mathrm{d}t$.

Similarly as in Siegel's paper (6) $f(t)$ is bounded from below and because the probability distribution on $\langle 0, 1\rangle$ is uniquely characterized by its moments it follows that $f(t)$ is the density function of the variable B. We remark that p is given by (5) because for another value of mixing probability $\mathsf{E}B^m$ are not the moments of an absolutely continuous distribution.

Lemma 4. The cumulative distribution of S is $F(t)$ given by

$$F(t) = 1 + \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \, .$$

$$. \sum_{l=1}^{k} \binom{k}{l} (-1)^{k+l+1} \sum_{v=0}^{lr-j_1-\dots-j_l-1} \frac{t^v (1 - ka - t)_+^{nr-j_1-\dots-j_k-v-1}}{v!\,(nr - j_1 - \dots - j_k - v - 1)!}$$

Proof. Since the function

$$F(t; k, l, j_1, \dots, j_k) = - \sum_{v=0}^{lr-j_1-\dots-j_l-1} \frac{t^v (1 - ka - t)_+^{nr-j_1-\dots-j_k-v-1}}{v!\,(nr - j_1 - \dots - j_k - v - 1)!} \, .$$

$$. \, (nr - j_1 - \dots - j_k - 1)!$$

is the primitive function of $f(t; k, l, j_1, \dots, j_k)$ then the function

$$F_0(t) = \frac{1}{1-p} \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!\,(nr - \sum j_i - 1)!} \, .$$

$$. \sum_{l=1}^{\min(k,n-1)} \binom{k}{l} (-1)^{k+l+1} F(t; k, l, j_1, \dots, j_k)$$

is the primitive function of $f(t)$. Therefore the distribution function of B is $F_B(t) = 1 + F_0(t)$. From the decomposition of S it follows that $F(t) = p\chi_{\{t \geqq (1-na)_+\}} + (1 - p)\,F_B(t)$. If $t \geqq (1 - na)_+$ then $(1 - na - t)_+ = 0$ and

$$F(t) = p + (1 - p)\left(1 + \frac{1}{1-p} \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \, .\right.$$

$$\left. . \sum_{l=1}^{\min(k,n-1)} \binom{k}{l} (-1)^{k+l+1} \sum_{v=0}^{lr-j_1-\dots-j_l-1} \frac{t^v (1 - ka - t)_+^{nr-j_1-\dots-j_k-v-1}}{v!\,(nr - j_1 - \dots - j_k - v - 1)!}\right) = (6)$$

If $na < 1$ and $t < (1 - na)_+$ then $p = \pi(n, a)$

$$F(t) = 1 - \pi(n, a) + \sum_{k=1}^{n} \binom{n}{k} (nr - 1)! \sum_{j_1=0}^{r-1} \dots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\dots+j_k}}{\prod j_i!} \, .$$

$$. \sum_{l=1}^{\min(k,n-1)} \binom{k}{l} (-1)^{k+l+1} \sum_{v=0}^{lr-j_1-\dots-j_l-1} \frac{t^v (1 - ka - t)_+^{nr-j_1-\dots-j_k-v-1}}{v!\,(nr - j_1 - \dots - j_k - v - 1)!} = (6)$$

5. CRITICAL VALUES FOR $T^{\lambda} = \sum(Y_p - \lambda u_g)_+$

Critical values are computed for $r = 2, 3$, $n = 20, 25, 30, \ldots, 50$ and significance levels $\alpha = 0{\cdot}01$, $0{\cdot}05$. The threshold values are chosen $0{\cdot}5\, u_g$ and $0{\cdot}7 u_g$, where u_g is the corresponding critical value of Fisher's test. The critical values were obtained from the approximation

$$\text{(9)} \qquad \mathsf{P}(T_\lambda > t) \doteq \sum_{k=1}^{5} \binom{n}{k} (nr-1)! \sum_{j_1=0}^{r-1} \ldots \sum_{j_k=0}^{r-1} \frac{a^{j_1+\ldots+j_k}}{\prod j_i!\,(nr - \sum j_i - 1)!} \cdot \\ \cdot \sum_{l=1}^{k} \binom{k}{l} (-1)^{k+l} F(t; k, l, j_1, \ldots, j_k)$$

Table 1.

	$r = 2$	$\alpha = 0{\cdot}01$			$r = 2$	$\alpha = 0{\cdot}05$	
n	u_g	$t_{0.5}$	$t_{0.7}$	n	u_g	$t_{0.5}$	$t_{0.7}$
20	0·2288	0·1297	0·0696	20	0·1921	0·1288	0·0613
25	0·1904	0·1116	0·0583	25	0·1600	0·1132	0·0518
30	0·1635	0·0985	0·0502	30	0·1376	0·1016	0·0451
35	0·1435	0·0886	0·0443	35	0·1209	0·0929	0·0401
40	0·1281	0·0810	0·0397	40	0·1081	0·0859	0·0361
45	0·1158	0·0745	0·0359	45	0·0978	0·0802	0·0329
50	0·1057	0·0695	0·0330	50	0·0894	0·0755	0·0303

	$r = 3$	$\alpha = 0{\cdot}01$			$r = 3$	$\alpha = 0{\cdot}05$	
n	u_g	$t_{0.5}$	$t_{0.7}$	n	u_g	$t_{0.5}$	$t_{0.7}$
20	0·1877	0·1157	0·0577	20	0·1601	0·1218	0·0528
25	0·1552	0·1001	0·0482	25	0·1326	0·1077	0·0446
30	0·1327	0·0890	0·0414	30	0·1136	0·0973	0·0387
35	0·1161	0·0804	0·0364	35	0·0995	0·0892	0·0343
40	0·1033	0·0736	0·0326	40	0·0886	0·0830	0·0311
45	0·0931	0·0683	0·0296	45	0·0800	0·0778	0·0283
50	0·0849	0·0635	0·0270	50	0·0730	0·0735	0·0261

6. EXAMPLE

In three points of a concrete fundament the vibrations were measured. The record was digitized so that we obtained 512 observations and the frequency interval $\Delta\omega = 4{\cdot}883$ Hz. For every point we calculated the periodogram using FFT. As we looked for the frequencies within the interval $(0, \Delta\omega\, 40)$ we considered the values of periodogram for $\lambda_p = 2\pi p/N$ $(p = 1, \ldots, 40)$ only. The results are given in Table 2.

Using Siegel's multidimensional test with $\lambda = 0{\cdot}7$ we get that two values $Y_{10} = 0{\cdot}6485$ (corresponding to frequency 48·83 Hz) and $Y_{40} = 0{\cdot}174$ (corresponding

Table 2.

p	$\|I(\lambda_p)\|$	p	$\|I(\lambda_p)\|$	p	$\|I(\lambda_p)\|$	p	$\|I(\lambda_p)\|$
1	0·0135	11	0·0012	21	0·0016	31	0·0015
2	0·0398	12	0·0010	22	0·0040	32	0·0008
3	0·0450	13	0·0790	23	0·0015	33	0·0009
4	0·0040	14	0·0042	24	0·0006	34	0·0004
5	0·0036	15	0·0193	25	0·0016	35	0·0015
6	0·0012	16	0·0024	26	0·0261	36	0·0015
7	0·0020	17	0·0015	27	0·0014	37	0·0147
8	0·0024	18	0·0014	28	0·0034	38	0·0132
9	0·0044	19	0·0038	29	0·0098	39	0·0249
10	1·9862	20	0·1973	30	0·0041	40	0·5356

to frequency 195·31 Hz) are greater than $0{\cdot}7\, u_g = 0{\cdot}0723$ and $(Y_{10} - 0{\cdot}0723) + (Y_{40} - 0{\cdot}0723) = 0{\cdot}679 > 0{\cdot}0326$. Using the Siegel's test with $\lambda = 0{\cdot}5$ we get that three values Y_{10}, Y_{40} and $Y_{20} = 0{\cdot}0644$ are greater than $0{\cdot}5 u_g = 0{\cdot}0516$ and $\sum_{i=10,20,40} (Y_i - 0{\cdot}0516) = 0{\cdot}733 > 0{\cdot}0736$. Further we omit the frequencies 48·83 Hz, 97·66 Hz, 146·49 Hz, 195·31 Hz corresponding to the frequency of the electric current and its harmonics as well as the frequency 190·44 Hz. After normalizing the new inference shows that the values Y_2 (9·77 Hz), Y_3 (14·65 Hz), Y_{13} (63·48 Hz), Y_{15}(73·24 Hz), Y_{26}(126·96 Hz) $> 0{\cdot}5\, u_g = 0{\cdot}0581$ and $\sum_{i=2,3,13,15,26} (Y_i - 0{\cdot}0581) = 0{\cdot}375 > 0{\cdot}0804$ or the values $Y_2, Y_3, Y_{13}, Y_{26} > 0{\cdot}7 u_g = 0{\cdot}0813$ and $\sum_{i=2,3,13,26} (Y_i - 0{\cdot}0813) = 0{\cdot}279 > 0{\cdot}0364$.

If we reject the null hypothesis because of some frequencies, we include these frequencies into the set Θ. We conclude that if we omit the frequencies corresponding to the electric current and its harmonics the vibration frequencies are 9·77 Hz and 14·65 Hz (it might be only one between them), 63·48 Hz, 73·24 Hz and 126·96 Hz.

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